Your Name / Ad - Soyad

Signature / İmza

Student ID # / Öğrenci No

(**mavi tükenmez!**)

Problem	1	2	3	4	Total
Points:	25	25	25	25	100
Score:					

Time Limit: 60 min.

1. (a) (9 Points) $\lim_{x\to 1^-} \frac{\sqrt{2x}\,(x-1)}{|x-1|} = ?$

Solution:

$$\lim_{x\to 1^-} \frac{\sqrt{2x}\,(x-1)}{|x-1|} = \lim_{x\to 1^-} \frac{\sqrt{2x}\,\cancel{(x-1)}}{-\cancel{(x-1)}} = \lim_{x\to 1^-}(-\sqrt{2x}) = \boxed{-\sqrt{2}}$$

p.90, pr.18(b)

(b) (8 Points) $\lim_{u\to 1} \frac{u^4-1}{u^3-1} = ?$

Solution:

$$\lim_{u\to 1} \frac{u^4-1}{u^3-1} = \lim_{u\to 1} \frac{\cancel{(u-1)}(u-1)(u+1)(u^2+1)}{\cancel{(u-1)}(u^2+u+1)} = \lim_{u\to 1} \frac{(u+1)(u^2+1)}{(u^2+u+1)} = \frac{(1+1)(1^2+1)}{(1^2+1+1)} = \boxed{\frac{4}{3}}$$

p.73, pr.27

(c) (8 Points) $\lim_{x\to 0} \frac{x\csc(2x)}{\cos(5x)} = ?$

Solution:

$$\lim_{x\to 0} \frac{x\csc(2x)}{\cos(5x)} = \lim_{x\to 0}\left(\frac{x}{\sin(2x)}\cdot\frac{1}{\cos(5x)}\right) = \left(\frac{1}{2}\lim_{x\to 0}\frac{2x}{\sin(2x)}\right)\left(\lim_{x\to 0}\frac{1}{\cos(5x)}\right) = \left(\frac{1}{2}\cdot 1\right)(1) = \boxed{\frac{1}{2}}$$

p.73, pr.27

2. (a) (14 Points) Show that $f(x) = x^3 - 15x + 1$ has three solutions between -4 and 4.

Solution: First note that f is a polynomial and so is continuous everywhere. Moreover $f(-4) = -3 < 0$ and $f(-3) = 19 > 0 \Rightarrow f$ has a root between -4 and -3 by the Intermediate Value Theorem. Similarly, $f(1) = -13 < 0$ implies that f has a solution between -3 and 1. Finally, since $f(1) < 0$ and $f(4) = 5 > 0$, Intermediate Value Theorem yields a third solution between 1 and 4. So all in all f has three solutions between -4 and $+4$. A polynomial of degree three cannot have more than 3 roots. Hence the given function has *precisely* three solutions in $[-4, +4]$.

p.98, pr.35(a)

(b) (11 Points) Find the limit $\lim_{t \to 0} \sin\left(\frac{\pi}{2}\cos(\tan t)\right)$. Is the functions continuous at the point being approached?

Solution:

$$\lim_{t \to 0} \sin\left(\frac{\pi}{2}\cos(\tan t)\right) = \sin\left(\frac{\pi}{2}\cos(\tan 0)\right) = \sin\left(\frac{\pi}{2}\cos(0)\right) = \sin\left(\frac{\pi}{2}\right) = 1,$$

and function continuous at $x = 0$.

3. (a) (13 Points) Find all asymptotes for $y = \dfrac{x^2+4}{x-3}$.

Solution: $y = \dfrac{x^2+4}{x-3}$ is undefined at $x=3$: $\lim\limits_{x\to 3^-} \dfrac{x^2+4}{x-3} = -\infty$ and $\lim\limits_{x\to 3^+} \dfrac{x^2+4}{x-3} = +\infty$, thus $\boxed{x=3}$ is a vertical asymptote.

Since $\lim\limits_{x\to\pm\infty} \dfrac{x^2+4}{x-3} = \mp\infty$, there is *no* horizontal asymtote.

For the oblique asymptote, the long division gives

$$\frac{x^2+4}{x-3} = (x+3) + \frac{13}{x-3}$$

Since $\lim\limits_{x\to\pm\infty} \dfrac{13}{x-3} = 0$, we see that the line $\boxed{y=x+3}$ is the oblique asymptote.

p.98, pr.47

(b) (12 Points) $\lim\limits_{x\to\infty} \left(\sqrt{9x^2-x} - 3x\right) = ?$

Solution:

$$\begin{aligned}
\lim_{x\to\infty} \left(\sqrt{9x^2-x} - 3x\right) &= \lim_{x\to\infty} \left(\sqrt{9x^2-x} - 3x\right) \frac{\sqrt{9x^2-x}+3x}{\sqrt{9x^2-x}+3x} \\
&= \lim_{x\to\infty} \frac{\cancel{9x^2} - x - \cancel{9x^2}}{\sqrt{9x^2-x}+3x} \\
&= \lim_{x\to\infty} \frac{-\cancel{x}}{\cancel{x}(\sqrt{9-1/x}+3)} \\
&= \lim_{x\to\infty} \frac{-1}{\sqrt{9-1/x}+3} \\
&= \frac{-1}{\sqrt{9-0}+3} \\
&= \boxed{-\frac{1}{6}}
\end{aligned}$$

p.94, pr.33

4. (a) (12 Points) Find an equation for the line tangent to thr graph of $y=\sqrt{x+1}$ at $(8,3)$.

Solution: By using the definition, we have

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{\sqrt{(8+h)+1}-\sqrt{8+1}}{h} = \lim_{h\to 0}\frac{\sqrt{9+h}-\sqrt{9}}{h}\,\frac{\sqrt{9+h}+\sqrt{9}}{\sqrt{9+h}+\sqrt{9}}\\ &= \lim_{h\to 0}\frac{\cancel{9}+h-\cancel{9}}{h(\sqrt{9+h}+3)}\\ &= \lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{9+h}+3)}\\ &= \lim_{h\to 0}\frac{1}{\sqrt{9+h}+3}\\ &= \frac{1}{\sqrt{9+0}+3}\\ &= \frac{1}{3+3}\\ &= \boxed{\frac{1}{6}}\end{aligned}$$

So $y-y_0=m(x-x_0)\Rightarrow y-3=\frac{1}{6}(x-9)\Rightarrow 6(y-3)=x-9\Rightarrow \boxed{x-6y=9}$ p.105, pr.25

(b) (13 Points) Use the formula $f'(x)=\lim_{z\to x}\frac{f(z)-f(x)}{z-x}$ to find $\frac{df}{dx}$ if $f(x)=x^2-3x+4$.

Solution:

$$\begin{aligned}f'(x)=\lim_{z\to x}\frac{f(z)-f(x)}{z-x} &= \lim_{z\to x}\frac{(z^2-3z+\cancel{4})-(x^2-3x+\cancel{4})}{z-x}=\lim_{z\to x}\frac{(z^2-x^2)-(3z-3x)}{z-x}\\ &= \lim_{z\to x}\frac{\cancel{(z-x)}(z+x-3)}{\cancel{(z-x)}}\\ &= \lim_{z\to x}(z+x-3)\\ &= (x+x-3)=\boxed{2x-3}\end{aligned}$$

p.147, pr.44