Your Name / Adınız - Soyadınız

Signature / İmza

Student ID # / Öğrenci No

(mavi tükenmez!)

Problem	1	2	3	4	Total
Points:	22	22	29	27	100
Score:					

1. (a) (11 Points) $\lim_{x\to 1^-} \dfrac{\sqrt{2x}\,(x-1)}{|x-1|} = ?$

Solution:

$$\lim_{x\to 1^-} \frac{\sqrt{2x}\,(x-1)}{|x-1|} = \lim_{x\to 1^-} \frac{\sqrt{2x}\,\cancel{(x-1)}}{-\cancel{(x-1)}} = \lim_{x\to 1^-}(-\sqrt{2x}) = \boxed{-\sqrt{2}}$$

p.90, pr.18(b)

(b) (11 Points) $\lim_{x\to 0} \dfrac{x\csc(2x)}{\cos(5x)} = ?$

Solution:

$$\lim_{x\to 0} \frac{x\csc(2x)}{\cos(5x)} = \lim_{x\to 0}\left(\frac{x}{\sin(2x)}\cdot\frac{1}{\cos(5x)}\right) = \left(\frac{1}{2}\lim_{x\to 0}\frac{2x}{\sin(2x)}\right)\left(\lim_{x\to 0}\frac{1}{\cos(5x)}\right) = \left(\frac{1}{2}\cdot 1\right)(1) = \boxed{\frac{1}{2}}$$

p.73, pr.27

2. (a) (11 Points) Use the *Intermediate Value Theorem* to show that $f(x) = x^3 - x - 1$ has a zero between -1 and 2.

Solution: First note that f is a polynomial and so continuous everywhere. Moreover $f(-1) = -1 < 0$ and $f(2) = 5 > 0 \Rightarrow f$ has a root between -1 and 2 by the Intermediate Value Theorem.

p.98, pr.35(a)

(b) (11 Points) Find the limit $\lim_{t\to 0} \sin\left(\dfrac{\pi}{2}\cos(\tan t)\right)$. Is the functions continuous at the point being approached?

Solution:

$$\lim_{t\to 0} \sin\left(\frac{\pi}{2}\cos(\tan t)\right) = \sin\left(\frac{\pi}{2}\cos(\tan 0)\right) = \sin\left(\frac{\pi}{2}\cos(0)\right) = \sin\left(\frac{\pi}{2}\right) = 1,$$

and function continuous at $x = 0$.

3. (a) (18 Points) Find all asymptotes for $y = \dfrac{x^2+4}{x-3}$.

Solution: $y = \dfrac{x^2+4}{x-3}$ is undefined at $x = 3$: $\lim_{x\to 3^-} \dfrac{x^2+4}{x-3} = -\infty$ and $\lim_{x\to 3^+} \dfrac{x^2+4}{x-3} = +\infty$, thus $\boxed{x=3}$ is a vertical asymptote.

Since $\lim_{x\to\pm\infty} \dfrac{x^2+4}{x-3} = \mp\infty$, there is *no* horizontal asymtote.

For the oblique asymptote, the long division gives

$$\frac{x^2+4}{x-3} = (x+3) + \frac{13}{x-3}$$

Since $\lim_{x\to\pm\infty} \dfrac{13}{x-3} = 0$, we see that the line $\boxed{y = x+3}$ is the oblique asymptote.

p.98, pr.47

(b) (11 Points) $\lim_{x\to\infty} \dfrac{\sqrt{x^2+1}}{x+1} = ?$

Solution:

$$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x+1} = \lim_{x\to\infty} \frac{\sqrt{x^2+1}/\sqrt{x^2}}{(x+1)/\sqrt{x^2}} = \lim_{x\to\infty} \frac{\sqrt{(x^2+1)/x^2}}{(x+1)/x} = \lim_{x\to\infty} \frac{\sqrt{1+1/x^2}}{(1+1/x)} = \frac{1+0}{(-1-0)} = -1.$$

p.94, pr.33

4. (a) (17 Points) Find equations of all lines having slope -1 that are tangent to the curve $y = \dfrac{1}{x-1}$.

Solution:

$$-1 = m = \lim_{h\to 0} \frac{\frac{1}{(x-h)-1} - \frac{1}{x-1}}{h} = \lim_{h\to 0} \frac{(x-1)-(x+h-1)}{h(x-1)(x+h-1)} = \lim_{h\to 0} \frac{-\not{h}}{\not{h}(x-1)(x+h-1)} = -\frac{1}{(x-1)^2}$$

Thus $(x-1)^2 = 1 \Rightarrow x^2 - 2x = 0 \Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0 \Rightarrow x = 0,\ x = 2$.
If $x = 0$, then $y = -1$ and $m = -1 \Rightarrow y = -1 - (x-0) = -(x+1)$. If $x = 2$, then $y = 1$ and $m = -1 \Rightarrow y = 1 - (x-2) = -(x-3)$. The lines are $\boxed{\text{y=-(x+1)}}$ and $\boxed{\text{y=-(x+3)}}$.

p.105, pr.25

(b) (10 Points) Find $\dfrac{dy}{dt}$ if $y = (1+\cos(t/2))^{-2}$.

Solution:

$$y = (1+\cos(t/2))^{-2} = -2\,(1+\cos(t/2))^{-3}\,(-\sin(t/2))\frac{1}{2} = (1+\cos(t/2))^{-3}\sin(t/2)$$

p.147, pr.44