Math 114 Summer 2018 **First Exam** July 11, 2018

Your Name / Ad - Soyad (**75 min.**) Signature / İmza

Student ID # / Öğrenci No (**mavi tükenmez!**)

Problem	1	2	3	4	Total
Points:	22	27	27	24	100
Score:					

Time limit is **75 minutes**. If you need more room on your exam paper, you may use the empty spaces on the paper. You have to show your. Answers without reasonable-even if your results are true- work will either get zero or very little credit.

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1. (a) (10 Points) Find the derivative $\dfrac{dy}{dz}$ if $y = \ln(\sinh z)$.

Solution:

$$\frac{dy}{dz} = \frac{d}{dz}(\ln(\sinh z)) = \frac{1}{\sinh z}\frac{d}{dz}(\sinh z)$$
$$= \frac{\cosh z}{\sinh z}$$
$$= \boxed{\coth z}$$

p.72, pr.15

(b) (12 Points) Evaluate the integral $\displaystyle\int_0^4 \frac{\mathrm{d}x}{\sqrt{4-x}}$.

Solution:

$$\int_0^4 \frac{\mathrm{d}x}{\sqrt{4-x}} = \lim_{b\to 4^-}\int_0^b \frac{\mathrm{d}x}{\sqrt{4-x}} = \lim_{b\to 4^-}\int_0^b (4-x)^{-1/2}\ \mathrm{d}x$$
$$= \lim_{b\to 4^-}\left[-\frac{(4-x)^{-1/2+1}}{-1/2+1}\right]_0^b$$
$$= -2\lim_{b\to 4^-}\left[(4-b)^{1/2} - (4-0)^{1/2}\right] = -2\left((4-4)^{1/2} - (4)^{1/2}\right) = \boxed{4}$$

p.94, pr.34

2. Evaluate the following integrals.

(a) (12 Points) $\int xe^{3x}\ \mathrm{d}x =$

Solution: We shall integrate by parts. Let $u = x$ and so $dv = e^{3x}\ dx$. Then $du = \mathrm{d}x$ and choose $v = \frac{1}{3}e^{3x}$. Therefore

$$\begin{aligned}\int xe^{3x}\,\mathrm{d}x &= \int u\,\mathrm{d}v = uv - \int v\,\mathrm{d}u \\ &= (x)\left(\frac{1}{3}e^{3x}\right) - \int\left(\frac{1}{3}e^{3x}\right)\ \mathrm{d}x \\ &= \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}\ \mathrm{d}x \\ &= \boxed{\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C}\end{aligned}$$

p.72, pr.8

(b) (15 Points) $\int \frac{x+3}{2x^3-8x}\ \mathrm{d}x =$

Solution: We decompose the integrand in the following way:

$$\frac{x+3}{2x^3-8x} = \frac{x+3}{2x(x^2-4)} = \frac{x+3}{2x(x-2)(x+2)} = \frac{1}{2}\left[\frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}\right]$$

Clearing the fractions changes to

$$x+3 = A(x-2)(x+2) + Bx(x+2)Cx(x-2) \Rightarrow A = -3/4, B = 5/8, C = 1/8.$$

Thus,

$$\begin{aligned}\int \frac{x+3}{2x^3-8x}\ \mathrm{d}x &= \frac{1}{2}\int\left(\frac{-3/4}{x} + \frac{5/8}{x-2} + \frac{1/8}{x+2}\right)\mathrm{d}x \\ &= \boxed{-\frac{3}{8}\ln|x| + \frac{5}{16}\ln|x-1| + \frac{1}{16}\ln|x+2| + C}\end{aligned}$$

p.83, pr.52

3. (a) (15 Points) Use a trigonometric substitution to evaluate the integral $\int \frac{6\ \mathrm{d}t}{(9t^2+1)^2}$.

Solution: For $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, let $t = \frac{1}{3}\tan\theta$, and so $6\,\mathrm{d}t = \frac{6}{3}\sec^2\theta\ \mathrm{d}\theta$ and also $(9t^2+1)^2 = (9(\frac{1}{3}\tan\theta)^2+1)^2 = (\tan\theta^2+1)^2 = \sec^4\theta$;

$$\begin{aligned}
\int \frac{6\ \mathrm{d}t}{(9t^2+1)^2} &= \int \frac{2\sec^2\theta\ \mathrm{d}\theta}{\sec^4\theta}\ \mathrm{d}\theta \\
&= 2\int \cos^2\theta\ \mathrm{d}\theta \\
&= \not{2}\int (\frac{1+\cos(2\theta)}{\not{2}}))\ \mathrm{d}\theta = \theta + \frac{1}{2}\sin(2\theta) + C = \theta + \frac{1}{2}2\sin(\theta)\cos(\theta) + C \\
&= \tan^{-1}(3t) + \frac{3t}{\sqrt{1+9t^2}}\frac{1}{\sqrt{1+9t^2}} + C \\
&= \boxed{\tan^{-1}(3t) + \frac{3t}{(1+9t^2)} + C}
\end{aligned}$$

p.95, pr.68

(b) (12 Points) If $a_n = \sqrt{\frac{2n}{n-1}}$, does the sequence $\{a_n\}$ converge? If it converges, find its limit.

Solution: The sequence converges and has limit 0, because

$$\begin{aligned}
\lim_{n\to\infty} a_n = \lim_{n\to\infty}\sqrt{\frac{2n}{n-1}} &= \sqrt{\lim_{n\to\infty}\frac{2n}{n-1}} = \sqrt{\lim_{n\to\infty}\frac{2}{1-1/n}} \\
&= \sqrt{\lim_{n\to\infty}\frac{2}{1-0}} = \boxed{\sqrt{2}}
\end{aligned}$$

p.112, pr.26

4. (a) (12 Points) Investigate the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n(n+1)}{(n+2)(n+3)}$.

◯ Converges. ◯ Diverges. Test Used: ______________

Solution: Let $a_n = \frac{n(n+1)}{(n+2)(n+3)}$ for each $n \geq 1$. We have

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n(n+1)}{(n+2)(n+3)} = \lim_{n\to\infty} \frac{n^2+n}{n^2+5n+6} \lim_{n\to\infty} \frac{1+1/n}{1+5/n+6/n^2} = \boxed{1 \neq 0}$$

p.72, pr.8

(b) (12 Points) Find the sum of the series $\sum_{n=0}^{\infty} \left(\frac{5}{2^n} - \frac{1}{3^n}\right)$.

◯ Converges. ◯ Diverges. Series' Sum: ______________

Solution: This is the difference of two convergent geometric series with sum

$$\sum_{n=0}^{\infty} \frac{5}{2^n} - \sum_{n=0}^{\infty} \frac{1}{3^n} = \frac{5}{1-1/2} - \frac{1}{1-1/3} = 10 - \frac{3}{2} = \frac{17}{2}$$

p.82, pr.35