Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- Hesap makinesi ve cep telefonunuzu kürsüye bırakınız.
- Bir sorudan tam puan alabilmek için, **işlemlerinizi açıklamak** zorundasınız. Bir cevapta "gidiş yolu" belirtilmemişse, sonucunuz doğru bile olsa, ya çok az puan verilecek ya da hiç puan verilmeyecek. **Limit, türev ve integral alırken nasıl yaptığınızı belirtiniz**.
- Cevabınızı kutu içine alınız.
- Kapak sayfasını **MAVİ tükenmez kalem** ile doldurunuz.
- Sınav süresi 70 dakikadır.

Yandaki tabloya hiçbir şey yazmayınız.

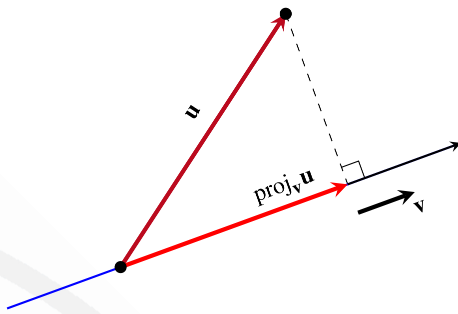
Soru	Puan	Sonuç
1	32	
2	35	
3	33	
Toplam	100	

1. (a) [10 Puan] $\mathbf{u} = \mathbf{i}+\mathbf{j}+\mathbf{k}$ vektörünün $\mathbf{v} = 5\mathbf{j}-3\mathbf{k}$ vektörü üzerine olan $\text{proj}_{\mathbf{v}}\mathbf{u}$ *izdüşüm vektörünü* bulunuz.

Solution: Projection formula is:

$$\begin{aligned}\text{proj}_{\mathbf{v}}\mathbf{u} &= \left(\frac{\mathbf{u}\bullet\mathbf{v}}{\mathbf{v}\bullet\mathbf{v}}\right)\mathbf{v}\\ &= \left(\frac{(1)(0)+(1)(5)+(1)(-3)}{(0)(0)+(5)(5)+(-3)(-3)}\right)\mathbf{v}\\ &= \frac{2}{34}\mathbf{v} = \frac{1}{17}(5\mathbf{j}-3\mathbf{k})\\ &= \boxed{\frac{5}{17}\mathbf{j}-\frac{3}{17}\mathbf{k}}\end{aligned}$$

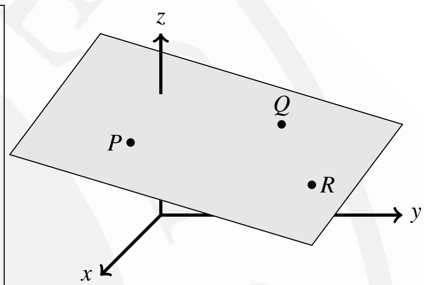
p.726, pr.5(d)



(b) [12 Puan] $P(1,-1,2)$, $Q(2,1,3)$ ve $R(-1,2,-1)$ noktalarından geçen düzlemin denklemini bulunuz.

Solution: First find a vector normal to the plane.

$$\begin{aligned}\mathbf{n} &= \vec{PQ}\times\vec{PR} = \langle 1,2,1\rangle\times\langle -2,3,-3\rangle\\ &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 2 & 1\\ -2 & 3 & -3\end{vmatrix}\\ &= \begin{vmatrix}2 & 1\\ 3 & -3\end{vmatrix}\mathbf{i} - \begin{vmatrix}1 & 1\\ -2 & -3\end{vmatrix}\mathbf{j} + \begin{vmatrix}1 & 2\\ -2 & 3\end{vmatrix}\mathbf{k}\\ &= -9\mathbf{i}+\mathbf{j}+7\mathbf{k}\end{aligned}$$



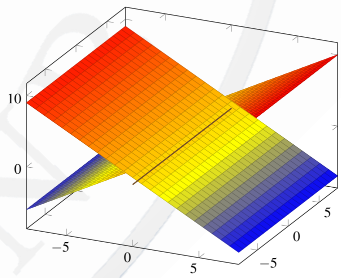
Hence the plane has equation

$$\begin{aligned}(-9\mathbf{i}+\mathbf{j}+7\mathbf{k})\bullet((x-1)\mathbf{i}+(y+1)\mathbf{j}+(z-2)\mathbf{k}) &= 0\\ \Rightarrow -9(x-1)+(y+1)+7(z-2) &= 0\\ \Rightarrow \boxed{-9x+y+7z=4}\end{aligned}$$

p.687, pr.17(a)

(c) [10 Puan] $x+2y+z=1$ ve $x-y+2z=-8$ düzlemlerinin kesiştiği doğrunun parametrik denklemlerini yazınız.

Solution: To find a point on this line we can for instance set $z=0$ and then use the above equations to solve for x and y.
In this case we get the equations $x+2y=1$ and $x-y=-8$ and solve these simultaneously and get $(-5,3,0)$ is a point on this line.
Also the direction of the line lives in both planes and so in particular is perpendicular to both normal vectors, therefore a vector which is parallel to the line is given by



$$\begin{aligned}\mathbf{n} &= \langle 1,2,1\rangle\times\langle 1,-1,2\rangle\\ &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 2 & 1\\ 1 & -1 & 2\end{vmatrix}\\ &= \begin{vmatrix}2 & 1\\ -1 & 2\end{vmatrix}\mathbf{i} - \begin{vmatrix}1 & 1\\ 1 & 2\end{vmatrix}\mathbf{j} + \begin{vmatrix}1 & 2\\ 1 & -1\end{vmatrix}\mathbf{k}\\ &= 5\mathbf{i}-\mathbf{j}-3\mathbf{k}\end{aligned}$$

Thus an equation of the line is given by the vector equation
$\langle x,y,z\rangle = \langle -5,3,0\rangle + t\langle 5,-1,-3\rangle$, or the parametric equations
$\boxed{x=-5+5t,\ y=3-t,\ z=-3t}$

p.687, pr.17(a)

2. (a) [14 Puan] xz-düzleminde 2 yarıçaplı ve $(0,0,0)$ merkezli çemberin denklemini yazınız.

Solution: $x^2+z^2=4$, $y=0$.

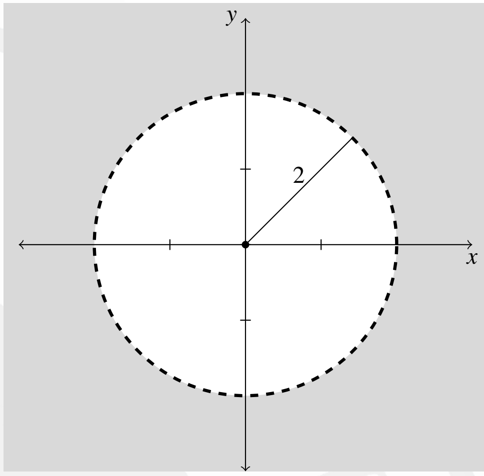
p.710, pr.28(b)

(b) [11 Puan] $f(x,y)=\ln(x^2+y^2-4)$ ise tanım kümesini bulunuz ve çiziniz.

Solution: Since the value of the argument of a natural log must be greater than zero, this implies that $x^2+y^2-4>0$. Here is the domain of this function.

$$D=\{(x,y)\in\mathbb{R}^2 \mid x^2+y^2>4\}$$

p.687, pr.17(a)



$\{(x,y) \mid x^2+y^2>2\}$

(c) [10 Puan] Varsa, $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^4-y^2}{x^4+y^2}$ limitini bulunuz.

Solution: The substitution $y=kx^2$ yields

$$\lim_{\substack{(x,y)\to(0,0)\\ y=kx^2}}\frac{x^4-(kx^2)^2}{x^4+(kx)^2}=\lim_{(x,kx^2)\to(0,0)}\frac{x^4-k^2x^4}{x^4+k^2x^4}=\lim_{x\to 0}\frac{\cancel{x^4}(1-k^2)}{\cancel{x^4}(1+k^2)}=\lim_{x\to 0}\frac{1-k^2}{1+k^2}$$
$$=\frac{1-k^2}{1+k^2}$$

This is the limit as $(x,y)\to(0,0)$ along the parabola. Different values of k (such as 1 and 0) give different values for the limit. Hence if $(x,y)\to(0,0)$, along the curve $y=x^2$ (where $k=1$) the limit is $\frac{1-1^2}{1+1^2}=0$, whereas if $(x,y)\to(0,0)$ along the line $y=0$ (where $k=0$) the limit is $\frac{1+0^2}{1+0^2}=1$. Therefore *the given limit does not exist.*

p.810, pr.43

3. (a) [11 Puan] $w = ye^{x^2-y}$ ise w_{xy} türevini bulunuz.

Solution:

$$w_x = \frac{\partial}{\partial x}(ye^{x^2-y}) = y\frac{\partial}{\partial x}(e^{x^2-y}) = ye^{x^2-y}(2x) = 2xye^{x^2-y}$$

$$w_{xy} = \frac{\partial}{\partial y}(2xye^{x^2-y}) = 2x\frac{\partial}{\partial y}(ye^{x^2-y}) = 2x\left[y\frac{\partial}{\partial y}(ye^{x^2-y}) + e^{x^2-y}\frac{\partial y}{\partial y}\right]$$

$$= 2x\left[ye^{x^2-y}(-1) + e^{x^2-y}\right]$$

$$= \boxed{2xe^{x^2-y}(1-y)}$$

p.821, pr.48

(b) [10 Puan] $xy + z^3x - 2yz = 0$ denklemi z'yi x ve y bağımsız değişkenlerinin fonksiyonu olarak tanımlıyorsa ve kısmi türev de mevcutsa, $\partial z/\partial x$ türevinin $(1,1,1)$ noktasındaki değerini bulunuz.

Solution:

$$\frac{\partial}{\partial x}(xy + z^3x - 2yz) = \frac{\partial}{\partial x}(0)$$

$$(3xz^2 - 2y)\frac{\partial z}{\partial x} - 2y\frac{\partial z}{\partial x} = -y - z^3$$

$$\frac{\partial z}{\partial x} = \frac{-y-z^3}{3xz^2-2y}$$

$$\left.\frac{\partial z}{\partial x}\right|_{(1,1,1)} = \left.\frac{-y-z^3}{3xz^2-2y}\right|_{(1,1,1)} = \frac{-1-1}{3-2} = \boxed{-2}$$

p.822, pr.65

(c) [12 Puan] $z = \sin(xy) + x\sin y$, $x = u^2 + v^2$, $y = uv$ olduğuna göre $u = 0$, $v = 1$ iken $\partial z/\partial u$ türevini bulunuz.

Solution:

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = (y\cos(xy) + \sin y)(2u) + (x\cos(xy) + \cos y)(v)$$

$$= [uv\cos(u^3v + uv^3) + \sin(uv)](2u) + [(u^2+v^2)\cos(uv)]$$

$$\Rightarrow \left.\frac{\partial z}{\partial u}\right|_{(u=0,v=1)} = [uv\cos(u^3v + uv^3) + \sin(uv)](2u) + [(u^2+v^2)\cos(uv)]\Big|_{(u=0,v=1)}$$

$$= 0 + (\cos 0 + \cos 0)(1) = \boxed{2}$$

p.831, pr.36