Your Name / Ad - Soyad (**süre:60**) Signature / İmza

Student ID # / Öğrenci No (**mavi tükenmez!**)

Soru	1	2	3	4	Toplam
Puan	22	22	26	31	101
Puanınız					

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1. Aşağıdaki fonksiyonların türevlerini bulunuz.

(a) (11 Puan) $p = \left(\frac{q^2+3}{12q}\right)\left(\frac{q^4-1}{q^3}\right)$,

Solution: First we multiply and rearrange the terms:

$$p = \left(\frac{q^2+3}{12q}\right)\left(\frac{q^4-1}{q^3}\right) = \left(\frac{q^2}{12q}+\frac{3}{12q}\right)\left(\frac{q^4}{q^3}-\frac{1}{q^3}\right) = \boxed{\frac{1}{12}q^2 - \frac{1}{12}q^{-2} + \frac{1}{4} - \frac{1}{4}q^{-4}}.$$

Now we can differentiate:

$$\frac{dp}{dq} = \frac{d}{dq}\left(\frac{1}{12}q^2 - \frac{1}{12}q^{-2} + \frac{1}{4} - \frac{1}{4}q^{-4}\right) = \boxed{\frac{1}{6}q + \frac{1}{6}q^{-3} + q^{-5}}$$

p.122, pr.39

(b) (11 Puan) $y = (1+\cos(t/2))^{-2}$ ise $\frac{dy}{dt}$ yi bulunuz.

Solution:

$$y = (1+\cos(t/2))^{-2} = -2\left(1+\cos(t/2)\right)^{-3}\left(-\sin(t/2)\right)\frac{1}{2} = (1+\cos(t/2))^{-3}\sin(t/2)$$

p.290, pr.27

2. Kapalı olarak tanımlı $x\sin 2y = y\cos 2x$ eğrisi verilsin.

(a) (12 Puan) Eğrinin $(\pi/4, \pi/2)$ noktasındaki teğetinin denklemini yazınız.

Solution:

$$x\sin 2y = y\cos 2x \Rightarrow x2(\cos 2y)y' + \sin 2y = -2y\sin 2x + y'\cos 2x \Rightarrow y'(2x\cos 2y - \cos 2x) = -\sin 2y - 2y\sin 2x$$

Hence

$$y' = \frac{\sin 2y + 2y\sin 2x}{\cos 2x - 2x\cos 2y};$$

the slope of the tangent line $m = y'\big|_{(\pi/4,\pi/2)} = \left.\frac{\sin 2y + 2y\sin 2x}{\cos 2x - 2x\cos 2y}\right|_{(\pi/4,\pi/2)} = \frac{\pi}{\frac{\pi}{2}} = 2 \Rightarrow$

the tangent line is $y - \frac{\pi}{2} = 2(x - \frac{\pi}{4}) \Rightarrow \boxed{y = 2x.}$

p.154, pr.36

(b) (10 Puan) Eğrinin $(\pi/4, \pi/2)$ noktasındaki normalinin denklemini yazınız.

Solution: the normal line is $y - \frac{\pi}{2} = -\frac{1}{2}(x - \frac{\pi}{4}) \Rightarrow \boxed{y = -\frac{1}{2}x + \frac{5\pi}{8}.}$

p.154, pr.36

3. (a) (11 Puan) $\sqrt{x^2+9}$'in $x = -4$'deki $L(x)$ lineerleştirmesini bulunuz.

Solution: First $f(2) = \sqrt{13}$ and

$$f'(x) = \frac{2x}{2\sqrt{x^2+9}} = \frac{x}{\sqrt{x^2+9}} \Rightarrow f'(x) = \frac{2}{\sqrt{2^2+9}} = \frac{2}{\sqrt{13}}$$

Hence the linearization is

$$L(x) = f(2) + f'(2)(x-2) = \boxed{\sqrt{13} + \frac{2}{\sqrt{13}}(x-2)}$$

p.282, pr.21

(b) (15 Puan)

$$g(x) = \begin{cases} x^2 - x, & -2 \leq x \leq -1 \\ 2x^2 - 3x + 3, & -1 < x \leq 0. \end{cases}$$

veriliyor. Buna göre: Verilen aralıkta Ortalama Değer Teoremi'nin hipotezini sağlar mı neden?

Solution: We begin with restating MVT. **The Mean Value Theorem:** *Suppose $y = g(x)$ is continuous on a closed interval $[a,b]$ and differentiable on the interval's interior (a,b). Then there is at least one point $c \in (a,b)$ at which* $\dfrac{g(b)-g(a)}{b-a} = g'(c)$. Here we have $a = -2$, $b = 2$. First note that $\lim_{x \to -2^+} g(x) = \lim_{x \to -2^+} (x^3) = (-2)^3 = -8 = g(-2)$ and so $g(x)$ is (right-)continuous at $a = -2$ and since $\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} (x^2) = (2)^2 = 4 = g(2)$ so that $g(x)$ is (left-)continuous at $x = 2$. By the solution of part (a), $g(x)$ is differentiable at $x = 0$, so is continuous there. Hence $g(x)$ is continuous on $[-2,2]$. Since x^2 and x^3 are differentiable functions and $g(x)$ is differentiable at $x = 0$, it follows that $g(x)$ is differentiable on $(-2,2)$ and so $g(x)$ satisfies the hypotheses of MVT. p.196, pr.6

4. $y = \dfrac{x^2-4}{x^2-2}$, $y' = \dfrac{4x}{(x^2-2)^2}$ ve $y'' = -\dfrac{12x^2+8}{(x^2-2)^3}$ veriliyor.

(a) (8 Puan) Asimptotları bulunuz.

Solution:

$$\lim_{x \to \pm\infty} \frac{5}{x^4+5} = \lim_{x \to \pm\infty} \frac{5/x^4}{1+5/x^4} = 0$$

Hence $y = 0$ is the only (horizontal) asymptote.

p.105, pr.25

(b) (8 Puan) Tüm maksimum ve minimum değerleri ve hangi noktalarda olduklarını bulunuz.

Solution: The curve is rising on $(-\infty, 0)$, and is falling on $(0, \infty)$. There is a local and absolute maximum at $x = 0$, and there is no local or absolute minimum.

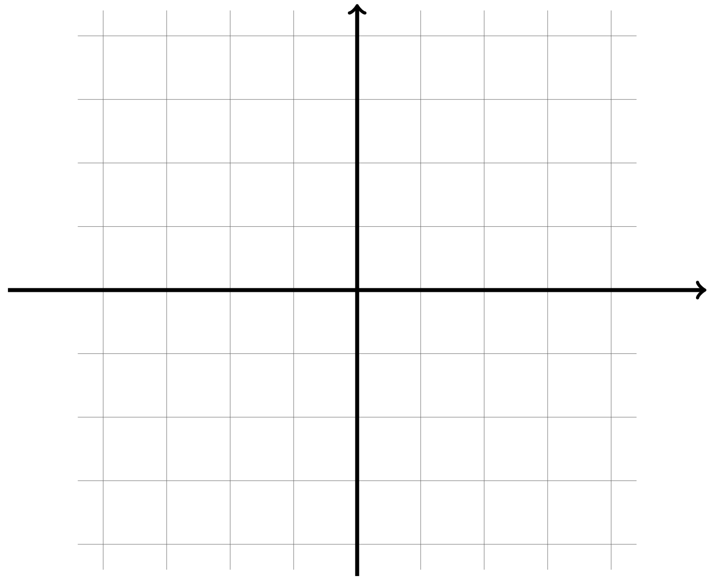
p.147, pr.44

(c) (5 Puan) Büküm noktalarını bulunuz.

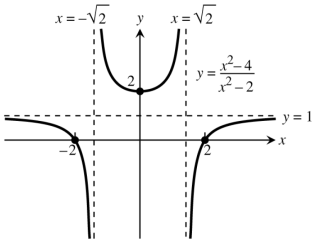
Solution: The curve is concave up on $(-\infty, -\sqrt[4]{3})$ and $(\sqrt[4]{3}, \infty)$, and concave down on $(-\sqrt[4]{3}, 0)$ and $(0, \sqrt[4]{3})$. There are points of inflection at $x = -\sqrt[4]{3}$ and $x = -\sqrt[4]{3}$.

p.147, pr.44

(d) (10 Puan) Fonksiyonun grafiğini çiziniz. Asimptotları, dönüm be büküm noktalarını belirtiniz.



Solution:



p.211, pr.44