Your Name / Ad - Soyad (**75 dak.**) Signature / İmza

Student ID # / Öğrenci No (**mavi tükenmez!**)

Soru	1	2	3	4	Toplam
Puan	24	26	26	24	100
Sonuç					

Sınav süresi **75 dakika**. Cevap için ayrılan yer yetmiyorsa, sınav kağıdınızdaki boş yerleri kullanabilirsiniz. Cevaplarınızı **AÇIKLAMALISINIZ**. Yeterince açıklanmamış cevaplar –sonuç doğru olsa bile– ya hiç puan alamayacak ya da çok az puan alacak.

1. (a) (8 Puan) $a = 0$ noktasında $f(x) = \sqrt{x+1} + \sin x$ fonksiyonunun lineerleştirmesini bulunuz.

Solution: We have

$$f(x) = \sqrt{x+1} + \sin x = (x+1)^{1/2} + \sin x \Rightarrow f'(x) = \left(\frac{1}{2}\right)(x+1)^{-1/2} + \cos x$$
$$\Rightarrow L_f(x) = f'(0)(x-0) + f(0) = \frac{3}{2}(x-0) + 1 \Rightarrow L_f(x) = \frac{3}{2}x + 1$$

$L_f(x) = f(a) + f'(a)(x-a)$.

(b) (8 Puan) $h(\theta) = 3\theta^{2/3}$ fonksiyonunun $-27 \leq \theta \leq 8$ aralığındaki mutlak maksimum/minimum değerlerlerini bulunuz. Hangi noktalarda gerçeklendiğinive belirleyiniz.

Solution: Computing the derivative, we have

$$h(\theta) = 3\theta^{2/3} \Rightarrow h'(\theta) = 2\theta^{-1/3}$$
$$\Rightarrow \text{a critical point at } \theta = 0;$$
$$\text{checking the endpoints}: h(-27) = 27,\ h(8) = 12$$
$$\Rightarrow \begin{cases} \text{absolute maximum is 27 at } \theta = -27 \\ \text{absolute minimum is 0 at } \theta = 0 \end{cases}$$

(c) (8 Puan) $f(x) = x + \frac{1}{x}$ fonksiyonu ve $\left[\frac{1}{2}, 1\right]$ aralığı için Ortalama Değer Teoremi sonucundaki $\frac{f(b)-f(a)}{b-a} = f'(c)$ eşitliğinin hangi c değerleri için sağlandığını bulunuz.

Solution: We have

$$\text{when } f(x) = x + \frac{1}{x} \quad \text{for} \quad \frac{1}{2} \leq x \leq 1,$$
$$\text{then } \frac{f(2) - f(1/2)}{2 - 1/2} = f'(c)$$
$$\Rightarrow 0 = 1 - \frac{1}{c^2} \Rightarrow c^2 = 1$$
$$c = \pm 1 \text{ but } c = -1 \notin \left[\frac{1}{2}, 1\right]$$
$$\boxed{c = 1}$$

p.73, pr.27

2. (a) (10 Puan) $\dfrac{x}{a}+\dfrac{y}{b}=1$ doğrusu üzerindeki *orijine en yakın noktayı* optimizasyon problemi olarak bulunuz.

Solution: Suppose $P(x,y)$ is the point on this line that is closest to the origin. Let $d=\sqrt{(x-0)^2+(y-0)^2}=\sqrt{x^2+y^2}$ and $\dfrac{x}{a}+\dfrac{y}{b}=1 \Rightarrow y=-\dfrac{b}{a}x+b$.

We can minimize d by minimizing $D=\left(\sqrt{x^2+y^2}\right)^2=x^2+\left(-\dfrac{b}{a}x+b\right)^2 \Rightarrow D'=2x+2\left(-\dfrac{b}{a}x+b\right)\left(-\dfrac{b}{a}\right)=2x+\dfrac{2b^2}{a^2}x-\dfrac{2b^2}{a}$. Hence $D'=0\Rightarrow 2\left(x+\dfrac{b^2}{a^2}x-\dfrac{b^2}{a}\right)=0\Rightarrow x=\dfrac{ab^2}{a^2+b^2}$ is the critical point $\Rightarrow y=-\dfrac{b}{a}\left(\dfrac{ab^2}{a^2+b^2}\right)+b=\dfrac{a^2b}{a^2+b^2}$. Thus $D''=2+\dfrac{2b^2}{a^2}\Rightarrow D''\left(\dfrac{ab^2}{a^2+b^2}\right)=2+\dfrac{2b^2}{a^2}>0\Rightarrow$ the critical point is local minimum by the Second Derivative Test, $\Rightarrow \boxed{\left(\dfrac{ab^2}{a^2+b^2},\dfrac{a^2b}{a^2+b^2}\right)}$ is the point on the line $\dfrac{x}{a}+\dfrac{y}{b}=1$ that is closest to the origin.

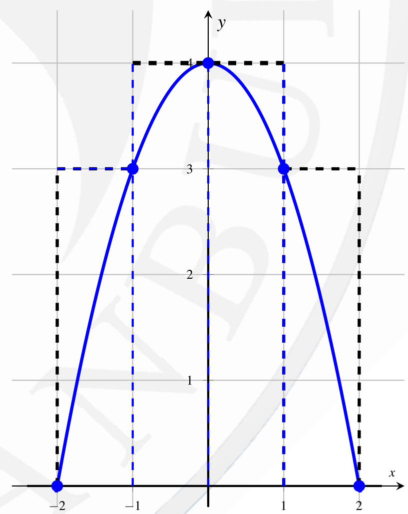
p.212, pr.85

(b) (8 Puan) $f(x)=\cos\left(\dfrac{\pi}{2}x\right)+\pi\cos x$ fonksiyonunun genel ters türevini bulunuz.

Solution: We have

$$f(x)=\cos\left(\frac{\pi}{2}x\right)-\pi\cos x\Rightarrow \boxed{F(x)=\frac{2}{\pi}\sin\left(\frac{\pi}{2}x\right)+\pi\sin x}$$

(c) (8 Puan) $f(x)=4-x^2$, $y=0$, $x=-2$ ve $x=2$ arasında kalan alanı eşit genişlikli dört dikdörtgen ile bir üst toplam yaklaşımı kullanarak tahmin ediniz.



Solution: Since f is increasing on $[-2,0]$ and decreasing on $[0,2]$, we use left endpoints on $[-2,0]$ and right endpoints on $[0,2]$ to obtain lower sums and use right endpoints on $[-2,0]$ and left endpoints on $[0,2]$ to obtain upper sums.

Here $a=-2$ and $b=2$ and $n=4$ and so $\Delta x=\dfrac{b-a}{n}=\dfrac{2-(-2)}{4}=1$ and $x_k=-2+k\Delta x=-2+k$. Therefore the partition is $P=\{x_0=-2,x_1=-1,x_2=0,x_3=1,x_4=2\}$. Therefore, an upper sum is

$$\begin{aligned}\sum_{k=0}^{3}(4-x_k^2)\cdot(1)&=\sum_{k=1}^{2}(4-x_k^2)\cdot(1)+\sum_{k=2}^{3}(4-x_k^2)\cdot(1)\\&=1(4-(-1)^2+4-(0)^2+(4-0^2)+(4-1^2))\\&=\boxed{14}\end{aligned}$$

(Page 253, problem 3d)

3. (a) (10 Puan) $\int_1^0 (3x^2+x-5)\,\mathrm{d}x$ integralini hesaplayınız.

Solution: By the Fundamental Theorem of Calculus and properties of definite integrals, we have

$$\begin{aligned}\int_1^0 (3x^2+x-5)\,\mathrm{d}x &= -\int_0^1 (3x^2+x-5)\,\mathrm{d}x \\ &= -\left[3\int_0^1 x^2\,\mathrm{d}x + \int_0^1 x\,\mathrm{d}x - \int_0^1 5\,\mathrm{d}x\right] = -\left[3\left(\frac{1^3}{3}-\frac{0^3}{3}\right)+\left(\frac{1^2}{2}-\frac{0^2}{2}\right)-5(1-0)\right] \\ &= -\left(\frac{3}{2}-5\right) = \boxed{\frac{7}{2}}\end{aligned}$$

(b) (8 Puan) $F(x) = \int_0^{\sin x} \frac{\mathrm{d}t}{\sqrt{1-t^2}}$, $|x| < \pi/2$ ise $F'(x)$ türevini hesaplayınız.

Solution: By the Fundamental Theorem of Calculus Part I, we have

$$F'(x) = \frac{d}{dx}\left(\int_0^{\sin x} \frac{\mathrm{d}t}{\sqrt{1-t^2}}\right) = \frac{1}{\sqrt{1-(\sin x)^2}}\frac{d}{dx}(\sin x) = \frac{\cos x}{\sqrt{1-\sin^2}} = \frac{\cos x}{\sqrt{\cos^2}} = \frac{\cos x}{|\cos x|} = \frac{\cos x}{\cos x} = \boxed{1}$$

(c) (8 Puan) $f(x) = -\frac{x^2}{2}$ fonksiyonunun $[0,3]$ aralığı üzerindeki ortalama değerini bulunuz.

Solution: using the definition, we have

$$\operatorname{av}(f) = \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x = \frac{1}{3-0}\int_0^3 \left(-\frac{x^2}{2}\right)\mathrm{d}x = \frac{1}{3}\left(-\frac{1}{2}\right)\int_0^3 x^2\,\mathrm{d}x = -\frac{1}{6}\left(\frac{3^3}{3}\right) = \boxed{-\frac{3}{2}}$$

4. $y = \dfrac{-x^2+2}{x^2-1} = -1 + \dfrac{1}{x^2-1}$ ve türevleri $y' = -\dfrac{2x}{(x^2-1)^2}$, $y'' = \dfrac{6x^2+2}{(x^2-1)^3}$ veriliyor.

(a) (7 Puan) *Asimptotları* bulunuz.

Solution: The graph has three asymptotes. Two are vertical, the other is horizontal. The lines $x=-1$ and $x=1$ are vertical asymptotes as $\lim\limits_{x\to\pm1^{\pm}} \dfrac{-x^2+2}{x^2-1} = \pm\infty$. Also since $\lim\limits_{x\to\pm\infty} \dfrac{-x^2+2}{x^2-1} = -1$, the line $y=-1$ is an horizontal asymptote.

p.212, pr.85

(b) (5 Puan) Fonksiyonun *arttığı* ve *azaldığı aralıkları* bulunuz. *Yerel maksimum* ve *minimum* değerleri bulunuz.

Solution: There is a critical point at $x=0$, where the function has a local maximum. The function is increasing on We have $y' = -\dfrac{2x}{(x^2-1)^2} = 0$ if and only if $2x=0$, that is iff $x=0$ is the critical point. Note that

$$y' \text{ is } \begin{cases} >0, & \text{on } (0,1)\cup(1,\infty) \qquad \text{y is incresing} \\ <0, & \text{on } (-\infty,-1)\cup(-1,0) \qquad \text{y is decresing} \end{cases}$$

Thus, y is decreasing on $(-\infty,-1)\cup(-1,0)$ and increasing on $(0,1)\cup(1,\infty)$. Moreover, the point $(0,-2)$ is a point of local maximum.

p.212, pr.85

(c) (5 Puan) Grafiğin *aşağı konkav* ve *yukarı konkav* olduğu *aralıkları* bulunuz. Varsa, *büküm noktalarını* yazınız.

Solution: We have $y'' = \dfrac{6x^2+2}{(x^2-1)^3}$ and so

$$y'' \begin{cases} > 0, & \text{on } (-\infty,-1)\cup(1,\infty) \qquad \text{y is concave up} \\ < 0, & \text{on } (-1,1) \qquad \text{y is concave down} \end{cases}$$

Hence f is concave up on $(-\infty,-1)\cup(1,\infty)$ and concave down on $(-\infty,-1)$. Also graph has *no point of inflection*.

p.212, pr.85

(d) (7 Puan) Fonksiyonun *grafiğini çiziniz*. Asimptotları, dönüm ve büküm noktalarını belirtiniz.