Math 114 Summer 2019 **First Exam** **July 3, 2019**

Your Name / Ad - Soyad (**70 min.**)

Signature / İmza

Student ID # / Öğrenci No (**mavi tükenmez!**)

Problem	1	2	3	4	Total
Points:	22	27	25	26	100
Score:					

Time limit is **70 minutes**. If you need more room on your exam paper, you may use the empty spaces on the paper. You have to show your. Answers without reasonable-even if your results are true- work will either get zero or very little credit.

1. (a) (10 Points) Evaluate the integral $\int_0^{\pi} \sqrt{1-\cos(2x)}\ \mathrm{d}x$.

Solution:

$$\begin{aligned}\int_0^{\pi} \sqrt{1-\cos(2x)}\ \mathrm{d}x &= \int_0^{\pi} \sqrt{\sin^2 x+\cos^2 x-(\cos^2 x-\sin^2 x)}\ \mathrm{d}x \\ &= \int_0^{\pi} \sqrt{2\sin^2 x}\ \mathrm{d}x = \int_0^{\pi} \sqrt{2}|\sin x|\ \mathrm{d}x \\ &= \int_0^{\pi} \sqrt{2}\sin x\ \mathrm{d}x \\ &= \sqrt{2}[-\cos x]_0^{\pi} = \sqrt{2}(-\cos\pi-(-\cos 0)) = \boxed{2\sqrt{2}}\end{aligned}$$

p.72, pr.15

(b) (12 Points) Evaluate the integral $\int_{-\infty}^{0} \theta e^{\theta}\ \mathrm{d}\theta$.

Solution:

$$\begin{aligned}\int_{-\infty}^{0} \theta e^{\theta}\ \mathrm{d}\theta &= \lim_{b\to-\infty}\int_b^0 \theta e^{\theta}\ \mathrm{d}\theta = \lim_{b\to-\infty}\left[\theta e^{\theta}-e^{\theta}\right]_b^0 \\ &= \lim_{b\to-\infty}\left[\left(0e^0-e^0\right)-\left(be^b-e^b\right)\right]_b^0 \\ &= -1-\lim_{b\to-\infty}\left(\frac{b-1}{e^{-b}}\right) = -1-\lim_{b\to-\infty}\left(\frac{1}{-e^{-b}}\right) = -1-0 = \boxed{-1}\end{aligned}$$

p.94, pr.34

2. Evaluate the following integrals.

(a) (9 Points) $\int \sec^2 x \tan^2 x \, dx =$

Solution: Let $u = \tan x$ and so $du = \sec^2 x dx$. Then

$$\begin{aligned} \int \sec^2 x \tan^2 x \, dx &= \int \tan^2 x \sec^2 x \, dx \\ &= \int u^2 \, du = \frac{1}{3}u^3 + C \\ &= \boxed{\frac{1}{3}\tan^3 x + C} \end{aligned}$$

p.72, pr.8

(b) (8 Points) $\int \sin(2x)\cos(4x) \, dx =$

Solution:

$$\begin{aligned} \int \sin(2x)\cos(4x) \, dx &= \frac{1}{2}\int (\sin(2x-4x)+\sin(2x+4x)) \, dx = \frac{1}{2}\int (\sin(-2x)+\sin(6x)) \, dx \\ &= \frac{1}{2}\int (-\sin(2x)+\sin(6x)) \, dx \\ &= \frac{1}{2}\left[\frac{1}{2}\cos(2x) - \frac{1}{6}\cos(6x)\right] + C = \boxed{\frac{1}{4}\cos(2x) - \frac{1}{12}\cos(6x) + C} \end{aligned}$$

p.83, pr.52

(c) (10 Points) $\int \frac{x+3}{2x^3-8x} \, dx =$

Solution: We decompose the integrand in the following way:

$$\frac{x+3}{2x^3-8x} = \frac{x+3}{2x(x^2-4)} = \frac{x+3}{2x(x-2)(x+2)} = \frac{1}{2}\left[\frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}\right]$$

Clearing the fractions changes to

$$x+3 = A(x-2)(x+2) + Bx(x+2)Cx(x-2) \Rightarrow A = -3/4, B = 5/8, C = 1/8.$$

Thus,

$$\begin{aligned} \int \frac{x+3}{2x^3-8x} \, dx &= \frac{1}{2}\int \left(\frac{-3/4}{x} + \frac{5/8}{x-2} + \frac{1/8}{x+2}\right) dx \\ &= \boxed{-\frac{3}{8}\ln|x| + \frac{5}{16}\ln|x-1| + \frac{1}{16}\ln|x+2| + C} \end{aligned}$$

p.83, pr.52

3. (a) (15 Points) Use a trigonometric substitution to evaluate the integral $\int \frac{8\ \mathrm{d}x}{x^2\sqrt{4-x^2}}$.

Solution: Let $x = 2\sin\theta$, $-\pi/2 < \theta < \pi/2$ and so $\mathrm{d}x = 2\cos\theta\ \mathrm{d}\theta$ and also $\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta} = \sqrt{4(1-\sin^2\theta)} = \sqrt{4\cos^2\theta} = 2\cos\theta$;

$$\int \frac{8\ \mathrm{d}x}{x^2\sqrt{4-x^2}} = \int \frac{8\cdot 2\cos\theta\ \mathrm{d}\theta}{4\sin^2\theta 2\cos\theta} = 2\int \frac{\mathrm{d}\theta}{\sin^2\theta} = -2\cot\theta + C = \boxed{-2\frac{\sqrt{4-x^2}}{x} + C}$$

p.95, pr.68

(b) (10 Points) Use a Comparison Test to investigate the convergence for the integral $\int_1^\infty \frac{e^x}{x}\ \mathrm{d}x$.

Solution: We use direct comparison test as follows.

$$0 < \frac{1}{x} < \frac{e^x}{x} \text{ for } x > 1 \text{ and } \int_1^\infty \frac{1}{x}\ \mathrm{d}x \text{ diverges } \Rightarrow \int_1^\infty \frac{e^x}{x}\ \mathrm{d}x \text{ diverges by Direct Comparison Test}$$

p.112, pr.26

4. (a) (8 Points) If $a_n = \left(1 - \frac{1}{n}\right)^n$, does the sequence $\{a_n\}$ converge? If it converges, find its limit.

Solution: The sequence converges and has limit 0, because

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{-1}{n}\right)^n = \boxed{e^{-1}}$$

The sequence converges and has limit e^{-1}.

p.112, pr.26

(b) (9 Points) Find the sum of the series $\sum_{n=1}^{\infty}\left(\frac{4}{3^n} - \frac{1}{4^{n-1}}\right)$.

○ Converges. ○ Diverges. Series' Sum: ______________

Solution: This is the difference of two convergent geometric series with sum

$$\sum_{n=1}^{\infty}\frac{4}{3^n} - \sum_{n=1}^{\infty}\frac{1}{4^{n-1}} = \frac{4/3}{1-1/3} - \frac{1}{1-1/4} = 2 - \frac{4}{3} = \frac{2}{3}$$

Series converges and has sum $2/3$.

p.82, pr.35

(c) (9 Points) $\frac{5}{1\cdot 2} + \frac{5}{2\cdot 3} + \frac{5}{3\cdot 4} + \cdots + \frac{5}{n(n+1)} + \cdots$

○ Converges. ○ Diverges. Series' Sum: ______________

Solution: First, by partial fraction decomposition of the general term, we have

$$\begin{aligned}
a_n &= \frac{5}{n(n+1)} = \frac{5}{n} - \frac{5}{n+1}\\
s_n &= a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n\\
&= \left(\frac{5}{1} - \frac{5}{2}\right) + \left(\frac{5}{2} - \frac{5}{3}\right) + \left(\frac{5}{3} - \frac{5}{4}\right) + \cdots + \left(\frac{5}{n} - \frac{5}{n-1}\right) + \left(\frac{5}{n} - \frac{5}{n+1}\right)\\
&= \frac{5}{1} - \frac{5}{n+1}\\
&\Rightarrow \lim_{n\to\infty} s_n = \lim_{n\to\infty}\left(\frac{5}{1} - \frac{5}{n+1}\right) = \boxed{5}
\end{aligned}$$

Series converges and has sum 5.

p.72, pr.8