Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- Calculators, cell phones off and away!.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place a box around your answer to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 90 min.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	25	
3	30	
4	25	
Total:	100	

1. (a) 10 Points Does the series $\sum_{n=1}^{\infty}\left(\frac{3n+1}{2n+1}\right)^n$ converge? Give reasons for your answer.

○ Converges. ○ Diverges. Test Used: ______________

Solution: Here $a_n = \left(\frac{3n+1}{2n+1}\right)^n > 0$. Use the Root Test.

$$\rho = \lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\sqrt[n]{\left(\frac{3n+1}{2n+1}\right)^n}$$
$$= \lim_{n\to\infty}\frac{3n+1}{2n+1} = \lim_{n\to\infty}\frac{3+1/n}{2+1/n} = \frac{3+0}{2+0} = \frac{3}{2} > 1$$

Hence by the Ratio Test, the series *diverges*.

p.491, pr.65

(b) 10 Points Evaluate the integral $\int \frac{x-5}{3x^3-12x}\,dx$.

Solution: we use partial fraction decomposition for the integrand.

$$\frac{x-5}{3x^3-12x} = \frac{x-5}{3x(x-2)(x+2)} = \frac{A}{3x}+\frac{B}{x-2}+\frac{C}{x+2} \Rightarrow x-5 = A(x-2)(x+2)+B(3x)(x+2)+C(3x)(x-2)$$

If $x=0$, we have $A=5/4$, if $x=2$, we have $-3=24B$ which implies $B=-1/8$ and if $x=-2$, we have $-7=24C$ so $C=-7/24$. Hence

$$\int \frac{x-5}{3x^3-12x} = \int \frac{5/4}{3x}\,dx + \int \frac{-1/8}{x-2}\,dx + \int \frac{-7/24}{x+2}\,dx$$
$$= \boxed{\frac{5}{12}\ln|x| + \frac{1}{12}\ln|x-2| - \frac{7}{24}\ln|x+2| + K}$$

p.532, pr.36

2. (a) 14 Points Find the distance between the planes $x+2y+6z=1$ and $x+2y+6z=10$.

Solution: The point $P(1,0,0)$ is on the first plane and $S(10,0,0)$ is a point on the second plane. Then $\vec{PS}=(10-1)\mathbf{i}+0\mathbf{j}+0\mathbf{k}=9\mathbf{i}$ and $\mathbf{n}=\mathbf{i}+2\mathbf{j}+6\mathbf{k}$ is normal to the first plane. Hence the distance from S to the first plane is

$$d=\left|\vec{PS}\bullet\frac{\mathbf{n}}{|\mathbf{n}|}\right|=\left|\frac{9}{\sqrt{1+4+36}}\right|=\frac{9}{\sqrt{41}},$$

which is also the distance between the planes.

p.749, pr.53

(b) 11 Points Find the equation for the plane $\mathscr{M}$ through $P(1,-1,2)$, $Q(2,1,3)$, and $R(-1,2,-1)$.

Solution: First we find a normal vector to the plane:

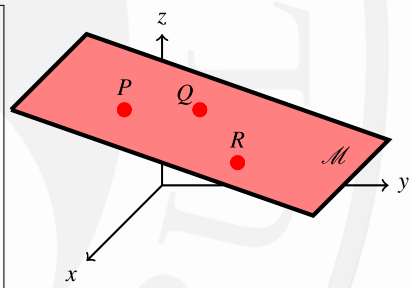
$$\begin{aligned}\vec{PQ}&=(2-1)\mathbf{i}+(1+1)\mathbf{j}+(3-2)\mathbf{k}\\&=\mathbf{i}+2\mathbf{j}+\mathbf{k}\\ \vec{PR}&=(-1-1)\mathbf{i}+(2+1)\mathbf{j}+(-1-2)\mathbf{k}\\&=-2\mathbf{i}+3\mathbf{j}-3\mathbf{k}\\ \Rightarrow \vec{PQ}\times\vec{PR}&=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&2&1\\-2&3&3\end{vmatrix}\\&=-9\mathbf{i}+\mathbf{j}+7\mathbf{k}\end{aligned}$$

is normal to the plane

$$\Rightarrow -9(x-1)+(y+1)+7(z-2)=0$$

hence $\boxed{-9x+y+7z=4}$ is the equation of the plane.

p.695, pr.23



3. (a) 15 Points Find $\left.\frac{dy}{dx}\right|_{P(0,1)}$ if $1-x-y^2-\sin(xy)=0$.

Solution: Let $F(x,y)=1-x-y^2-\sin(xy)=0$. Then

$$\begin{aligned}\frac{dy}{dx}&=-\frac{\partial F/\partial x}{\partial F/\partial y}\\&=-\frac{-1-y\cos(xy)}{-2y-x\cos(xy)}=\frac{1+y\cos(xy)}{-2y-x\cos(xy)}\\ \Rightarrow \left.\frac{dy}{dx}\right|_{P(0,1)}&=\frac{1+1}{-2}=\boxed{-1}\end{aligned}$$

p.879, pr.39

(b) 15 Points Find parametric equations for the line tangent to the curve of intersection of the surfaces $x^2+2y+2z=4$ and $y-1=0$ at the point $P_0(1,1,1/2)$.

Solution: Let $f(x,y,z)=x^2+2y+2z-4=0$ and $g(x,y,z)=y-1$. Then

$$\nabla f=2x\mathbf{i}+2\mathbf{j}+2\mathbf{k}\Rightarrow\nabla f(1,1,1/2)=2\mathbf{i}+2\mathbf{j}+2\mathbf{k};$$

$$\nabla g=\mathbf{j}\Rightarrow\nabla g(1,1,1/2)=\mathbf{j};$$

$$\Rightarrow\mathbf{v}=\nabla f\times\nabla g=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&2&2\\0&1&0\end{vmatrix}=2\mathbf{i}+2\mathbf{k}$$

$$\Rightarrow \text{ Tangent Line: }\boxed{x=1-2t,\quad y=1,\quad z=\frac{1}{2}+t}$$

p.192, pr.87

4. (a) [12 Points] Find the local maxima, the local minima, and the saddle points for $f(x,y) = x^3 + y^3 - 3xy + 15$. Find function's value at these points.

Solution:

$$f_x = 3x^2 - 3y = 0 \qquad f_y = 3y^2 - 3x = 0$$
$$3(y^2)^2 - 3y = 0 \qquad 3y^2 = 3x$$
$$3y^4 - 3y = 0 \qquad x = y^2$$
$$3y(y^3 - 1) = 0$$
$$y = 0 \quad y = 1$$
$$x = 0 \quad x = 1$$

The critical points for this function are $(0,0)$ and $(1,1)$. Now we have

$$f_{xx} = 6x, \qquad f_{yy} = 6y, \qquad f_{xy} = -3, \qquad f_{xx}f_{yy} - (f_{xy})^2 = (6x)(6y) - (-3y)^2 = 36xy - 9y^2.$$

At $(0,0)$, we have

$$f_{xx}(0,0)f_{yy}(0,0) - (f_{xy}(0,0))^2 = (6(0))(6(0)) - (-3)^2 = 36(0)(0) - 9 = -9 < 0.$$

So f has a saddle point at $(0,0)$ and $f(0,0) = 15$.
At $(1,1)$, we have

$$f_{xx}(1,1)f_{yy}(1,1) - (f_{xy}(1,1))^2 = (6(1))(6(1)) - (-3)^2 = 36(1)(1) - 9 = 27 > 0 \quad \text{and} \quad f_{xx}(1,1) = 6 > 0$$

So f has a local minimum at $(1,1)$ and $f(1,1) = 14$.

p.588, pr.4

(b) [13 Points] If a and b are constants, $w = u^2 + \tanh u + \cos u$ and $u = ax + by$, calculate the right hand side of the following equation.

$$a\frac{\partial w}{\partial y} - b\frac{\partial w}{\partial x} = ?$$

.

Solution:

$$\frac{\partial u}{\partial y} = b \text{ and } \frac{\partial u}{\partial x} = a \Rightarrow \frac{\partial w}{\partial x} = \frac{dw}{du}\frac{\partial u}{\partial x} = a\frac{dw}{du} \text{ and}$$
$$\frac{\partial w}{\partial y} = \frac{dw}{du}\frac{\partial u}{\partial y} \Rightarrow b\frac{dw}{du} \Rightarrow \frac{1}{a}\frac{\partial u}{\partial x} = \frac{dw}{du} \text{ and}$$
$$\frac{1}{b}\frac{\partial w}{\partial y} = \frac{dw}{du} \Rightarrow \frac{1}{a}\frac{\partial u}{\partial x} = \frac{1}{b}\frac{\partial w}{\partial y} \Rightarrow b\frac{\partial w}{\partial x} = a\frac{\partial w}{\partial y}$$

p.880, pr.75