Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place a box around your answer to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 70 min.

Do not write in the table to the right.

Problem	Points	Score
1	24	
2	27	
3	27	
4	22	
Total:	100	

1. (a) 12 Points Does the series $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n^2}$ converge absolutely, conditionally, or diverge? Justify your answer.

○ Converges absolutely. ○ Converges conditionally. ○ Diverges. Test Used: ______

Solution: This is an alternating series of the form $\sum_{n=1}^{\infty} (-1)^n u_n$ where $u_n = \frac{2^n}{n^2}$. Then since

- $\lim_{n\to\infty} \frac{2^n}{n^2} = \infty \Rightarrow \lim_{n\to\infty} (-1)^n \frac{2^n}{n^2} =$ does not exist,

the series diverges by nth Term Test.

p.573, pr.36

(b) 12 Points Find the radius and interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n^3 3^n}$.

Radius of Convergence: ______. Interval of Convergence: ______

Solution: Let $u_n = \frac{(x-1)^n}{n^3 3^n}$. Then

$$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| < 1 \Rightarrow \lim_{n\to\infty} \left| \frac{\frac{(x-1)^{n+1}}{(n+1)^3 3^{n+1}\sqrt{n+1}}}{\frac{(x-1)^n}{n^3 3^n}} \right| < 1 \Rightarrow \lim_{n\to\infty} \left| \frac{(x-1)^{n+1}}{(n+1)^3 3^{n+1}} \frac{n^3 3^n}{(x-1)^n} \right| < 1 \Rightarrow |x-1| \lim_{n\to\infty} \left(\frac{n^2}{3(n+1)^2}\right) < 1$$

$$\Rightarrow \frac{1}{3}|x-1| < 1$$

$$\Rightarrow -3 < x-1 < 3$$

$$\Rightarrow -2 < x < 4$$

When $x=-2$, we have $\sum_{n=1}^{\infty} \frac{(-3)^n}{n^2 3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$, a absolutely convergent series.

When $x=4$, we have $\sum_{n=1}^{\infty} \frac{(3)^n}{n^2 3^n} = \sum_{n=1}^{\infty} \frac{1}{n^2}$, a convergent series. So the radius of convergence is $R=3$; the interval of convergence is $-2 \leq x \leq 2$.

p.583, pr.17

2. (a) 12 Points Find the center and radius r for the sphere $x^2+y^2+z^2-6y+8z=0$.

Solution: We complete the squares:

$$x^2+y^2+z^2-6y+8z=0 \Rightarrow x^2+\left(y^2-6y+9\right)+\left(z^2+8z+16\right)=9+16$$
$$\Rightarrow x^2+(y-3)^2+(z+4)^2=25$$

Hence the center is at $(0,3,-4)$ and the radius is 5. p.491, pr.86

(b) 15 Points Write the inequalities to describe the half-space consisting of the points on and below the xy-plane.

Solution: The inequality is $z \leq 0$.

p.491, pr.86

3. (a) 14 Points Find $\cos\theta$ if θ is the angle between the vectors $\mathbf{u} = \mathbf{i} + \sqrt{2}\mathbf{j} - \sqrt{2}\mathbf{k}$ and $\mathbf{v} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$.

Solution: We have

$$\cos\theta = \frac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{u}||\mathbf{v}|} = \frac{(1)(-1) + (\sqrt{2})(1) + (-\sqrt{2})(1)}{\sqrt{(1)^2 + (\sqrt{2})^2 + (-\sqrt{2})^2}\sqrt{(-1)^2 + (1)^2 + (1)^2}} = \frac{-1}{\sqrt{5}\sqrt{3}} = -\frac{1}{\sqrt{15}}$$

p.583, pr.17

(b) 13 Points Find the area of the triangle $\triangle(KLM)$ determined by the points $K(1,1,1)$, $L(2,1,3)$, and $M(3,-1,1)$.

Solution: We have $\vec{KL} = (2-1)\mathbf{i} + (1-1)\mathbf{j} + (3-1)\mathbf{k} = \mathbf{i} + 0\mathbf{j} + 2\mathbf{k}$ and $\vec{KM} = (3-1)\mathbf{i} + (-1-1)\mathbf{j} + (1-1)\mathbf{k} = 2\mathbf{i} - 2\mathbf{j} + 0\mathbf{k}$. Hence

$$\vec{KL} \times \vec{KM} = \langle 1,0,2\rangle \times \langle 2,-2,0\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 2 \\ 2 & -2 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 2 \\ -2 & 0 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & 0 \\ 2 & -2 \end{vmatrix}\mathbf{k}$$

$$= 4\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$$

$$\Rightarrow \text{Area } = \frac{1}{2}|\vec{KL} \times \vec{KM}| = \frac{1}{2}\sqrt{16+16+4} = 3$$

p.94, pr.34

4. (a) 12 Points Parametrize the line segment joining the points $P(1,0,-1)$ and $Q(0,3,0)$.

Solution: The direction $\vec{PQ} = (0-1)\mathbf{i} + (3-0)\mathbf{j} + (0+1)\mathbf{k} = -\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and using $P(1,0,-1)$, we have

$$L: \begin{cases} x = 1 - t, \\ y = 0 + 3t, \quad 0 \le t \le 1 \\ z = -1 + t. \end{cases}$$

p.573, pr.38

(b) 10 Points Find an equation of the plane through the points $P(1,-1,2)$, $Q(2,1,3)$, and $R(-1,2,-1)$.

Solution: First find a vector normal to the plane.

$$\begin{aligned} \mathbf{n} &= \vec{PQ} \times \vec{PR} = \langle 1,2,1 \rangle \times \langle -2,3,-3 \rangle \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 1 \\ -2 & 3 & -3 \end{vmatrix} \\ &= \begin{vmatrix} 2 & 1 \\ 3 & -3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 1 \\ -2 & -3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & 2 \\ -2 & 3 \end{vmatrix} \mathbf{k} \\ &= -9\mathbf{i} + \mathbf{j} + 7\mathbf{k} \end{aligned}$$

Hence the plane has equation

$$\begin{aligned} (-9\mathbf{i} + \mathbf{j} + 7\mathbf{k}) \bullet ((x-1)\mathbf{i} + (y+1)\mathbf{j} + (z-2)\mathbf{k}) &= 0 \\ \Rightarrow -9(x-1) + (y+1) + 7(z-2) &= 0 \\ \Rightarrow \boxed{-9x + y + 7z = 4} \end{aligned}$$

p.687, pr.17(a)