Math 113 Summer 2017 **Second Exam** July 24, 2017

Your Name / Ad - Soyad (**75 min.**) Signature / İmza

Student ID # / Öğrenci No (**use a blue pen!**)

Problem	1	2	3	4	Total
Points:	30	20	22	28	100
Score:					

Time limit is **75 minutes**. If you need more room on your exam paper, you may use the empty spaces on the paper. You have to show your. Answers without reasonable-even if your results are true- work will either get zero or very little credit.

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1. (a) (12 Points) If $x^2y^3=\dfrac{4}{27}$ and $\dfrac{dy}{dt}=\dfrac{1}{2}$, then what is $\dfrac{dx}{dt}$ when $x=2$?

Solution: By differentiating implicitly, we have

$$3x^2y^2\frac{dy}{dt}+2xy^3\frac{dx}{dt}=0;$$
$$x=2\Rightarrow(2)^2y^3=\frac{4}{27}\Rightarrow y=\frac{1}{3}$$
$$\text{Thus } 3(2)^2\left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right)+2(2)\left(\frac{1}{3}\right)^3\frac{dx}{dt}=0$$
$$\boxed{\frac{dx}{dt}=-\frac{9}{2}}$$

(Page 160, problem 8)

(b) (8 Points) $\displaystyle\int\frac{t\sqrt{t}+\sqrt{t}}{t^2}\,dt=?$

Solution:

$$\int\frac{t\sqrt{t}+\sqrt{t}}{t^2}\,dt=\int\left(\frac{t^{3/2}}{t^2}+\frac{t^{1/2}}{t^2}\right)dt$$
$$=\int\left(t^{-1/2}+t^{-3/2}\right)dt=\frac{t^{-1/2+1}}{-1/2+1}+\frac{t^{-3/2+1}}{-3/2+1}+C$$
$$=\boxed{2\sqrt{t}-\frac{2}{\sqrt{t}}+C}$$

(Page 236, problem 33)

(c) (10 Points) Suppose

$$\begin{cases}\dfrac{d^2y}{dx^2} &=2-6x\\ y'(0)&=4\\ y(0)&=1.\end{cases}$$

Find $y(x)$.

Solution:

$$\frac{d^2y}{dx^2}=2-6x\Rightarrow\frac{dy}{dx}=2x-3x^2+C_1$$
$$\Rightarrow\ \text{At } \frac{dy}{dx}=4 \text{ and } x=0 \text{ we have } 4=2(0)-3(0)^2+C_1\Rightarrow C_1=4$$
$$\Rightarrow\frac{dy}{dx}=2x-3x^2+4\Rightarrow y=x^2-x^3+4x+C_2;$$
$$\text{at } y=1 \text{ and } x=0 \text{ we have } 1=0^2-0^3+4(0)+C_2\Rightarrow C_2=1$$
$$\Rightarrow\boxed{y=x^2-x^3+4x+1}$$

(Page 238, problem 81)

2. (a) (10 Points) Find the linearization of $f(x) = \sqrt{1+x} + \sin x - 0.5$ at $x = 0$.

Solution:

$$f(x) = \sqrt{1+x} + \sin x - 0.5 = (1+x)^{1/2} + \sin x - \frac{1}{2} \Rightarrow f(0) = 1 - \frac{1}{2} = \frac{1}{2}$$
$$\Rightarrow f'(x) = \frac{1}{2}(1+x)^{-1/2} + \cos x \Rightarrow f'(0) = \frac{1}{2} + 1 = \frac{3}{2}$$
$$\Rightarrow L(x) = f(0) + f'(0)(x-0) = \frac{1}{2} + \frac{3}{2}x$$
$$\Rightarrow \boxed{L(x) = \frac{1}{2}(1+3x)}$$

(Page179, problem 119)

(b) (10 Points) For $f(x) = \sqrt{x-1}$ and $[1,3]$, find the value(s) of c that satisfy the equation

$$\frac{f(b) - f(a)}{b-a} = f'(c)$$

in the conclusion of the Mean Value Theorem

Solution: First, since $f'(x) = \frac{d}{dx}(x-1)^{1/2} = \frac{1}{2}(x-1)^{-1/2}$ and so $f'(x)$

$$\frac{f(b) - f(a)}{b-a} = f'(c) \Rightarrow \frac{f(3) - f(1)}{3-1} = f'(c)$$
$$\Rightarrow \frac{\sqrt{2} - \sqrt{0}}{3-1} = \frac{1}{2\sqrt{c-1}}$$
$$\Rightarrow 2\sqrt{c-1} = \frac{2}{\sqrt{2}} \Rightarrow \sqrt{c-1} = \frac{1}{\sqrt{2}}$$
$$\Rightarrow c - 1 = \frac{1}{2} \Rightarrow \boxed{c = \frac{3}{2} \in [1,3]}$$

(Page 196, problem 4)

3. (a) (12 Points) A right triangle whose hypotenuse is $\sqrt{3}$ m long is revolved about one of its legs to generate a right circular cone. Find the radius, height and volume of the cone of greatest volume that can be made this way.

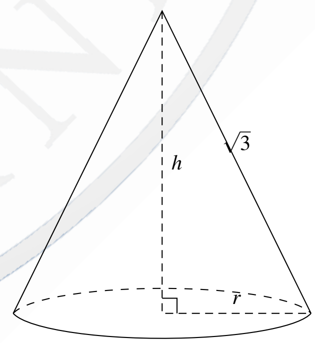
Solution: First, notice that $h^2 + r^2 = 3$ and so $r = \sqrt{3 - h^2}$. Then the volume is given by

$$V = \frac{\pi}{3} r^2 h = \frac{\pi}{3}(3 - h^2)h = \pi h - \frac{\pi}{3}h^3 \text{ for } 0 < h < \sqrt{3},$$
$$\Rightarrow \frac{dV}{dh} = \pi - \pi h^2 = \pi(1 - h^2) = 0 \Rightarrow h = \pm 1$$

but $h > 0 \Rightarrow h = 1$ is the only critical point.

To classify this critical point, note that for $0 < h < 1$, we have $\frac{dV}{dh} > 0$ and for $1 < h < \sqrt{3}$, we have $\frac{dV}{dh} < 0$, so the critical point correspond to the maximum volume. The maximum volume cone has radius $\sqrt{2}$ m, height 1 m and volume $\frac{2\pi}{3}$ m^3.

(Page 222, problem 27)

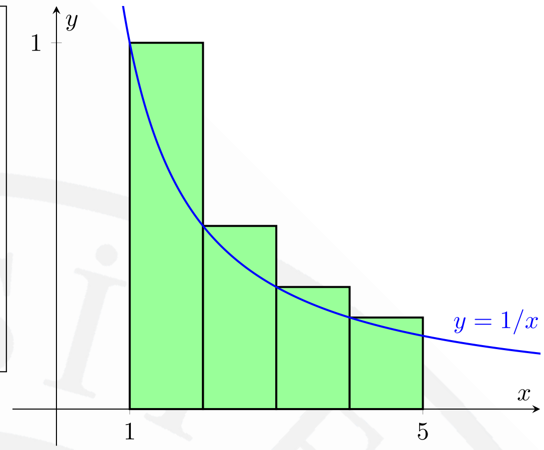


(b) (10 Points) Use finite approximation to estimate the area under the graph of $f(x) = \frac{1}{x}$ between $x = 1$ and $x = 5$ using an *upper sum* with *four rectangles* of equal width.

Solution: Here $a = 1$ and $b = 5$ and $n = 4$ and so $\Delta x = \dfrac{b-a}{n} = \dfrac{5-1}{4} = 1$ and $x_k = k\Delta x = 1 + k$. Therefore the partition is $P = \{x_0 = 1, x_1 = 2, x_2 = 3, x_3 = 4, x_4 = 5\}$. Therefore, an upper sum is

$$\sum_{k=0}^{3} \frac{1}{x_k}.(1) = 1\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = \frac{25}{12}$$

(Page 253, problem 3d)



4. Consider the function $y = \dfrac{x^2 - x + 1}{x - 1}$. You may assume that $y' = \dfrac{x^2 - 2x}{(x-1)^2}$ and $y'' = \dfrac{2}{(x-1)^3}$. Use this information to graph the function.

(a) (3 Points) Identify the *domain* of f.

Solution: The domain of f is $\boxed{(-\infty, 1) \cup (1, +\infty) = \mathbb{R} - \{1\}}$.

p.241, pr.45

(b) (7 Points) Give the *asymptotes*.

Solution: We have $\lim\limits_{x \to 1^+} \dfrac{x^2 - x + 1}{x - 1} = +\infty$, $\lim\limits_{x \to 1^-} \dfrac{x^2 - x + 1}{x - 1} = -\infty$. From these we see that the graph has *one vertical asymptote at* $x = 1$. Next *there is no horizontal asymptote* as $\lim\limits_{x \to \pm\infty} \dfrac{x^2 - x + 1}{x - 1} = \pm\infty$. But as

$$\frac{x^2 - x + 1}{x - 1} = x + \frac{1}{x - 1}$$

the line $y = x$ is an oblique asymptote.

p.212, pr.85

(c) (5 Points) Find the *intervals* where the graph is *increasing* and *decreasing*. Find the *local maximum* and *minimum* values.

Solution: We have $y' = 1 - \dfrac{1}{(x-1)^2} = 0$ if and only if $(x-1)^2 = 1$, that is iff $x = 0$ and $x = 2$ are the critical points. Note that

$$y' \text{ is } \begin{cases} > 0, & \text{on } (-\infty, 0) \cup (2, +\infty) \quad \text{y is incresing} \\ < 0, & \text{on } (0, 1) \cup (1, 2) \quad \text{y is decresing} \end{cases}$$

Thus, y is decreasing on $(0, 1) \cup (1, 2)$ and increasing on $(-\infty, 0) \cup (2, +\infty)$. Moreover, the point $(0, -1)$ is a point of local maximum and $(2, 3)$ is a point of local minimum. p.212, pr.85

(d) (5 Points) Determine where the graph is *concave up* and *concave down*, and find any *inflection points*.

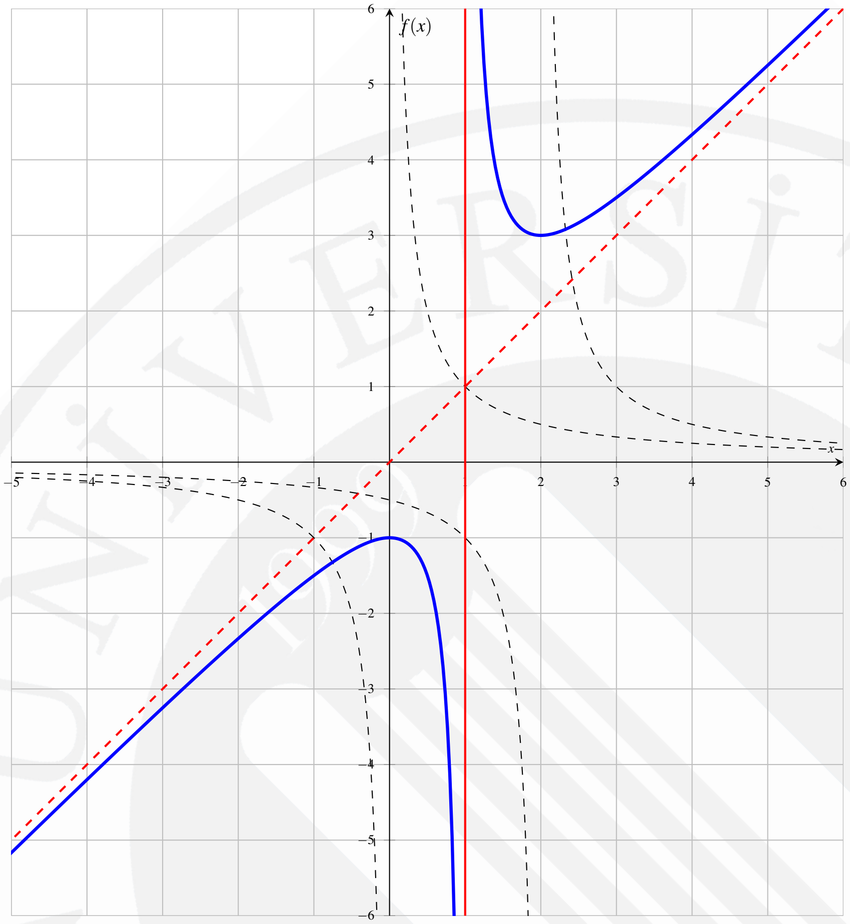
Solution: We have $y'' = \dfrac{2}{(x-1)^3}$ and so

$$y'' \begin{cases} > 0, & \text{on } (1, +\infty) \quad \text{y is concave up} \\ < 0, & \text{on } (-\infty, 1) \quad \text{y is concave down} \end{cases}$$

Hence f is concave up on $(1, +\infty)$ and concave down on $(-\infty, 1)$. Also graph has *no point of inflection* there is no tangent line at $x = 1$. p.212, pr.85

(e) (8 Points) *Sketch the graph* of the function. Label the asymptotes, critical points and the inflection points.

Solution:



p.212, pr.85