Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi

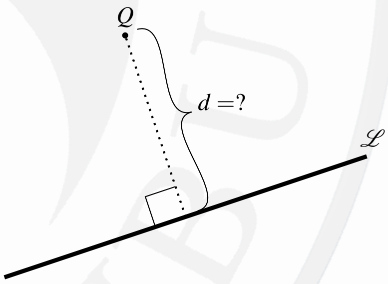
Your Department / Bölüm

- Calculators, cell phones off and away!.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place a box around your answer to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 70 min.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	25	
3	30	
4	25	
Total:	100	

1. (a) 10 Points Find the distance from the point $Q(0,4,1)$ to the line $\mathscr{L}: x=2+t,\ y=2+t,\ z=t$.



Solution: We shall use the distance formula $d=\dfrac{|\vec{PQ}\times\mathbf{v}|}{|\mathbf{v}|}$. Here (by letting $t=0$) $P(2,2,0)$ is a point on $\mathscr{L}$ and $\mathbf{v}=\mathbf{i}+\mathbf{j}+\mathbf{k}$ is a vector that is parallel to $\mathscr{L}$. Now we have $\vec{PQ}=-2\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and so

$$\vec{PQ}\times\mathbf{v}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -2 & 2 & 1\\ 1 & 1 & 1\end{vmatrix}=\mathbf{i}\begin{vmatrix}2 & 1\\ 1 & 1\end{vmatrix}-\mathbf{j}\begin{vmatrix}-2 & 1\\ 1 & 1\end{vmatrix}+\mathbf{k}\begin{vmatrix}-2 & 2\\ 1 & 1\end{vmatrix}$$
$$=\mathbf{i}+3\mathbf{j}-4\mathbf{k}$$

Therefore, we have

$$d=\frac{|\vec{PQ}\times\mathbf{v}|}{|\mathbf{v}|}=\frac{\sqrt{(1)^2+(3)^2+(-4)^2}}{\sqrt{(1)^2+(1)^2+(1)^2}}=\boxed{\frac{\sqrt{26}}{\sqrt{3}}}$$

p.695, pr.37

(b) 10 Points Write the equation of the circle in which the plane through $(1,1,3)$ perpendicular to the z-axis meets the sphere of radius 5 centered at $(0,0,0)$.

Solution: $x^2+y^2+z^2=25$, $z=3\Rightarrow x^2+y^2+(3)^2=25$ so the circle has equation $x^2+y^2=16$ in the $z=3$ plane.

p.695, pr.37

2. (a) 14 Points Find the point in which the line through the origin perpendicular to the plane $2x - y - z = 4$ meets the plane $3x - 5y + 2z = 6$.

Solution: $x = 2t,\ y = -t,\ z = -t$ represents a line containing the origin and perpendicular to the plane $2x - y - z = 4$; this line intersects the plane $3x - 5y + 2z = 6$ when t is the solution of $3(2t) - 5(-t) + 2(-t) = 6 \Rightarrow t = \frac{2}{3} \Rightarrow \left(\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}\right)$ is the point of intersection.

p.748, pr.40

(b) 11 Points Find an equation for the plane through $A(-2,0,-3)$ and $B(1,-2,1)$ that lies parallel to the line through $C(-2,-13/5,26/5)$ and $D(16/5,-13/5,0)$.

Solution: The vector

$$\vec{AB} \times \vec{CD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 4 \\ \frac{26}{5} & 0 & -\frac{26}{5} \end{vmatrix} = \mathbf{i}\begin{vmatrix} -2 & 4 \\ 0 & -\frac{26}{5} \end{vmatrix} - \mathbf{j}\begin{vmatrix} 3 & 4 \\ \frac{26}{5} & -\frac{26}{5} \end{vmatrix} + \mathbf{k}\begin{vmatrix} 3 & -2 \\ \frac{26}{5} & 0 \end{vmatrix}$$
$$= \frac{26}{5}(2\mathbf{i} + 7\mathbf{j} + 2\mathbf{k})$$

is normal to the plane and $A(-2,0,-3)$ lies on the plane $\Rightarrow 2(x+2) + 7(y-0) + 2(z+3) = 0 \Rightarrow \boxed{2x + 7y + 2z + 10 = 0}$ is an equation of the plane.

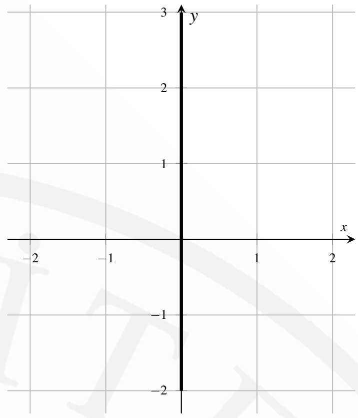
p.749, pr.59

3. (a) 15 Points Find and sketch the domain for $f(x,y) = \sqrt{x^2 + y}$. Sketch the level curve through the point $(2,1)$.

Solution: Since the value of the argument of a square root must be non-negative, this implies that $x^2 + y \geq 0$. Here is the domain of this function.

$$D = \{(x,y) \in \mathbb{R}^2 \mid -x^2 \leq y\}$$

p.687, pr.17(a)



(b) 15 Points Let $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ for $(x,y) \neq (0,0)$. Is it possible to define $f(0,0)$ in a way that makes f continuous at the origin? Why?

Solution: Let $y = kx$. Then

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2} = \lim_{(x,y)\to(0,0)} \frac{x^2 - k^2x^2}{x^2 + k^2x^2} = \frac{1 - k^2}{1 + k^2}$$

which gives different limits for different values of $k \Rightarrow$ the limit does not exist so $f(0,0)$ cannot be defined in a way that makes f continuous at the origin.

p.878, pr.17

4. (a) 12 Points Find all second order partial derivatives of $g(x,y) = y - xy - 8y^3 + \ln(y^2-1)$

Solution:

$$\frac{\partial g}{\partial x} = -y, \quad \frac{\partial g}{\partial y} = 1 - x - 24y^2 + \frac{1}{y^2-1}2y$$

$$\Rightarrow \frac{\partial^2 g}{\partial x^2} = 0, \quad \frac{\partial^2 g}{\partial y^2} = -48y + 2\left[\frac{(y^2-1)(1) - y(2y)}{(y^2-1)^2}\right] = -48y - 2\frac{y^2+1}{(y^2-1)^2}$$

$$\Rightarrow \frac{\partial^2 g}{\partial x \partial y} = \frac{\partial^2 g}{\partial y \partial x} = -1$$

p.878, pr.27

(b) 13 Points Find the limit $\displaystyle\lim_{\substack{(x,y)\to(1,1)\\ x\neq 1}} \frac{xy - y - 2x + 2}{x-1}$.

Solution:

$$\lim_{\substack{(x,y)\to(1,1)\\ x\neq 1}} \frac{xy - y - 2x + 2}{x-1} = \lim_{\substack{(x,y)\to(1,1)\\ x\neq 1}} \frac{y(x-1) - 2(x-1)}{x-1} = \lim_{\substack{(x,y)\to(1,1)\\ x\neq 1}} \frac{\cancel{(x-1)}(y-2)}{\cancel{x-1}} = \lim_{\substack{(x,y)\to(1,1)\\ x\neq 1}} (y-2) = 1 - 2 = \boxed{-1}$$

p.573, pr.18