Your Name / Adınız - Soyadınız (**süre:60**) Signature / İmza

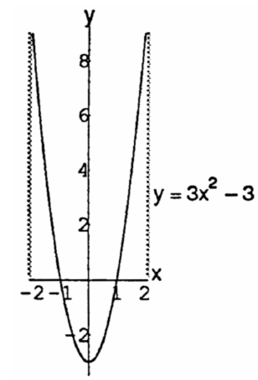
Student ID # / Öğrenci No (**mavi tükenmez!**)

Soru	1	2	3	4	Toplam
Puan	22	20	22	37	101
Puanınız					

1. (a) (11 Puan) $y = 3x^2 - 3, \quad -2 \le x \le 2$ bölgesi ve x-ekseni arasındaki toplam alanı bulunuz.

Solution: $y = 3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$; because of symmetry about the y-axis,

$$\begin{aligned} \text{AREA} &= 2\left(-\int_0^1 (3x^2-3)\,dx + \int_1^2 (3x^2-3)\,dx\right) \\ &= 2\left(-\left[x^3-3x\right]_0^1 + \left[x^3-3x\right]_1^2\right) \\ &= 12 \end{aligned}$$



p.282, pr.42

(b) (11 Puan) $\displaystyle\int r^2\left(\frac{r^3}{18}-1\right)^5 dr = ?$

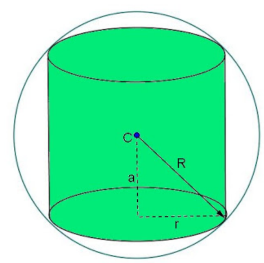
2. (20 Puan) Yarıçapı 10 cm olan bir kürenin içine yerleştirilebilen maksimum hacimli dik dairesel silindirin boyutlarını bulunuz. Maksimum hacim kaçtır?

Solution: Let the radius of the cylinder be r cm, $0 < r < 10$. Then the height is $2\sqrt{100-r^2}$ and the volume is $V(r) = 2\pi r^2\sqrt{100-r^2}$ cm^3. Then,

$$\begin{aligned} V'(r) = &\; 2\pi r^2\left(\frac{1}{\sqrt{100-r^2}}\right)(-2r) + \left(2\pi\sqrt{100-r^2}\right)(2r) \\ = &\; \frac{-2\pi r^3 + 4\pi r(100-r^2)}{\sqrt{100-r^2}} = \frac{2\pi r(200-3r^2)}{\sqrt{100-r^2}}. \end{aligned}$$

The critical point for $0 < r < 10$ occurs at $r = \sqrt{\frac{200}{3}} = 10\sqrt{\frac{2}{3}}$. Since $V'(r) > 0$ for $0 < r < 10\sqrt{\frac{2}{3}}$ and $V'(r) < 0$ for $10\sqrt{\frac{2}{3}} < r < 10$, the critical point corresponds to the maximum volume. The dimensions are $r = 10\sqrt{\frac{2}{3}} \approx 8.16$ and $h = \frac{20}{\sqrt{3}} \approx 11.55$ cm, and the volume is $\frac{4000\pi}{3\sqrt{3}} \approx 2418.40$ cm^3.

p.221, pr.19



3. (a) (11 Puan) $\displaystyle\int_{\sqrt{2}}^1 \left(\frac{u^7}{2} - \frac{1}{u^5}\right) du = ?$

Solution:

$$\int_{\sqrt{2}}^1 \left(\frac{u^7}{2} - \frac{1}{u^5}\right) du = \int_{\sqrt{2}}^1 \left(\frac{u^7}{2} - u^{-5}\right) du = \left[\frac{u^8}{16} + \frac{u^{-4}}{4}\right]_{\sqrt{2}}^1 = \left(\frac{1}{16}+\frac{1}{4}\right) - \left(\frac{16}{16} + \frac{1}{4(\sqrt{2})^2}\right) = -\frac{3}{4}$$

p.282, pr.21

(b) (11 Puan) $y = \int_{\sqrt{x}}^{0} \sin(t^2)\, dt \Rightarrow \frac{dy}{dx} = ?$

Solution:

$$\frac{dy}{dx} = \frac{d}{dx}\int_{\sqrt{x}}^{0} \sin(t^2)\, dt = -\sin((\sqrt{x})^2)\frac{d}{dx}(\sqrt{x}) = -\frac{1}{2\sqrt{x}}\sin x.$$

p.282, pr.35

4. $y = \frac{5}{x^4+5}$, $y' = \frac{-20x^3}{(x^4+5)^2}$ ve $y'' = \frac{100x^2(x^4-3)}{(x^4+5)^3}$ veriliyor.

(a) (7 Puan) Asimptotları bulunuz.

Solution:

$$\lim_{n\to\pm\infty} \frac{5}{x^4+5} = \lim_{n\to\pm\infty} \frac{5/x^4}{1+5/x^4} = 0$$

Hence $y = 0$ is the only (horizontal) asymptote.

p.105, pr.25

(b) (10 Puan) Tüm maksimum ve minimum değerleri ve hangi noktalarda olduklarını bulunuz.

Solution: The curve is rising on $(-\infty, 0)$, and is falling on $(0, \infty)$. There is a local and absolute maximum at $x = 0$, and there is no local or absolute minimum.

p.147, pr.44

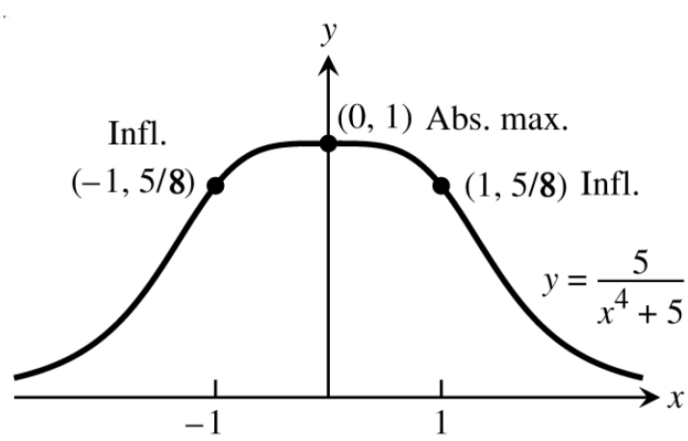
(c) (10 Puan) Büküm noktalarını bulunuz.

Solution: The curve is concave up on $(-\infty, -\sqrt[4]{3})$ and $(\sqrt[4]{3}, \infty)$, and concave down on $(-\sqrt[4]{3}, 0)$ and $(0, \sqrt[4]{3})$. There are points of inflection at $x = -\sqrt[4]{3}$ and $x = -\sqrt[4]{3}$.

p.147, pr.44

(d) (10 Puan) Fonksiyonun grafiğini çiziniz. Asimptotları, dönüm be büküm noktalarını belirtiniz.

Solution:



p.211, pr.44