Math 113 Summer 2017 **First Exam** July 10, 2017

Your Name / Ad - Soyad (**75 min.**) Signature / İmza

Student ID # / Öğrenci No (**use a blue pen!**)

Problem	1	2	3	4	Total
Points:	30	20	25	25	100
Score:					

Time limit is **75 minutes**. If you need more room on your exam paper, you may use the empty spaces on the paper. You have to show your. Answers without reasonable-even if your results are true- work will either get zero or very little credit.

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1. Find the following limits.

(a) (12 Points) $\displaystyle\lim_{x\to-1}\frac{\sqrt{x^2+8}-3}{x+1}$.

You are not allowed to use L'Hôpital's rule.

Solution: We have

$$\lim_{x\to-1}\frac{\sqrt{x^2+8}-3}{x+1}=\lim_{x\to-1}\frac{\sqrt{x^2+8}-3}{x+1}\cdot\frac{\sqrt{x^2+8}+3}{\sqrt{x^2+8}+3}=\lim_{x\to-1}\frac{x^2+8-9}{(x+1)\left(\sqrt{x^2+8}+3\right)}$$

$$=\lim_{x\to-1}\frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}\left(\sqrt{x^2+8}+3\right)}=\lim_{x\to-1}\frac{x-1}{\sqrt{x^2+8}+3}=\frac{-1-1}{\sqrt{(-1)^2+8}+3}==\frac{-2}{6}=\boxed{-\frac{1}{3}}$$

p.72, pr.15

(b) (8 Points) Find y' if $y=\dfrac{(x+1)(x+2)}{(x-1)(x-2)}$.

Solution:

$$y=\frac{(x+1)(x+2)}{(x-1)(x-2)}=\frac{x^2+3x+2}{x^2-3x+2}$$

$$\frac{dy}{dx}=\frac{(x^2-3x+2)(2x+3)-(x^2+3x+2)(2x-3)}{(x^2-3x+2)^2}=\boxed{\frac{-6x^2+12}{(x^2-3x+2)^2}}$$

p.94, pr.10

(c) (10 Points) Suppose $q=\sin\left(\dfrac{t}{\sqrt{t+1}}\right)$. Find $\dfrac{dq}{dt}$

Solution: We have by the Chain Rule,

$$\frac{dq}{dt}=\cos\left(\frac{t}{\sqrt{t+1}}\right)\cdot\frac{d}{dt}\left(\frac{t}{\sqrt{t+1}}\right)=\cos\left(\frac{t}{\sqrt{t+1}}\right)\cdot\frac{(\sqrt{t+1})(1)-t\frac{d}{dt}(\sqrt{t+1})}{(\sqrt{t+1})^2}$$

$$=\cos\left(\frac{t}{\sqrt{t+1}}\right)\cdot\frac{\sqrt{t+1}-\dfrac{1}{2\sqrt{t+1}}}{t+1}=\cos\left(\frac{t}{\sqrt{t+1}}\right)\cdot\frac{2(t+1)-t}{2(t+1)^{3/2}}=\boxed{\left(\frac{t+2}{2(t+1)^{3/2}}\right)\cos\left(\frac{t}{\sqrt{t+1}}\right)}$$

p.94, pr.34

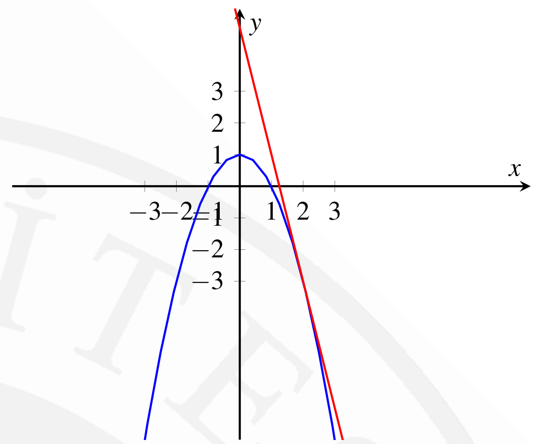
2. (a) (10 Points) Find an equation for the tangent to the curve $y = 1 - x^2$ at point $(2, -3)$. Then sketch this curve and tangent together.

Solution: First the slope for the line at $(2, -3)$ is

$$\begin{aligned} m &= \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{[1-(2+h)^2]-(-3)}{h} \\ &= \lim_{h\to 0} \frac{(1-4-4h-h^2)+3}{h} = \lim_{h\to 0} \frac{-\not{h}(4+h)}{\not{h}} \\ &= \lim_{h\to 0} -(4+h) = -4 \end{aligned}$$

Then the equation for the tangent is $y + 3 = -4(x-2) \Rightarrow$ $\boxed{y = -4x + 5}$.

p.72, pr.8



(b) (10 Points) For what value(s) of a and b is

$$f(x) = \begin{cases} -2 & x \le -1 \\ ax - b & -1 < x < 1 \\ 3, & x \ge 1 \end{cases}$$

is continuous at every x.

Solution: Clearly f is continuous if $x \neq -1$ and for $x \neq 1$ for if $x < -1$ or if $-1 < x < 1$ or if $x > 1$, f is a polynomial, regardless the values of a and b. For continuity at $x = -1$, we require that the one-sided limits of $f(x)$ at $x = -1$ be equal. But $\lim\limits_{x\to -1^-} f(x) = -2$ and $\lim\limits_{x\to -1^+} f(x) = a(-1) - b = -a - b$.
Similarly, for continuity at $x = 1$, we require that the one-sided limits of $f(x)$ at $x = 1$ be equal. But $\lim\limits_{x\to 1^-} f(x) = a(1) - b = a - b$ and $\lim\limits_{x\to 1^+} f(x) = 3$.
Equality of one-sided limits is equivalent to

$$\begin{aligned} -2 = -a - b &\text{ and } a - b = 3 \\ \Longrightarrow a = \frac{5}{2} &\text{ and } b = -\frac{1}{2}. \end{aligned}$$

p.83, pr.40

3. (a) (12 Points) Suppose $f(x) = 2x-2$, $L = -6$, $x_0 = -2$, $\varepsilon = 0.02$. Find an open interval about x_0 on which the inequality $|f(x) - L| < \varepsilon$ holds. Then, using the given information, give a value for $\delta > 0$ such that for all x satisfying $0 < |x - x_0| < \delta$ the inequality $|f(x) - L| < \varepsilon$ holds.

Solution: For the required interval, we want $|(2x-2)-6| < 0.02$. In other words, we need $|2x+4| < 0.02$. This is the same as writing $-0.02 < 2x+4 < 0.02$. Solving this for x, we get

$$-4.02 < 2x < -3.98$$

and hence by dividing by 2 gives

$$-2.01 < x < -1.99,$$

that is an interval we want is then

$$\boxed{x \in (-2.01, -1.99).}$$

For the second part, we want to find a $\delta > 0$ such that

$$\begin{aligned} |x-(-2)| < \delta &\Rightarrow -\delta < x+2 < \delta \\ &\Rightarrow -\delta - 2 < x < \delta - 2 \\ &\Rightarrow x \in (-\delta - 2, \delta - 2) \end{aligned}$$

Since $x \in (-2.01, -1.99)$, we can have $\delta - 2 = -1.99$ and $-\delta - 2 = -2.01$ and so solving these we can have $\boxed{\delta = 0.01}$.

p.72, pr.8

(b) (13 Points) Find $\dfrac{dy}{dx}$ if $x^2 - xy + y^3 = 5$ defines y implicitly as a function of x.

Solution: We have

$$\begin{aligned} \frac{d}{dx}\left(x^2 - xy + y^3\right) &= \frac{d}{dx}(5) \\ \frac{d}{dx}(x^2) - \frac{d}{dx}(xy) + \frac{d}{dx}(y^3) &= \frac{d}{dx}(5) \\ 2x - y - x\frac{dy}{dx} + 3y^2\frac{dy}{dx} &= 0 \\ \left(-x + 3y^2\right)\frac{dy}{dx} &= -2x + y \\ \frac{dy}{dx} &= \boxed{\frac{-2x+y}{-x+3y^2}} \end{aligned}$$

Use Implicit Differentiation.

p.83, pr.52

4. (a) (13 Points) Find all the asymptotes and graph $y = \dfrac{2x}{x+1}$.

Solution: First notice that

$$y = \frac{2x}{x+1} = 2\left(\frac{x+1-1}{x+1}\right) = 2\left(1 - \frac{1}{x+1}\right).$$

Then

$$\lim_{x\to\pm\infty} \frac{2x}{x+1} = \lim_{x\to\pm\infty} 2\left(1 - \frac{1}{x+1}\right) = 2(1-0) = 2$$

and so $\boxed{y=2}$ *is the (only) horizontal asymptote*. Moreover, since

$$\lim_{x\to -1^+} \frac{2x}{x+1} = \lim_{x\to -1^+} 2\left(1 - \frac{1}{x+1}\right) = 2(1-\infty) = -\infty$$

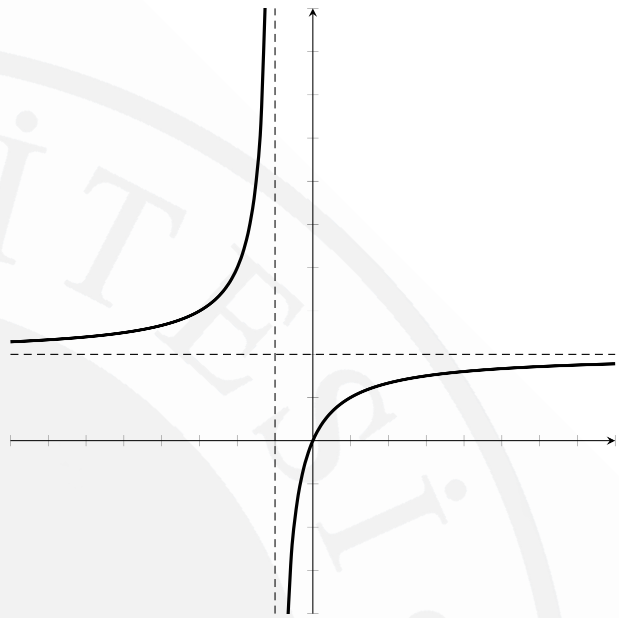
and

$$\lim_{x\to -1^-} \frac{2x}{x+1} = \lim_{x\to -1^-} 2\left(1 - \frac{1}{x+1}\right) = 2(1+\infty) = +\infty,$$

the graph has *only one vertical asymptote*, and it is $\boxed{x=-1}$.

p.95, pr.68

$y = f(x) = \dfrac{2x}{x+1}$



(b) (12 Points) Use the formula $f'(x) = \lim_{z\to x} \dfrac{f(z)-f(x)}{z-x}$ to find the derivative of $g(x) = 1+\sqrt{x}$.

Solution: Here $f(z) = g(z) = 1+\sqrt{z}$ and $f(x) = g(x) = 1+\sqrt{x}$ and so

$$\begin{aligned}
f'(x) &= \lim_{z\to x} \frac{f(z)-f(x)}{z-x} = \lim_{z\to x} \frac{g(z)-g(x)}{z-x} \\
&= \lim_{z\to x} \frac{\cancel{1}+\sqrt{z}-(\cancel{1}+\sqrt{x})}{z-x} \\
&= \lim_{z\to x} \frac{\cancel{\sqrt{z}-\sqrt{x}}}{\cancel{(\sqrt{z}-\sqrt{x})}(\sqrt{z}+\sqrt{x})} \\
&= \lim_{z\to x} \frac{1}{(\sqrt{z}+\sqrt{x})} = \frac{1}{(\sqrt{x}+\sqrt{x})} = \boxed{\frac{1}{2\sqrt{x}}}
\end{aligned}$$

p.112, pr.26