Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- Calculators, cell phones off and away!.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place a box around your answer to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 70 min.

Do not write in the table to the right.

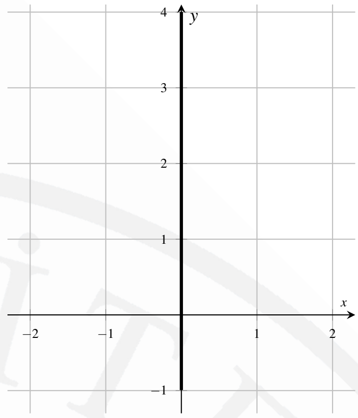
Problem	Points	Score
1	32	
2	35	
3	33	
Total:	100	

1. (a) 10 Points Find and sketch the domain for $f(x,y) = \sqrt{x^2 - y}$. Sketch the level curve through the point $(2,1)$.

Solution: Since the value of the argument of a square root must be non-negative, this implies that $x^2 - y \geq 0$. Here is the domain of this function.

$$D = \{(x,y) \in \mathbb{R}^2 \mid x^2 \geq y\}$$

p.687, pr.17(a)



(b) 12 Points Let $f(x,y) = \dfrac{x^3y^3 - 1}{xy - 1}$. Find the limit $\lim\limits_{(x,y)\to(1,1)} f(x,y)$.

Solution: Using the identity $A^3 - B^3 = (A-B)(A^2 + AB + B^2)$, we have

$$\lim_{(x,y)\to(1,1)} f(x,y) = \lim_{(x,y)\to(1,1)} \frac{x^3y^3 - 1}{xy - 1} = \lim_{(x,y)\to(1,1)} \frac{\cancel{(xy-1)}(x^2y^2 + xy + 1)}{\cancel{xy-1}}$$
$$= \lim_{(x,y)\to(1,1)} (x^2y^2 + xy + 1) = (1)^2(1)^2 + (1)(1)y + 1 = 3$$

p.573, pr.38

(c) 10 Points How can $f(1,1)$ be defined so that $f(x,y)$ is continuous at $(1,1)$?

Solution: For f to be continuous at $(1,1)$, we must have $f(1,1) = \lim\limits_{(x,y)\to(0,0)} f(x,y)$, that is, we must define $f(1,1) = 3$.

p.573, pr.18

2. (a) 10 Points Use the definition $\left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} = \lim_{h\to 0}\frac{f(x_0+h,y_0)-f(x_0,y_0)}{h}$ to find the value of $\left.\frac{\partial f}{\partial x}\right|_{(1,2)}$, if $f(x,y)=(xy-1)^2$.

Solution: Substituting the ingredients into the given definition, we have

$$\left.\frac{\partial f}{\partial x}\right|_{(1,2)} = \lim_{h\to 0}\frac{f(1+h,2)-f(1,2)}{h} = \lim_{h\to 0}\frac{((1+h)2-1)^2-((1)(2)-1)^2}{h} = \lim_{h\to 0}\frac{(2+2h-1)^2-(2-1)^2}{h}$$
$$= \lim_{h\to 0}\frac{\cancel{1}+4h+4h^2-\cancel{1}}{h}$$
$$= \lim_{h\to 0}\frac{\cancel{h}(4+4h)}{\cancel{h}} = \lim_{h\to 0}(4+4h) = 4+(4)(0) = \boxed{4}$$

p.491, pr.86

(b) 10 Points Find $\frac{\partial f}{\partial x}$ if $f(x,y)=\frac{1}{2}\ln(x^2-y^2)+\sin^{-1}\frac{y}{x}$.

Solution:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\frac{1}{2}\ln(x^2-y^2)+\sin^{-1}\frac{y}{x}\right) = \frac{\partial}{\partial x}\left(\frac{1}{2}\ln(x^2-y^2)\right)+\frac{\partial}{\partial x}\left(\sin^{-1}\frac{y}{x}\right)$$
$$= \frac{1}{2}\frac{2x}{x^2-y^2}+\frac{-y/x^2}{\sqrt{1-(y/x)^2}} = \boxed{\frac{x}{x^2-y^2}-\frac{y}{\sqrt{x^4-x^2y^2}}}$$

p.695, pr.37

(c) 15 Points Find dw/dt at $t=1$ if $w=xe^y+y\sin z-\cos z$, $x=2\sqrt{t}$, $y=t-1+\ln t$, and $z=\pi t$.

Solution:

$$\frac{\partial w}{\partial x}=e^y,\quad \frac{\partial w}{\partial y}=xe^y+\sin z,\quad \frac{\partial w}{\partial z}=y\cos z+\sin z$$
$$\frac{dx}{dt}=t^{-1/2},\quad \frac{dy}{dt}=1+\frac{1}{t},\quad \frac{dz}{dt}=\pi,$$
$$\frac{dw}{dt}=\frac{\partial w}{\partial x}\frac{dx}{dt}+\frac{\partial w}{\partial y}\frac{dy}{dt}+\frac{\partial w}{\partial z}\frac{dz}{dt}$$
$$=(e^y)\left(t^{-1/2}\right)+(xe^y+\sin z)\left(1+\frac{1}{t}\right)+(y\cos z+\sin z)(\pi)$$
$$t=1\Rightarrow x=2, y=0, \text{ and } z=\pi$$
$$\Rightarrow \left.\frac{dw}{dt}\right|_{t=1}=(1)(1)+((2)(1)-0)(2)+(0+0)(\pi)=\boxed{5}$$

p.583, pr.17

3. (a) [10 Points] Find the value of $\partial x/\partial z$ at the point $(1,-1,-3)$ if the equation

$$xz + y\ln x - x^2 + 4 = 0$$

defines x as a function of two independent variables y and z and the partial derivative exists.

Solution: Differentiating both sides with respect to z gives:

$$\frac{\partial}{\partial z}(xz + y\ln x - x^2 + 4) = \frac{\partial}{\partial z}(0)$$

$$x + z\frac{\partial x}{\partial z} + y\frac{1}{x}\frac{\partial x}{\partial z} - 2x\frac{\partial x}{\partial z} = 0 \Rightarrow (z + \frac{y}{x} - 2x)\frac{\partial x}{\partial z} = 0$$

$$\frac{\partial x}{\partial z} = \frac{-x}{z + \frac{y}{x} - 2x}$$

$$\left.\frac{\partial x}{\partial z}\right|_{(1,-1,-3)} = \left.\frac{-x}{z + \frac{y}{x} - 2x}\right|_{(1,-1,-3)} = \frac{-1}{-3-1-2} = \boxed{\frac{1}{6}}$$

p.822, pr.65

(b) [11 Points] Find an equation for the plane tangent to the surface $x^2 - y - 5z = 0$ at the point $P_0(2,-1,1)$. Also find the parametric equations for the line that is normal to the surface at P_0.

Solution: We need a vector normal to the tangent plane and a vector parallel to the normal line.

$f(x,y,z) = x^2 - y - 5z = 0 \Rightarrow \nabla f = 2x\mathbf{i} - \mathbf{j} - 5\mathbf{k}$;

at $(2,-1,1)$ we get $\nabla f|_{(2,-1,1)} = 4\mathbf{i} - \mathbf{j} - 5\mathbf{k}$

Tangent Plane : $4(x-2) - (y+1) - 5(z-1) = 0 \Rightarrow 4x - y - 5z = 4$;

Normal Line : $\begin{cases} x = 2 + 4t \\ y = -1 - t \\ z = 1 - 5t \end{cases}$

p.695, pr.37

(c) [12 Points] What is the largest value that the directional derivative of $f(x,y,z) = xyz$ can have at the point $(1,1,1)$?

Solution: We know that at points P, the rate of change is largest in the direction of ∇f and in this direction derivative has value $|\nabla f|$.

$f(x,y,z) = xyz \Rightarrow \nabla f = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$;

at $(1,1,1)$ we get $\nabla f(1,1,1) = \mathbf{i} + \mathbf{j} + \mathbf{k}$

maximum value of $D_{\mathbf{u}}f|_{(1,1,1)} = |\nabla f(1,1,1)| = |\mathbf{i} + \mathbf{j} + \mathbf{k}| = \boxed{\sqrt{3}}$

p.583, pr.17