

Exam

Name _____

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Answer the question.

- 1) Write an "area" word problem for which finding the solution would involve evaluating the double integral 1) _____

$$\int_0^2 \int_{x^2}^{2x} dy dx.$$

Provide an appropriate response.

- 2) Give a geometric interpretation of the triple integral $\int_0^{2\pi} \int_0^{\sqrt{3}} \int_0^{3-r^2} r dz dr d\theta$. 2) _____

Answer the question.

- 3) Write a "volume" word problem for which finding the solution would involve evaluating the double integral 3) _____

$$\int_1^2 \int_4^6 (y+x) dx dy.$$

- 4) Write an "area" word problem for which finding the solution would involve evaluating the double integral 4) _____

$$\int_0^5 \int_0^{x^2} dy dx.$$

- 5) Write a "volume" word problem for which finding the solution would involve evaluating the double integral 5) _____

$$\int_1^2 \int_4^6 (y+x) dy dx.$$

Provide an appropriate response.

- 6) Give a geometric interpretation of the triple integral 6) _____
 $\int_0^{\pi/4} \int_0^\pi \int_0^1 \rho^2 \sin \phi \, d\rho d\phi d\theta$. Describe the three-dimensional region associated with this integral in detail.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Write an equivalent double integral with the order of integration reversed.

- 7) $\int_0^3 \int_0^x dy dx$ 7) _____
A) $\int_0^3 \int_3^y dx dy$ B) $\int_0^3 \int_y^3 dx dy$ C) $\int_0^x \int_0^3 dx dy$ D) $\int_0^3 \int_{-3}^y dx dy$

Find the volume of the indicated region.

- 8) The solid cut from the first octant by the surface $z = 25 - x^2 - y$ 8) _____
A) $\frac{12500}{9}$ B) $\frac{2500}{3}$ C) $\frac{6250}{9}$ D) $\frac{3125}{3}$

Solve the problem.

- 9) Set up the triple integral for the volume of the sphere $\rho = 2$ in rectangular coordinates. 9) _____

A) $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dz dy dx$
B) $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dz dy dx$
C) $8 \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} dz dy dx$
D) $8 \int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dz dy dx$

- 10) Write an iterated triple integral in the order $dy dz dx$ for the volume of the region in the first octant enclosed by the cylinder $x^2 + z^2 = 81$ and the plane $y = 2$. 10) _____

A) $\int_{-9}^9 \int_{-\sqrt{81-z^2}}^{\sqrt{81-z^2}} \int_0^{2-z} dy dz dx$ B) $\int_{-9}^9 \int_{-\sqrt{81-x^2}}^{\sqrt{81-x^2}} \int_0^z dy dz dx$
C) $\int_{-9}^9 \int_{-\sqrt{81-x^2}}^{\sqrt{81-x^2}} \int_0^2 dy dz dx$ D) $\int_{-9}^9 \int_{-\sqrt{81-z^2}}^{\sqrt{81-z^2}} \int_0^2 dy dz dx$

Find the volume of the indicated region.

- 11) The region bounded by the paraboloid $z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 16$ 11) _____
A) $\frac{256}{3}\pi$ B) 128π C) 384π D) $\frac{1024}{3}\pi$

Evaluate the integral.

$$12) \int_1^2 \int_0^{\ln x} e^y dy dx$$

A) $\frac{1}{4}$

B) $\frac{9}{4}$

C) $\frac{1}{2}$

D) $\frac{9}{2}$

12) _____

Solve the problem.

- 13) Find the centroid of the region in the first quadrant bounded by the curve $y = 5 \cos x$ and the axes. 13) _____

A) $\bar{x} = \frac{\pi - 2}{2}, \bar{y} = \frac{5\pi}{8}$

B) $\bar{x} = \frac{5\pi - 10}{2}, \bar{y} = \frac{25\pi}{8}$

C) $\bar{x} = \frac{5\pi}{8}, \bar{y} = \frac{\pi - 2}{2}$

D) $\bar{x} = \frac{\pi}{2}, \bar{y} = \frac{5\pi}{4}$

Determine the order of integration and then evaluate the integral.

$$14) \int_0^1 \int_{x/2}^1 5y^2 \cos xy dy dx$$

14) _____

A) $5(1 - \cos 2)$

B) $\frac{5}{4}(1 - \cos 2)$

C) $\frac{11}{2}(1 - \cos 2)$

D) $\cos 2 - 1$

Find the volume of the indicated region.

- 15) The region enclosed by the cone $z^2 = x^2 + y^2$ between the planes $z = 5$ and $z = 7$ 15) _____

A) $\frac{109}{2}$

B) $\frac{218}{3}$

C) $\frac{218}{3}\pi$

D) $\frac{109}{2}\pi$

Find the average value of the function over the region.

- 16) $f(r, \theta, z) = r^2$ over the region bounded by the cylinder $r = 4$ between the planes $z = -6$ and $z = 6$ 16) _____

A) 8

B) $\frac{8}{3}$

C) $\frac{1}{8}$

D) 768π

Solve the problem.

- 17) Let D be the region that is bounded below by the cone $\phi = \frac{\pi}{4}$ and above by the sphere $\rho = 4$. Set up 17) _____

the triple integral for the volume of D in cylindrical coordinates.

A) $\int_0^{2\pi} \int_0^4 \int_0^{\sqrt{16 - r^2}} r dz dr d\theta$

B) $\int_0^{2\pi} \int_0^4 \int_0^{\sqrt{16 - r^2}} r dz dr d\theta$

C) $\int_0^{2\pi} \int_0^4 \int_r^{\sqrt{16 - r^2}} r dz dr d\theta$

D) $\int_0^{2\pi} \int_0^4 \int_r^{\sqrt{16 - r^2}} r dz dr d\theta$

Evaluate the integral.

$$18) \int_0^{\pi/10} \int_0^{\cos 5x} \sin 5x dy dx$$

18) _____

A) $\frac{1}{20}$

B) $\frac{\pi}{20}$

C) $\frac{1}{10}$

D) $\frac{\pi}{10}$

Find the average value of $F(x, y, z)$ over the given region.

- 19) $F(x, y, z) = x^2 + y^2 + z^2$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 4, y = 9, z = 7$ 19) _____

A) $\frac{178}{3}$

B) $\frac{146}{3}$

C) $\frac{244}{3}$

D) $\frac{308}{3}$

Solve the problem.

- 20) A solid right circular cylinder is bounded by the cylinder $r = 2$ and the planes $z = 0$ and $z = 4$. Find the radius of gyration about the z-axis if the density is $\delta = 3z$. 20) _____

A) 2

B) 192π

C) $\frac{\sqrt{2}}{2}$

D) $\frac{2}{\sqrt{2}}$

Integrate the function f over the given region.

- 21) $f(x, y) = e^{2x} + 3y$ over the rectangle $0 \leq x \leq 1, 0 \leq y \leq 1$ 21) _____

A) $\frac{1}{6}(e^5 - e^3 - e^2 + 1)$

B) $\frac{1}{4}(e^5 - e^3 - e^2 - 1)$

C) $\frac{1}{6}(e^5 - e^3 - e^2 - 1)$

D) $\frac{1}{4}(e^5 - e^3 - e^2 + 1)$

Solve the problem.

- 22) The centers of n nonoverlapping spheres having the same mass are located at the points $(1, i, i^2)$, $1 \leq i \leq n$. Find their center of mass (assume that each sphere has uniform density). 22) _____

A) $\bar{x} = \frac{n}{2}, \bar{y} = \frac{n(n+1)}{6}, \bar{z} = \frac{n(n+1)(2n+1)}{24}$

B) $\bar{x} = 1, \bar{y} = n, \bar{z} = n^2$

C) $\bar{x} = n, \bar{y} = \frac{n(n+1)}{2}, \bar{z} = \frac{n(n+1)(2n+1)}{6}$

D) $\bar{x} = 1, \bar{y} = \frac{n+1}{2}, \bar{z} = \frac{(n+1)(2n+1)}{6}$

- 23) Find the moment of inertia I_z of the rectangular solid of density $\delta(x, y, z) = xyz$ defined by $0 \leq x \leq 6, 0 \leq y \leq 2, 0 \leq z \leq 10$. 23) _____

A) 122400

B) 36000

C) 93600

D) 126000

- 24) Evaluate 24) _____

$$\int_R \int \int (2x + y - z)(z - x + 8y) dV$$

where R is the parallelepiped enclosed by the planes $2x + y - z = 1, 2x + y - z = 3, -x + 8y + z = -2, -x + 8y + z = 5, x - y + 10z = -1, x - y + 10z = 6$.

A) $\frac{5}{4}$

B) $\frac{49}{30}$

C) $\frac{64}{45}$

D) $\frac{9}{5}$

- 25) Find the average height of the paraboloid $z = 4x^2 + 8y^2$ above the disk $x^2 + y^2 \leq 9$ in the xy-plane. 25) _____

A) 45

B) 27

C) 18

D) 36

- 26) Find the center of mass of a thin infinite region in the first quadrant bounded by the coordinate axes and the curve $y = e^{-8x}$ if $\delta(x, y) = xy$. 26) _____

A) $\bar{x} = \frac{1}{8}, \bar{y} = \frac{2}{9}$

B) $\bar{x} = \frac{1}{12}, \bar{y} = \frac{2}{9}$

C) $\bar{x} = \frac{1}{12}, \bar{y} = \frac{8}{27}$

D) $\bar{x} = \frac{1}{8}, \bar{y} = \frac{8}{27}$

- 27) Find the average height of the paraboloid $z = x^2 + y^2$ above the disk $x^2 + y^2 \leq 49$ in the xy-plane. 27) _____
- A) $\frac{49}{2}$ B) $\frac{245}{2}$ C) $\frac{147}{2}$ D) $\frac{49}{3}$

Determine the order of integration and then evaluate the integral.

$$28) \int_0^{9\sqrt{\ln 6}} \int_{y/9}^{\sqrt{\ln 6}} e^{x^2} dx dy \quad 28) _____$$

A) $\frac{63}{2}$ B) 30 C) 24 D) $\frac{45}{2}$

Evaluate the improper integral.

$$29) \int_1^{50} \int_2^{731} \frac{dy dx}{\sqrt{(x-1)(y-2)^2/3}} \quad 29) _____$$

A) 189 B) 315 C) 378 D) 252

Solve the problem.

30) Write $\int_0^4 \int_0^y f(x, y) dx dy + \int_0^4 \int_y^4 f(x, y) dx dy$
as a single iterated integral.

- A) $\int_0^4 \int_0^4 f(x, y) dx dy$
B) $\int_0^4 \int_{-y}^y f(x, y) dx dy$
C) $\int_{-4}^4 \int_{-4}^4 f(x, y) dx dy$
D) $\int_0^4 \int_{-y/4}^{y/4} f(x, y) dx dy$

Use the given transformation to evaluate the integral.

31) $u = 2x + y - z, v = -x + y + z, w = -x + y + 2z;$ 31) _____

$$\int_R \int \int (2x + y - z)(z - x + y)(2z - x + y) dx dy dz,$$

where R is the parallelepiped bounded by the planes $2x + y - z = 7, 2x + y - z = 10, -x + y + z = 7, -x + y + z = 8, -x + y + 2z = 2, -x + y + 2z = 3$
A) $\frac{1275}{2}$ B) $\frac{11475}{16}$ C) $\frac{11475}{8}$ D) $\frac{1275}{8}$

Find the area of the region specified in polar coordinates.

- 32) The region enclosed by the curve $r = 8 \cos 3\theta$ 32) _____
- A) 32π B) $\frac{16}{3}\pi$ C) $\frac{32}{3}\pi$ D) 16π

Solve the problem.

- 33) Evaluate 33) _____

$$\int \int \int \sqrt{1 - \left(\frac{x^2}{16} + \frac{y^2}{49} + \frac{z^2}{9} \right)} dx dy dz,$$

R

where R is the interior of the ellipsoid $\frac{x^2}{16} + \frac{y^2}{49} + \frac{z^2}{9} = 1$. Hint: Let $x = 4u, y = 7v, z = 3w$, and then convert to spherical coordinates.

- A) 21π B) $21\pi^2$ C) 28π D) $28\pi^2$

Find the average value of the function f over the region R.

- 34) $f(x, y) = \frac{1}{xy}$ 34) _____
R: $1 \leq x \leq 5, 1 \leq y \leq 5$
A) $\left[\frac{\ln 5}{5} \right]^2$ B) $\left(\frac{\ln 5}{4} \right)^2$ C) $\frac{\ln 5}{25}$ D) $\frac{\ln 5}{16}$

Solve the problem.

- 35) Write 35) _____

$$\int_0^4 \int_0^y f(x, y) dx dy + \int_4^8 \int_0^{8-y} f(x, y) dx dy$$

as a single iterated integral.

- A) $\int_0^4 \int_4^{8-x} f(x, y) dy dx$
B) $\int_{-4}^4 \int_x^{8-x} f(x, y) dy dx$
C) $\int_0^4 \int_x^{8-x} f(x, y) dy dx$
D) $\int_{-4}^4 \int_{-4}^{8-x} f(x, y) dy dx$

- 36) Find the radius of gyration about the x-axis of the thin semicircular region of constant density $\delta = 4$ 36) _____
bounded by the x-axis and the curve $y = \sqrt{36 - x^2}$.

- A) 9 B) 3π C) 3 D) 9π

Provide an appropriate response.

- 37) What does the graph of the equation $\phi = 0$ look like? 37) _____
A) The yz-plane B) The y-axis C) The xy-plane D) The z-axis

Set up the iterated integral for evaluating

$$\int \int \int_D f(r, \theta, z) dz r dr d\theta$$

over the given region D.

- 38) D is the solid right cylinder whose base is the region in the xy-plane that lies inside the cardioid 38)

$r = 8 - 3 \sin \theta$ and outside the circle $r = 5$, and whose top lies in the plane $z = 5$.

$$\begin{array}{ll} A) \int_0^{2\pi} \int_0^{8-3 \sin \theta} \int_0^5 f(r, \theta, z) dz r dr d\theta \\ B) \int_0^{\pi} \int_5^{8-3 \sin \theta} \int_0^5 f(r, \theta, z) dz r dr d\theta \\ C) \int_0^{\pi} \int_0^{8-3 \sin \theta} \int_0^5 f(r, \theta, z) dz r dr d\theta \\ D) \int_0^{2\pi} \int_5^{8-3 \sin \theta} \int_0^5 f(r, \theta, z) dz r dr d\theta \end{array}$$

Solve the problem.

- 39) The center of a sphere of mass m is located at the point $(2, 0, 0)$, the center of a sphere of mass $2m$ is 39)

located at the point $(0, 6, 0)$, and the center of a sphere of mass $3m$ is located at the point $(0, 0, 3)$.

Find their center of mass (assume that the spheres are nonoverlapping and that each has uniform density).

$$\begin{array}{ll} A) \bar{x} = \frac{1}{2}, \bar{y} = 2, \bar{z} = 1 & B) \bar{x} = \frac{3}{2}, \bar{y} = 2, \bar{z} = \frac{1}{3} \\ C) \bar{x} = \frac{1}{3}, \bar{y} = 2, \bar{z} = \frac{3}{2} & D) \bar{x} = 1, \bar{y} = 2, \bar{z} = \frac{1}{2} \end{array}$$

Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ or $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ (as appropriate) using the given equations.

- 40) $x = 3u \cosh 10v, y = 3u \sinh 10v, z = 2w$ 40)

$$A) 180u \quad B) 600v \quad C) 600u \quad D) 180v$$

Use a CAS integration utility to evaluate the triple integral of the given function over the specified solid region.

- 41) $F(x, y, z) = -2x + 5y^2 + 10z^3$ over the rectangular solid $0 \leq x \leq 9, 0 \leq y \leq 8, 0 \leq z \leq 2$ 41)

$$A) 774 \quad B) 688 \quad C) \frac{2064}{5} \quad D) 516$$

Find the volume of the indicated region.

- 42) The region enclosed by the paraboloids $z = x^2 + y^2 - 4$ and $z = 28 - x^2 - y^2$ 42)

$$A) 1024\pi \quad B) 256\pi \quad C) 768\pi \quad D) 512\pi$$

Solve the problem.

- 43) Evaluate

$$\int_{-8}^8 \int_{-\sqrt{64-x^2}}^{\sqrt{64-x^2}} \int_{\sqrt{x^2+y^2}}^8 dz dy dx$$

by transforming to cylindrical or spherical coordinates.

$$A) 256\pi \quad B) 128\pi \quad C) \frac{512}{3}\pi \quad D) \frac{256}{3}\pi$$

Evaluate the cylindrical coordinate integral.

$$44) \int_0^{8\pi} \int_0^7 \int_0^r (5r^2 + 7z^2) dz r dr d\theta$$

$$A) \frac{739508}{3}\pi \quad B) \frac{1479016}{5}\pi \quad C) \frac{2958032}{9}\pi \quad D) \frac{2958032}{15}\pi$$

Evaluate the integral.

$$45) \int_1^3 \int_0^y x^2 y^2 dx dy$$

$$A) \frac{350}{3} \quad B) \frac{364}{3} \quad C) \frac{350}{9} \quad D) \frac{364}{9}$$

Use the given transformation to evaluate the integral.

$$46) u = -2x + y, v = 6x + y;$$

$$\int \int (y - 2x) dx dy,$$

R where R is the parallelogram bounded by the lines $y = 2x + 9, y = 2x + 10, y = -6x + 7, y = -6x + 9$

$$A) 152 \quad B) \frac{19}{4} \quad C) 304 \quad D) \frac{19}{8}$$

Find the volume of the indicated region.

- 47) The region that lies under the plane $z = 2x + 7y$ and over the triangle with vertices at $(1, 1), (2, 1)$, and $(1, 2)$ 47)

$$A) \frac{32}{3} \quad B) 6 \quad C) \frac{22}{3} \quad D) 12$$

Solve the problem.

- 48) Let D be the region bounded below by the xy-plane, above by the sphere $x^2 + y^2 + z^2 = 100$, and on the sides by the cylinder $x^2 + y^2 = 9$. Set up the triple integral in cylindrical coordinates that gives the volume of D using the order of integration $d\theta dz dr$ 48)

$$\begin{array}{ll} A) \int_0^3 \int_0^{\sqrt{100-r^2}} \int_0^{2\pi} r d\theta dz dr & B) \int_0^{2\pi} \int_0^{\sqrt{100-r^2}} \int_0^{10} r d\theta dz dr \\ C) \int_0^{2\pi} \int_0^{\sqrt{100-r^2}} \int_0^3 r d\theta dz dr & D) \int_0^{10} \int_0^{\sqrt{9-r^2}} \int_0^{2\pi} r d\theta dz dr \end{array}$$

- 49) A cubical box in the first octant is bounded by the coordinate planes and the planes $x = 5$, $y = 6$, $z = 7$. The box is filled with a liquid of density $\delta(x, y, z) = x + y + z + 1$. Find the work done by (constant) gravity g in moving the liquid in the container down to the the xy -plane.
- A) $\frac{16415}{2}g$ B) $\frac{32830}{3}g$ C) $\frac{49245}{4}g$ D) $6566g$

Provide an appropriate response.

- 50) What does the graph of the equation $\phi = \frac{\pi}{3}$ look like?
- 50) _____
- A) A cylinder B) A cone C) A plane D) A line

Find the area of the region specified in polar coordinates.

- 51) The region inside both $r = 10 \sin \theta$ and $r = 10 \cos \theta$
- A) $25(\pi - 1)$ B) $25(\pi - 2)$ C) $\frac{25}{2}(\pi - 1)$ D) $\frac{25}{2}(\pi - 2)$

Solve the problem.

- 52) Find the center of mass of the thin semicircular region of constant density $\delta = 4$ bounded by the x -axis and the curve $y = \sqrt{121 - x^2}$.
- A) $\bar{x} = 0, \bar{y} = \frac{88}{3\pi}$ B) $\bar{x} = 0, \bar{y} = \frac{44}{3\pi}$ C) $\bar{x} = 0, \bar{y} = \frac{22}{3\pi}$ D) $\bar{x} = 0, \bar{y} = \frac{11}{3\pi}$

Evaluate the integral.

- 53) $\int_0^\pi \int_0^\pi \int_0^{10 \sin \phi} p^2 \sin \phi \, dp \, d\theta \, d\phi$
- 53) _____
- A) $\frac{500}{3}\pi^2$ B) $125\pi^2$ C) $\frac{1000}{9}\pi^2$ D) $\frac{1000}{3}\pi^2$

Determine the order of integration and then evaluate the integral.

- 54) $\int_0^{72} \int_{y/9}^8 \frac{\cos x}{x} \, dx \, dy$
- 54) _____
- A) $8 \sin 9$ B) $9 \sin 8$ C) $9 \cos 8$ D) $8 \cos 9$

Evaluate the cylindrical coordinate integral.

- 55) $\int_7^9 \int_8^{10} \int_0^{4r} z \, dz \, r \, dr \, d\theta$
- 55) _____
- A) 23616 B) 15744 C) 236160 D) 129888

Solve the problem.

- 56) Find the mass of the region of density $\delta(x, y, z) = \frac{1}{64 - x^2 - y^2}$ bounded by the hemisphere $z = \sqrt{64 - x^2 - y^2}$ and the xy -plane.
- A) 64π B) 8π C) 16π D) 32π

49) _____

57) What domain D in \mathbb{R}^3 minimizes the value of the integral

$$\int \int \int_D \left(1 - \frac{x^2}{9} - \frac{y^2}{9} - \frac{z^2}{9}\right) dV$$

- A) $D = \mathbb{R}^3$
 B) D = the boundary of the sphere $x^2 + y^2 + z^2 = 9$.
 C) D = the boundary and interior of the sphere $x^2 + y^2 + z^2 = 9$.
 D) No such maximum domain exists.

57) _____

Provide an appropriate response.

- 58) What does the graph of the equation $\theta = 0$ look like?

- A) The yz -plane B) The x -axis C) The y -axis D) The xz -plane

Solve the problem.

- 59) Find the mass of the rectangular solid of density $\delta(x, y, z) = xyz$ defined by $0 \leq x \leq 3$, $0 \leq y \leq 7$,

$$0 \leq z \leq 5.$$

A) $\frac{3675}{4}$ B) 735 C) $\frac{11025}{8}$ D) $\frac{3675}{2}$

- 60) Let R be a thin triangular region cut off from the first quadrant by the line $x + y = c$, where $c > 0$. If the density of R is given by $\delta(x, y) = 4x + 2y$, for what value of c will the mass of R be the same as that of an identical region with constant density $\delta(x, y) = 1$?

A) $\frac{1}{2}$ B) $\frac{3}{10}$ C) $\frac{1}{4}$ D) $\frac{3}{8}$

Use the given transformation to evaluate the integral.

- 61) $u = x + y, v = -2x + y;$

$$\int \int_R (5x - 5y) \, dx \, dy,$$

where R is the parallelogram bounded by the lines $y = -x + 1$, $y = -x + 4$, $y = 2x + 2$, $y = 2x + 5$

A) $-\frac{75}{4}$ B) $-\frac{75}{2}$ C) $-\frac{95}{2}$ D) $-\frac{95}{4}$

Find the average value of the function over the region.

- 62) $f(r, \theta, z) = r$ over the region bounded by the cylinder $r = 6$ between the planes $z = -3$ and $z = 3$

A) 9 B) 864π C) 6 D) 4

Use the given transformation to evaluate the integral.

- 63) $u = -2x + y, v = 9x + y;$

$$\int \int_R (y - 2x)(9x + y) \, dx \, dy,$$

where R is the parallelogram bounded by the lines $y = 2x + 6$, $y = 2x + 7$, $y = -9x + 5$, $y = -9x + 6$

A) $\frac{1573}{2}$ B) $\frac{13}{2}$ C) $\frac{1573}{4}$ D) $\frac{13}{4}$

Evaluate the cylindrical coordinate integral.

$$64) \int_0^{\pi/2} \int_9^{18} \int_{1/r^2}^{1/r} \cos \theta \, dz \, r \, dr \, d\theta$$

- A) $18 - \ln 2$
B) $9\pi + \ln 2$
C) $9 - \ln 2$
D) $18\pi + \ln 2$

64) _____

Find the volume of the indicated region.

65) The region bounded by the paraboloid $z = x^2 + y^2$, the cylinder $x^2 + y^2 = 36$, and the xy-plane
 A) 324π
B) 216π
C) 648π
D) 432π

65) _____

Determine the order of integration and then evaluate the integral.

$$66) \int_0^1 \int_{5y}^5 x^4 e^{x^2 y} \, dx \, dy$$

- A) $\frac{5e^{25} - 130}{3}$
B) $5e^{25} - 130$
C) $e^{25} - 1$
D) $e^{25} - \frac{130}{3}$

66) _____

Use the given transformation to evaluate the integral.

$$67) u = x + y, v = -2x + y;$$

$$\int \int_R -5x \, dx \, dy,$$

where R is the parallelogram bounded by the lines $y = -x + 1$, $y = -x + 4$, $y = 2x + 2$, $y = 2x + 5$
 A) 10
B) -10
C) -5
D) 5

67) _____

Determine the order of integration and then evaluate the integral.

$$68) \int_0^2 \int_{\sqrt{x/2}}^1 ey^3 \, dy \, dx$$

- A) $\frac{2}{3}(2e - 1)$
B) $\frac{2}{3}(e - 1)$
C) $\frac{1}{3}(e - 1)$
D) $\frac{1}{3}((2e - 1))$

68) _____

Solve the problem.

69) Find the center of mass of the solid enclosed between the cone with equation $z = \frac{4}{3}\sqrt{x^2 + y^2}$ and the plane with equation $z = 4$ if the density at any point is proportional to the distance from that point to the axis of the cone.

- A) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{8}{3}\right)$
 B) $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 3)$
 C) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{5}{2}\right)$
 D) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{16}{5}\right)$

69) _____

70) Evaluate

$$\int \int \int_R (5x + y - z) \, dV$$

where R is the parallelepiped enclosed by the planes $5x + y - z = 1$, $5x + y - z = 3$, $-x + 6y + z = -2$, $-x + 6y + z = 5$, $x - y + 10z = -1$, $x - y + 10z = 6$.

- A) $\frac{196}{321}$
B) $\frac{75}{107}$
C) $\frac{176}{321}$
D) $\frac{72}{107}$

70) _____

Determine the order of integration and then evaluate the integral.

$$71) \int_0^{56} \int_{y/8}^7 e^{x^2} \, dx \, dy$$

- A) $4e^{49}$
B) $4(e^{49} - 1)$
C) $2(e^{49} - 1)$
D) $2e^{49}$

71) _____

Solve the problem.

72) Let D be the region bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $z = \sqrt{81 - x^2 - y^2}$. Set up the triple integral in cylindrical coordinates that gives the volume of D using the order of integration $dr \, dz \, d\theta$.

- A) $\int_0^{2\pi} \int_0^{9/\sqrt{2}} \int_0^z r \, dr \, dz \, d\theta + \int_0^{2\pi} \int_{9/\sqrt{2}}^9 \int_0^{\sqrt{162 - z^2}} r \, dr \, dz \, d\theta$
 B) $\int_0^{2\pi} \int_0^{9/2} \int_0^z r \, dr \, dz \, d\theta + \int_0^{2\pi} \int_{9/2}^9 \int_0^{\sqrt{81 - z^2}} r \, dr \, dz \, d\theta$
 C) $\int_0^{2\pi} \int_0^{9/\sqrt{2}} \int_0^z r \, dr \, dz \, d\theta + \int_0^{2\pi} \int_{9/\sqrt{2}}^9 \int_0^{\sqrt{81 - z^2}} r \, dr \, dz \, d\theta$
 D) $\int_0^{2\pi} \int_0^{9/2} \int_0^z r \, dr \, dz \, d\theta + \int_0^{2\pi} \int_{9/2}^9 \int_0^{\sqrt{162 - z^2}} r \, dr \, dz \, d\theta$

72) _____

Use a spherical coordinate integral to find the volume of the given solid.

- 73) The solid between the sphere $\rho = \cos \phi$ and the hemisphere $\rho = 5$, $z \geq 0$
 A) $\frac{499}{6}\pi$
B) $\frac{499}{4}\pi$
C) $\frac{499}{2}\pi$
D) $\frac{499}{3}\pi$

73) _____

Solve the problem.

- 74) Evaluate

$$\int_0^\infty \frac{e^{-2x} - e^{-4x}}{x} \, dx$$

by writing the integrand as an integral.

- A) $\ln 2$
B) $\frac{\ln 4}{\ln 2}$
C) $\frac{\ln 2}{\ln 4}$
D) $\ln \frac{1}{2}$

74) _____

Use the given transformation to evaluate the integral.

- 75) $u = x + y, v = -2x + y;$

$$\int \int_R 5y \, dx \, dy,$$

75) _____

where R is the parallelogram bounded by the lines $y = -x + 1$, $y = -x + 4$, $y = 2x + 2$, $y = 2x + 5$

- A) $\frac{65}{4}$
B) $\frac{85}{4}$
C) $\frac{85}{2}$
D) $\frac{65}{2}$

Solve the problem.

- 76) Find the mass of a thin plate covering the region inside the curve $r = 8 + 2 \cos \theta$ if $\delta(x, y) = 6$.

- A) 98π
B) 600π
C) 264π
D) 396π

76) _____

Use the given transformation to evaluate the integral.

77) $x = 5u, y = 7v, z = 4w;$

$$\int \int \int_R x^2 y^2 dx dy dz,$$

where R is the interior of the ellipsoid $\frac{x^2}{25} + \frac{y^2}{49} + \frac{z^2}{16} = 1$

A) 7π

B) $\frac{28}{3}\pi$

C) $\frac{16}{3}\pi$

D) $\frac{28}{5}\pi$

77) _____

Solve the problem.

- 78) Write an iterated triple integral in the order $dx dy dz$ for the volume of the rectangular solid in the first octant bounded by the planes $x = 9, y = 4$, and $z = 5$.

A) $\int_0^9 \int_0^{4-x} \int_0^{5-y-x} dx dy dz$

B) $\int_0^9 \int_0^4 \int_0^5 dx dy dz$

C) $\int_0^5 \int_0^{4-x} \int_0^{9-y-x} dx dy dz$

D) $\int_0^5 \int_0^4 \int_0^9 dx dy dz$

78) _____

- 79) Find the centroid of the first-octant portion of a solid ball of radius 5 centered at the origin.

A) $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{25}{8}, \frac{25}{8}, \frac{25}{8}\right)$

B) $(\bar{x}, \bar{y}, \bar{z}) = (3, 3, 3)$

C) $(\bar{x}, \bar{y}, \bar{z}) = (2, 2, 2)$

D) $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{15}{8}, \frac{15}{8}, \frac{15}{8}\right)$

79) _____

Use the given transformation to evaluate the integral.

80) $u = x + y, v = x - y;$

$$\int \int_R (x+y) \sin(x-y) dx dy,$$

where R is the parallelogram bounded by the lines $y = x, y = x - 8, y = -x, y = -x + 2$

A) $1(1 + \cos 8)$

B) $2(1 - \cos 8)$

C) $2(1 + \cos 8)$

D) $1(1 - \cos 8)$

80) _____

Solve the problem.

- 81) Evaluate

$$\int_0^\infty \frac{\sin x}{x} dx$$

by integrating

$$\int_0^\infty \int_0^\infty e^{-xy} \sin x dA.$$

A) $\frac{\pi}{2}$

B) $\frac{\pi}{4}$

C) $\frac{\pi}{3}$

D) $\frac{\pi}{6}$

81) _____

Use a spherical coordinate integral to find the volume of the given solid.

- 82) The solid bounded below by the sphere $\rho = 7 \cos \phi$ and above by the cone $z = \sqrt{x^2 + y^2}$

A) $\frac{343}{25}\pi$

B) $\frac{343}{20}\pi$

C) $\frac{343}{24}\pi$

D) $\frac{343}{18}\pi$

82) _____

Evaluate the improper integral.

$$83) \int_0^\infty \int_0^\infty \frac{dx dy}{(x^2 + 4)(y^2 + 9)}$$

A) $\frac{\pi}{24}$

B) $\frac{\pi}{36}$

C) $\frac{\pi^2}{24}$

D) $\frac{\pi^2}{36}$

83) _____

Change the Cartesian integral to an equivalent polar integral, and then evaluate.

$$84) \int_{-7}^7 \int_{-\sqrt{49-y^2}}^{\sqrt{49-y^2}} (x^2 + y^2)^{5/2} dx dy$$

A) 117649π

B) 235298π

C) 33614π

D) 16807π

84) _____

Solve the problem.

- 85) Let R be a thin triangular region cut off from the first quadrant by the line $x + y = 13$. If the density of R is given by $\delta(x, y) = ax + \frac{y}{a}$ where $a > 0$, for what value of a will the moment of inertia of R about the y-axis be a minimum?

A) $\sqrt{13}$

B) $\sqrt{3}$

C) $\frac{1}{\sqrt{13}}$

D) $\frac{1}{\sqrt{3}}$

85) _____

- 86) A rectangular solid is defined by $0 \leq x \leq 7, 0 \leq y \leq 3, 0 \leq z \leq 4$. An adjacent rectangular solid is defined by $7 \leq x \leq 10, 0 \leq y \leq 8, 0 \leq z \leq 9$. If the solids have the same uniform density, find their center of mass.

A) $\bar{x} = \frac{103}{50}, \bar{y} = \frac{597}{200}, \bar{z} = \frac{19}{5}$

B) $\bar{x} = \frac{103}{75}, \bar{y} = \frac{199}{100}, \bar{z} = \frac{38}{15}$

C) $\bar{x} = \frac{103}{150}, \bar{y} = \frac{199}{200}, \bar{z} = \frac{19}{15}$

D) $\bar{x} = \frac{103}{100}, \bar{y} = \frac{597}{400}, \bar{z} = \frac{19}{10}$

86) _____

Integrate the function f over the given region.

- 87) $f(x, y) = 4x \sin xy$ over the rectangle $0 \leq x \leq \pi, 0 \leq y \leq 1$

A) π

B) $\frac{\pi}{4}$

C) $4\pi - 4$

D) 4π

87) _____

Evaluate the integral.

$$88) \int_5^{10} \int_0^{\pi/2} \int_0^{\pi/2} (\rho \sin \phi)^2 d\theta d\phi d\rho$$

A) $\frac{875}{24}\pi^2$

B) $\frac{875}{18}\pi$

C) $\frac{875}{24}\pi$

D) $\frac{875}{18}\pi^2$

88) _____

Solve the problem.

- 89) Let D be the region bounded below by the xy-plane, on the side by the cylinder $r = 8 \cos \theta$, and on top by the paraboloid $z = 6r^2$. Set up the triple integral in cylindrical coordinates that gives the volume of D using the order of integration $dz\ dr\ d\theta$. 89) _____

A) $\int_0^{\pi/2} \int_0^{8 \cos \theta} \int_0^{6r^2} r\ dz\ dr\ d\theta$

B) $\int_0^{\pi} \int_0^{8 \cos \theta} \int_0^{6r^2} r\ dz\ dr\ d\theta$

C) $\int_0^{\pi/4} \int_0^{8 \cos \theta} \int_0^{6r^2} r\ dz\ dr\ d\theta$

D) $\int_0^{2\pi} \int_0^{8 \cos \theta} \int_0^{6r^2} r\ dz\ dr\ d\theta$

- 90) Find the mass of a sphere of radius 8 if $\delta = kp$, k a constant.

A) $512k\pi$

B) $4096k\pi$

C) $64k\pi$

D) $8192k\pi$

90) _____

Evaluate the integral.

- 91) $\int_8^9 \int_{6\pi}^{8\pi} \int_0^z \frac{r}{z}\ dr\ d\theta\ dz$ 91) _____

A) $\frac{17}{2}\pi$

B) $\frac{17}{3}\pi$

C) 17π

D) $\frac{34}{3}\pi$

Solve the problem.

- 92) Find the average height of the paraboloid $z = x^2 + y^2$ above the annular region $49 \leq x^2 + y^2 \leq 64$ in the xy-plane. 92) _____

A) $\frac{177}{2}$

B) $\frac{113}{2}$

C) 81

D) $\frac{241}{2}$

- 93) Find the radius of gyration R_L of a tetrahedron of constant density bounded by the coordinate planes and the plane $\frac{x}{9} + \frac{y}{10} + \frac{z}{5} = 1$, where L is the line through the points $(0, 1, 3)$, $(0, 3, 3)$. 93) _____

A) 1470

B) 1050

C) $\frac{1815}{2}$

D) $\frac{3315}{2}$

Provide an appropriate response.

- 94) What form do planes perpendicular to the z-axis have in spherical coordinates? 94) _____

A) $\rho = a \sec \phi$

B) $\rho = a \csc \phi$

C) $\rho = a \cos \phi$

D) $\rho = a \sin \phi$

Find the area of the region specified in polar coordinates.

- 95) One petal of the rose curve $r = 6 \cos 3\theta$ 95) _____

A) 3π

B) 9π

C) 18π

D) 6π

Change the Cartesian integral to an equivalent polar integral, and then evaluate.

- 96) $\int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} dx\ dy$ 96) _____

A) 2π

B) 8π

C) 4π

D) 16π

Find the area of the region specified by the integral(s).

97) $\int_0^7 \int_{7-x}^{e^x} dy\ dx$

A) $e^7 - 25.5$

B) $e^7 - 17.33$

C) $e^7 - 33.67$

D) $e^7 - 13.25$

97) _____

Integrate the function f over the given region.

- 98) $f(x, y) = y^2 e^{x^4}$ over the triangular region in the first quadrant bounded by the lines $x = y/\sqrt{a}$, $x = 1$, $y = 0$, and $y = 7$ 98) _____

A) $\frac{343}{3}(e-1)$

B) $\frac{343}{12}e$

C) $\frac{343}{12}(e-1)$

D) $343(e+1)$

Solve the problem.

- 99) Find the centroid of the region bounded above by the sphere $x^2 + y^2 + z^2 = 49$ and below by the plane $z = 3$, if the density is constant. 99) _____

A) $x = 0, y = 0, z = \frac{507}{68}$

B) $x = 0, y = 0, z = \frac{75}{17}$

C) $x = 0, y = 0, z = \frac{51}{4}$

D) $x = 0, y = 0, z = \frac{60}{17}$

Find the area of the region specified by the integral(s).

100) $\int_0^{10} \int_0^{\ln x} dy\ dx$

A) $\ln 10$

B) $10 \ln 10$

C) $10(\ln 10 - 1)$

D) $\ln 10 + 1$

100) _____

Use a spherical coordinate integral to find the volume of the given solid.

- 101) The solid bounded below by the xy-plane, on the sides by the sphere $\rho = 9$, and above by the cone 101) _____

$\phi = \frac{\pi}{3}$

A) $\frac{729}{4}\pi$

B) 243π

C) $\frac{729}{2}\pi$

D) $\frac{729}{5}\pi$

Evaluate the integral.

102) $\int_6^9 \int_{4\pi}^{6\pi} \int_0^r rz\ dr\ d\theta\ dz$

A) $-7.2704e+12\pi^2$

B) $-\frac{1.8176e+13}{3}\pi^2$

C) $-6.816e+12\pi^2$

D) $-9.088e+12\pi^2$

102) _____

Evaluate the improper integral.

103) $\int_0^{\infty} \int_0^{\infty} \frac{dx\ dy}{(x+4)^2(y+9)^2}$

A) $\frac{1}{144}$

B) $\frac{1}{108}$

C) $\frac{1}{72}$

D) $\frac{1}{36}$

103) _____

Solve the problem.

- 104) Find the moment of inertia about the y -axis of the region enclosed by the curve $r = 8 + 2 \sin \theta$ if

$$\delta(x, y) = \frac{1}{r}$$

A) $\frac{920}{3}\pi$ B) 136π C) $\frac{536}{3}\pi$ D) $\frac{664}{3}\pi$

104) _____

Find the volume of the indicated region.

- 105) The region that lies under the paraboloid $z = x^2 + y^2$ and above the triangle enclosed by the lines

$$x = 2, y = 0, \text{ and } y = 3x$$

- A) 24 B) 48 C) 12 D) 68

105) _____

Integrate the function f over the given region.

- 106) $f(x, y) = \frac{xy}{\ln y}$ over the square $6 \leq x \leq 8, 6 \leq y \leq 8$

A) $\left(\ln \frac{3}{4}\right)^2$ B) $\left(\frac{\ln 6}{\ln 8}\right)^2$ C) $\frac{\ln 6}{\ln 8}$ D) $\ln \frac{3}{4}$

106) _____

Evaluate the integral.

- 107) $\int_{-4}^{-5} \int_7^8 xy^2 dx dy$

A) $\frac{6893}{6}$ B) $\frac{305}{2}$ C) $-\frac{6893}{6}$ D) $-\frac{305}{2}$

107) _____

Solve the problem.

- 108) The centers of four nonoverlapping spheres having the same mass are located at the points $(2, 9, 8)$, $(6, 2, 9)$, $(8, 6, 2)$, and $(9, 8, 6)$. Find their center of mass (assume that each sphere has uniform density).

A) $\bar{x} = \frac{25}{2}, \bar{y} = \frac{25}{2}, \bar{z} = \frac{25}{2}$
 B) $\bar{x} = \frac{25}{4}, \bar{y} = \frac{25}{4}, \bar{z} = \frac{25}{4}$
 C) $\bar{x} = \frac{25}{3}, \bar{y} = \frac{25}{3}, \bar{z} = \frac{25}{3}$
 D) $\bar{x} = \frac{25}{6}, \bar{y} = \frac{25}{6}, \bar{z} = \frac{25}{6}$

108) _____

Use a spherical coordinate integral to find the volume of the given solid.

- 109) The solid enclosed by the cardioid of revolution $\rho = 9 + 2 \cos \phi$

A) 255π B) 1020π C) 510π D) 2040π

109) _____

Write an equivalent double integral with the order of integration reversed.

- 110) $\int_0^1 \int_0^{\tan^{-1} x} dy dx$

A) $\int_0^{\pi/4} \int_0^1 \tan^{-1} y dx dy$
 B) $\int_0^{\pi/4} \int_{\tan^{-1} y}^{\pi/2} dx dy$
 C) $\int_0^{\pi/4} \int_0^{\pi/2} dx dy$
 D) $\int_0^{\pi/4} \int_0^1 dx dy$

110) _____

Change the Cartesian integral to an equivalent polar integral, and then evaluate.

- 111) $\int_0^4 \int_0^{\sqrt{16-x^2}} e^{-(x^2+y^2)} dy dx$

A) $\frac{\pi(1-e^{-16})}{2}$ B) $\frac{\pi(1-e^{-16})}{4}$ C) $\frac{\pi(1+e^{-16})}{2}$ D) $\frac{\pi(1+e^{-16})}{4}$

111) _____

Provide an appropriate response.

- 112) What does the graph of the equation $\rho = \sec \phi$ look like?

A) A line B) A cylinder C) A sphere D) A plane

112) _____

Solve the problem.

- 113) Write an iterated triple integral in the order $dz dy dx$ for the volume of the tetrahedron cut from the first octant by the plane $\frac{x}{6} + \frac{y}{10} + \frac{z}{3} = 1$.

A) $\int_0^6 \int_0^{1-y/10} \int_0^{1-x/6-y/10} dz dy dx$
 B) $\int_0^6 \int_0^{6(1-y/10)} \int_0^{3(1-x/6-y/10)} dz dy dx$
 C) $\int_0^6 \int_0^{10(1-x/6)} \int_0^{3(1-x/6-y/10)} dz dy dx$
 D) $\int_0^6 \int_0^{1-x/6} \int_0^{1-x/6-y/10} dz dy dx$

113) _____

Use a CAS integration utility to evaluate the triple integral of the given function over the specified solid region.

- 114) $F(x, y, z) = 2x + 9y + 4z$ over the tetrahedron bounded by the coordinate planes and the plane $x + y + z = 1$

A) $\frac{5}{6}$ B) $\frac{5}{8}$ C) $\frac{5}{12}$ D) $\frac{5}{9}$

114) _____

Evaluate the spherical coordinate integral.

- 115) $\int_0^{\pi/2} \int_0^{\pi/3} \int_{\sec \phi}^5 \rho^3 \sin \phi d\rho d\phi d\theta$

A) $\frac{1861}{24}\pi$ B) $\frac{467}{6}\pi$ C) $\frac{467}{12}\pi$ D) $\frac{1861}{48}\pi$

115) _____

Solve the problem.

- 116) Integrate $f(x, y) = \frac{\ln(x^2+y^2)}{\sqrt{x^2+y^2}}$ over the region $0 \leq x^2 + y^2 \leq 9$.

A) $12\pi(2 \ln 3 - 1)$ B) $6\pi(\ln 3 - 1)$ C) $12\pi(\ln 3 - 1)$ D) $6\pi(2 \ln 3 - 1)$

116) _____

Find the average value of the function f over the region R .

- 117) $f(x, y) = 5x + 8y$
R is the triangle with vertices $(0, 0)$, $(2, 0)$, and $(0, 4)$.

A) $\frac{14}{3}$ B) $\frac{23}{3}$ C) 14 D) $\frac{10}{3}$

117) _____

Find the volume of the indicated region.

- 118) The region in the first octant bounded by the coordinate planes and the surface $z = 4 - x^2 - y$

A) $\frac{32}{3}$ B) 8 C) $\frac{64}{9}$ D) $\frac{128}{15}$

118) _____

Solve the problem.

- 119) A cubical box in the first octant is bounded by the coordinate planes and the planes $x = 1$, $y = 1$, $z = 1$. The box is filled with a liquid of density $\delta(x, y, z) = 3x + 2y + 10z + 1$. Find the work done by (constant) gravity g in moving the liquid in the container down to the the xy -plane.

A) $\frac{71}{12}g$ B) $\frac{21}{4}g$ C) $\frac{61}{12}g$ D) $\frac{67}{12}g$

119) _____

Find the area of the region specified by the integral(s).

- 120) $\int_{\pi/4}^{\pi/2} \int_{4 \cos x}^{4 \sin x} dy dx$

A) 8 B) 1 C) 4 D) $4\sqrt{2}$

120) _____

Use a spherical coordinate integral to find the volume of the given solid.

- 121) The solid between the spheres $\rho = \cos \phi$ and $\rho = 10$

A) $\frac{133}{2}\pi$ B) $\frac{2000}{3}\pi$ C) $\frac{4000}{3}\pi$ D) $\frac{7999}{6}\pi$

121) _____

Find the average value of $F(x, y, z)$ over the given region.

- 122) $F(x, y, z) = xyz$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 9$, $y = 9$, $z = 9$

A) $\frac{243}{2}$ B) $\frac{243}{4}$ C) $\frac{729}{8}$ D) 243

122) _____

Solve the problem.

- 123) What region in the xy -plane minimizes the value of

$$\int \int_R (49x^2 + 16y^2 - 784) dA ?$$

- A) The ellipse $49x^2 + 16y^2 = 1$ B) The ellipse $\frac{x^2}{16} + \frac{y^2}{49} = 1$
C) The ellipse $\frac{x^2}{49} + \frac{y^2}{16} = 1$ D) The ellipse $16x^2 + 49y^2 = 784$

123) _____

Evaluate the integral.

$$124) \int_0^5 \int_0^{\sqrt{25-y^2}} \int_0^{5x+10y} dz dx dy$$

A) 25 B) 125 C) 3125 D) 625

124) _____

Set up the iterated integral for evaluating

$$\int \int \int_D f(r, \theta, z) dz r dr d\theta$$

over the given region D .

- 125) D is the solid right cylinder whose base is the region between the circles $r = 5 \cos \theta$ and $r = 6 \cos \theta$, and whose top lies in the plane $z = 6 - y$.

- A) $\int_0^\pi \int_{5 \cos \theta}^{6 \cos \theta} \int_0^{6 - \sin \theta} f(r, \theta, z) dz r dr d\theta$
B) $\int_0^\pi \int_{5 \cos \theta}^{6 \cos \theta} \int_0^{6 - r \sin \theta} f(r, \theta, z) dz r dr d\theta$
C) $\int_0^{2\pi} \int_{5 \cos \theta}^{6 \cos \theta} \int_0^{6 - \sin \theta} f(r, \theta, z) dz r dr d\theta$
D) $\int_0^{2\pi} \int_{5 \cos \theta}^{6 \cos \theta} \int_0^{6 - r \sin \theta} f(r, \theta, z) dz r dr d\theta$

Write an equivalent double integral with the order of integration reversed.

$$126) \int_0^6 \int_{y^2}^{36} 3y dy dx$$

- A) $\int_0^{36} \int_0^{\sqrt{x}} 3y dy dx$ B) $\int_0^6 \int_0^{\sqrt{x}} 3y dy dx$
C) $\int_0^{36} \int_0^{\sqrt{x}} 3y dy dx$ D) $\int_0^6 \int_0^{\sqrt{x}} 3y dy dx$

126) _____

Provide an appropriate response.

- 127) What form do planes perpendicular to the y -axis have in spherical coordinates?

A) $\rho = a \csc \theta$ B) $\rho = a \csc \phi$ C) $\rho = a \sin \phi \sin \theta$ D) $\rho = a \csc \phi \csc \theta$

127) _____

Evaluate the integral.

$$128) \int_0^1 \int_0^{v^4} v du dv$$

A) $\frac{2}{5}$ B) $\frac{2}{6}$ C) $\frac{1}{6}$ D) $\frac{1}{5}$

128) _____

Solve the problem.

- 129) The centers of three nonoverlapping spheres having the same mass are located at the points $(5, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 4)$. Find their center of mass (assume that each sphere has uniform density).

A) $\bar{x} = \frac{5}{4}$, $\bar{y} = \frac{1}{2}$, $\bar{z} = 1$
 B) $\bar{x} = \frac{10}{3}$, $\bar{y} = \frac{4}{3}$, $\bar{z} = \frac{8}{3}$
 C) $\bar{x} = \frac{5}{3}$, $\bar{y} = \frac{2}{3}$, $\bar{z} = \frac{4}{3}$
 D) $\bar{x} = \frac{5}{2}$, $\bar{y} = 1$, $\bar{z} = 2$

129) _____

- 130) Write

$$\int_0^8 \int_0^y f(x, y) dx dy + \int_8^{16} \int_0^{16-y} f(x, y) dx dy + \int_0^8 \int_y^8 f(x, y) dx dy + \int_0^8 \int_8^{16-y} f(x, y) dx dy$$

130) _____

as a single iterated integral with the order of integration reversed.

A) $\int_8^{16} \int_8^{16-x} f(x, y) dy dx$
 B) $\int_8^{16} \int_0^{16-x} f(x, y) dy dx$
 C) $\int_0^{16} \int_8^{16-x} f(x, y) dy dx$
 D) $\int_0^{16} \int_0^{16-x} f(x, y) dy dx$

Change the Cartesian integral to an equivalent polar integral, and then evaluate.

131) $\int_{-5}^5 \int_0^{\sqrt{25-x^2}} dy dx$
 A) $\frac{5\pi}{2}$
 B) $\frac{\pi}{2}$
 C) $\frac{25\pi}{2}$
 D) $\frac{125\pi}{2}$

131) _____

Solve the problem.

- 132) Find the radius of gyration R_X of the rectangular solid of density $\delta(x, y, z) = xyz$ defined by $0 \leq x \leq 6$, $0 \leq y \leq 7$, $0 \leq z \leq 10$.

A) $\sqrt{\frac{149}{3}}$
 B) $\sqrt{\frac{149}{2}}$
 C) $\sqrt{\frac{149}{6}}$
 D) $\sqrt{\frac{149}{4}}$

132) _____

- 133) Let V_t be the volume of the tetrahedron bounded by the coordinate planes and the plane

$$\frac{x}{4} + \frac{y}{2} + \frac{z}{5} = 1, \text{ and let } V_e \text{ be the volume of the ellipsoid } \frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{25} = 1. \text{ Find the quotient } \frac{V_e}{V_t}.$$

A) 8π
 B) $\frac{2\pi}{3}$
 C) $\frac{4\pi}{3}$
 D) 4π

133) _____

- 134) Solve for a :

$$\int_0^a \int_0^1 \int_0^{10(1-x/a)} dz dy dx = 15$$

A) $a = \frac{5}{2}$
 B) $a = 2$
 C) $a = \frac{18}{5}$
 D) $a = 3$

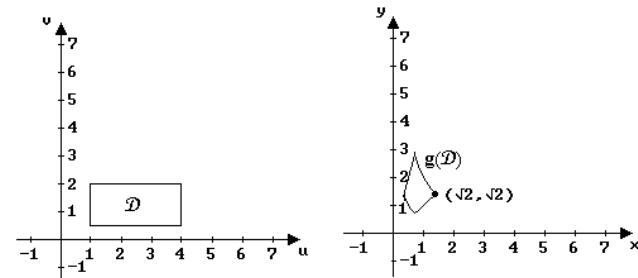
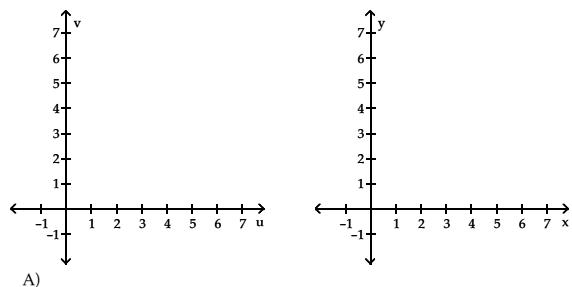
134) _____

- 135) Find the centroid of a hemispherical shell having outer radius 10 and inner radius 3 if the density is constant. 135) _____

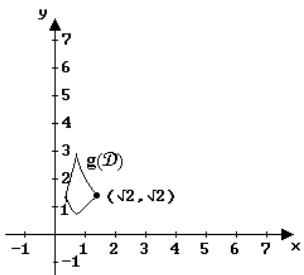
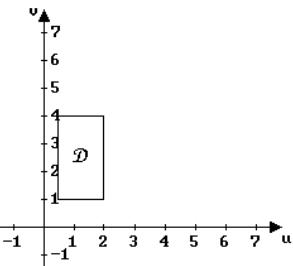
A) $x = 0, y = 0, z = \frac{1417}{417}$
 B) $x = 0, y = 0, z = \frac{4251}{1112}$
 C) $x = 0, y = 0, z = \frac{1417}{556}$
 D) $x = 0, y = 0, z = \frac{2834}{417}$

Sketch D and $g(D)$ from the description of D and change of variables $(x, y) = g(u, v)$.

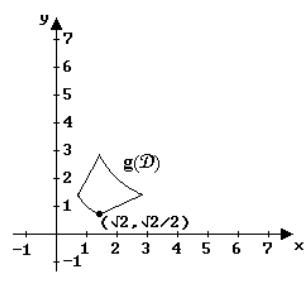
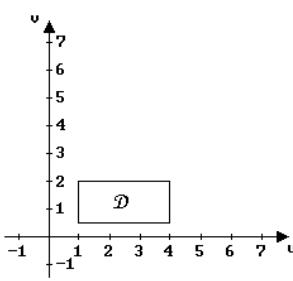
136) $u = xy, v = \frac{y}{x}$ where D is the rectangle $1 \leq x \leq 4, \frac{1}{2} \leq y \leq 2$ 136)



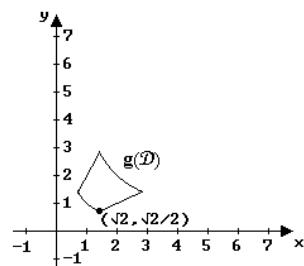
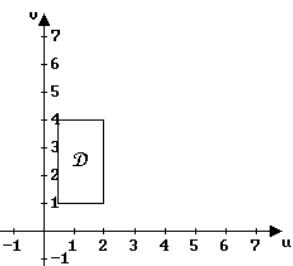
B)



C)



D)



Determine the order of integration and then evaluate the integral.

$$137) \int_0^{27} \int_{\sqrt[3]{x}}^3 \frac{1}{y^4+1} dy dx$$

A) $\frac{\ln 82 - 1}{4}$

B) $\ln 3$

C) $\ln 3 - 1$

D) $\frac{\ln 82}{4}$

137) _____

Find the area of the region specified by the integral(s).

$$138) \int_0^4 \int_0^y dx dy + \int_4^8 \int_0^{8-y} dx dy$$

A) 4

B) 2

C) 32

D) 16

138) _____

Use the given transformation to evaluate the integral.

139) $x = 8u, y = 7v, z = 10w;$

$$\int \int \int_R z^2 dx dy dz,$$

where R is the interior of the ellipsoid $\frac{x^2}{64} + \frac{y^2}{49} + \frac{z^2}{100} = 1$

A) 224π

B) $\frac{560}{3}\pi$

C) 140π

D) $\frac{448}{3}\pi$

139) _____

Solve the problem.

140) Find the centroid of the region cut from the first quadrant by the line $\frac{x}{8} + \frac{y}{6} = 1$.

A) $\bar{x} = \frac{3}{2}, \bar{y} = 2$

B) $\bar{x} = \frac{8}{3}, \bar{y} = 2$

C) $\bar{x} = 2, \bar{y} = \frac{8}{3}$

D) $\bar{x} = 2, \bar{y} = \frac{3}{2}$

140) _____

Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ or $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ (as appropriate) using the given equations.

141) $x = 3u^2, y = 10uv$

A) $30u^2$

B) $60u^2$

C) $30v^2$

D) $60v^2$

141) _____

Find the area of the region specified in polar coordinates.

142) The smaller loop of the curve $r = 2 + 4 \sin \theta$

A) $2(2\pi - 3\sqrt{3})$

B) $2(2\pi + 3\sqrt{3})$

C) $1(2\pi + 3\sqrt{3})$

D) $1(2\pi - 3\sqrt{3})$

142) _____

Find the average value of $F(x, y, z)$ over the given region.

143) $F(x, y, z) = 4x + 6y + 5z$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 3, y = 3, z = 3$

A) $\frac{45}{2}$

B) $\frac{63}{2}$

C) $\frac{45}{4}$

D) $\frac{45}{8}$

143) _____

Solve the problem.

144) Evaluate

$$\int_0^{\infty} \frac{\tan^{-1} \frac{x}{5} - \tan^{-1} \frac{x}{9}}{x} dx$$

by writing the integrand as an integral.

A) $\ln \frac{5}{9}$

B) $\frac{\pi}{2} \ln \frac{9}{5}$

C) $\frac{\pi}{4} \ln \frac{9}{5}$

D) $\frac{\pi}{4} \ln \frac{9}{10}$

144) _____

Evaluate the integral.

$$145) \int_6^7 \int_{5\pi}^{7\pi} \int_0^z \frac{r}{z} d\theta dz dr$$

A) 13π

B) $\frac{13}{2}\pi$

C) $\frac{13}{3}\pi$

D) $\frac{26}{3}\pi$

145) _____

Solve the problem.

146) Find the moment of inertia about the x-axis of a thin disk bounded by the circle $r = 9 \cos \theta$ if the disk's density is given by $\delta(x, y) = x^2 + y^2$.

A) $\frac{885735}{256}\pi$

B) $\frac{531441}{256}\pi$

C) $\frac{531441}{128}\pi$

D) $\frac{885735}{128}\pi$

146) _____

147) Write

$$\int_0^4 \int_0^y f(x, y) dx dy + \int_4^8 \int_0^{8-y} f(x, y) dx dy + \int_0^4 \int_y^4 f(x, y) dx dy + \int_0^4 \int_4^{8-y} f(x, y) dx dy$$

as a single iterated integral with the same order of integration.

A) $\int_4^8 \int_0^{8-y} f(x, y) dx dy$

B) $\int_0^8 \int_4^{8-y} f(x, y) dx dy$

C) $\int_4^8 \int_4^{8-y} f(x, y) dx dy$

D) $\int_0^8 \int_0^{8-y} f(x, y) dx dy$

147) _____

Evaluate the integral.

$$148) \int_0^8 \int_0^2 (5x^2y - 8xy) dy dx$$

A) $\frac{448}{3}$

B) $\frac{1792}{3}$

C) $\frac{224}{3}$

D) $\frac{3584}{3}$

148) _____

Use the given transformation to evaluate the integral.

149) $x = 8u, y = 10v, z = 7w;$

$$\int \int \int \left[\frac{x^2}{64} + \frac{y^2}{100} + \frac{z^2}{49} \right]^3 dx dy dz,$$

where R is the interior of the ellipsoid $\frac{x^2}{64} + \frac{y^2}{100} + \frac{z^2}{49} = 1$

A) $\frac{1120}{3}\pi$

B) 140π

C) $\frac{560}{3}\pi$

D) 420π

149) _____

Integrate the function f over the given region.

150) $f(x, y) = \sqrt{x} + \sqrt{y}$ over the rectangle $0 \leq x \leq 1, 0 \leq y \leq 1$

A) $\frac{8}{3}$

B) $\frac{4}{3}$

C) $\frac{1}{3}$

D) $\frac{2}{3}$

150) _____

Solve the problem.

151) Find the moment of inertia about the z-axis of a thick-walled right circular cylinder bounded on the inside by the cylinder $r = 1$, on the outside by the cylinder $r = 4$, and on the top and bottom by the planes $z = 3$ and $z = 7$.

A) 1020π

B) 512π

C) 255π

D) 510π

151) _____

Write an equivalent double integral with the order of integration reversed.

$$152) \int_5^9 \int_5^{14-y} dx dy$$

A) $\int_5^4 \int_9^{14-x} dy dx$

B) $\int_5^9 \int_5^{14-x} dy dx$

C) $\int_5^4 \int_5^{14-x} dy dx$

D) $\int_5^9 \int_9^{14-x} dy dx$

152) _____

Solve the problem.

153) What domain D in \mathbb{R}^3 maximizes the value of the integral

$$\int \int \int \left(\frac{x^2}{16} + \frac{y^2}{36} + \frac{z^2}{64} - 1 \right) dV?$$

A) $D = \mathbb{R}^3$

B) $D = \text{the boundary of the ellipsoid } \frac{x^2}{16} + \frac{y^2}{36} + \frac{z^2}{64} = 1$

C) $D = \text{the boundary and interior of the ellipsoid } \frac{x^2}{16} + \frac{y^2}{36} + \frac{z^2}{64} = 1$

D) No such minimum domain exists.

153) _____

Find the average value of the function over the region.

154) $f(\rho, \phi, \theta) = \rho \cos \phi$ over the solid lower ball $\rho \leq 9, \pi/2 \leq \phi \leq \pi$

A) $\frac{8}{27}$

B) $-\frac{27}{8}$

C) $-\frac{6561}{4}\pi$

D) $\frac{27}{16}$

154) _____

Evaluate the integral.

$$155) \int_0^{10} \int_0^3 \int_0^{9(1-x/10-y/3)} xyz \, dz \, dy \, dx$$

- A) $\frac{405}{2}$ B) $\frac{405}{4}$ C) 225 D) $\frac{1215}{4}$

155) _____

Evaluate the spherical coordinate integral.

$$156) \int_0^{2\pi} \int_0^\pi \int_{(1-\sin\phi)/2}^{(1-\cos\phi)/2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

- A) $\frac{1}{48}\pi(15\pi - 8)$ B) $\frac{1}{96}\pi(15\pi - 16)$ C) $\frac{1}{96}\pi(15\pi - 8)$ D) $\frac{1}{48}\pi(15\pi - 16)$

156) _____

Evaluate the integral.

$$157) \int_0^\pi \int_{(1-\sin\phi)/2}^{(1-\cos\phi)/2} \int_0^{3\pi} \rho^2 \sin\phi \, d\theta \, d\rho \, d\phi$$

- A) $\frac{1}{64}\pi(15\pi - 8)$ B) $\frac{1}{64}\pi(15\pi - 16)$ C) $\frac{1}{32}\pi(15\pi - 8)$ D) $\frac{1}{32}\pi(15\pi - 16)$

157) _____

Find the area of the region specified in polar coordinates.

$$158) \text{ The region enclosed by the curve } r = 8 + \cos\theta$$

- A) 65π B) $\frac{65}{2}\pi$ C) $\frac{129}{2}\pi$ D) 43π

158) _____

Solve the problem.

159) Let D be the region bounded below by the xy-plane, on the side by the cylinder $r = 6 \sin\theta$, and on top by the paraboloid $z = 10r^2$. Set up the triple integral in cylindrical coordinates that gives the volume of D using the order of integration $dz \, dr \, d\theta$.

- A) $\int_0^{\pi/2} \int_0^{6 \sin\theta} \int_0^{10r^2} r \, dz \, dr \, d\theta$
B) $\int_0^\pi \int_0^{6 \sin\theta} \int_0^{10r^2} r \, dz \, dr \, d\theta$
C) $\int_0^{2\pi} \int_0^{6 \sin\theta} \int_0^{10r^2} r \, dz \, dr \, d\theta$
D) $\int_0^{\pi/4} \int_0^{6 \sin\theta} \int_0^{10r^2} r \, dz \, dr \, d\theta$

159) _____

160) Find the center of mass of the solid enclosed between the cone with equation $z = 7\sqrt{x^2 + y^2}$ and the plane with equation $z = 7$ if the density at any point is proportional to the distance from that point to the plane $z = 7$.

- A) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{49}{12}\right)$
B) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{14}{3}\right)$
C) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{35}{8}\right)$
D) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{21}{5}\right)$

160) _____

Use a CAS integration utility to evaluate the triple integral of the given function over the specified solid region.

$$161) F(x, y, z) = (1 + x + y + z)^2 \text{ over the rectangular cube } 0 \leq x, y, z \leq 9$$

- A) 171315 B) $\frac{349191}{2}$ C) 164754 D) $\frac{336069}{2}$

161) _____

Change the Cartesian integral to an equivalent polar integral, and then evaluate.

$$162) \int_{-8}^8 \int_{-\sqrt{64-y^2}}^0 \frac{\sqrt{x^2+y^2}}{1+\sqrt{x^2+y^2}} \, dx \, dy$$

- A) $\frac{\pi(48+\ln 9)}{4}$ B) $\frac{\pi(48+\ln 9)}{2}$ C) $\frac{\pi(48+2\ln 9)}{4}$ D) $\frac{\pi(48+2\ln 9)}{2}$

162) _____

Solve the problem.

163) Find the mass of a thin plate covering the region inside the curve $r = 7 + 2 \cos\theta$ if $\delta(x, y) = \frac{1}{r}$.

- A) 28π B) 14π C) 7π D) 21π

163) _____

164) Find the average distance from a point $P(x, y)$ in the first quadrant of the disk $x^2 + y^2 \leq 36$ to the origin.

- A) 2 B) 4 C) 3 D) $\frac{3}{2}$

164) _____

Use the given transformation to evaluate the integral.

$$165) u = 2x + y - z, v = -x + y + z, w = -x + y + 2z; \int \int \int_R dx \, dy \, dz,$$

R

where R is the parallelepiped bounded by the planes $2x + y - z = 5$, $2x + y - z = 8$, $-x + y + z = 3$, $-x + y + z = 6$, $-x + y + 2z = 8$, $-x + y + 2z = 9$

- A) 27 B) 6 C) $\frac{27}{2}$ D) 3

165) _____

Solve the problem.

166) Find the mass of a thin plate bounded by $|x| + |y| = 7$ if $\delta(x, y) = x^2y^2$.

- A) $\frac{117649}{45}$ B) $\frac{16807}{8}$ C) $\frac{117649}{25}$ D) $\frac{117649}{40}$

166) _____

Evaluate the integral.

$$167) \int_0^1 \int_0^y e^x + y \, dx \, dy$$

- A) $\frac{1}{2}(e^2 - e)^2$ B) $\frac{1}{3}(e - 1)^2$ C) $\frac{1}{2}(e - 1)^2$ D) $\frac{1}{e}(e^2 - e)^2$

167) _____

Solve the problem.

168) Find the moment of inertia about the y-axis of the thin infinite region of constant density $\delta = 10$ in the first quadrant bounded by the coordinate axes and the curve $y = e^{-3x}$.

- A) $\frac{10}{9}$ B) $\frac{10}{81}$ C) $\frac{20}{27}$ D) $\frac{5}{27}$

168) _____

Integrate the function f over the given region.

169) $f(x, y) = \frac{x}{3} + \frac{y}{6}$ over the trapezoidal region bounded by the x -axis, y -axis, line $x = 3$, and line 169) _____

$$y = -\frac{4}{3}x + 10$$

A) $\frac{82}{3}$

B) $\frac{112}{3}$

C) $\frac{172}{3}$

D) $\frac{67}{3}$

Solve the problem.

170) Find the radius of gyration about the x -axis of the thin infinite region of constant density $\delta = 5$ in the first quadrant bounded by the coordinate axes and the curve $y = e^{-9x}$. 170) _____

A) $\frac{1}{3}$

B) $\frac{1}{2}$

C) $\frac{\pi}{2}$

D) $\frac{\pi}{3}$

171) Find the center of mass of a tetrahedron of constant density bounded by the coordinate planes and the plane 171) _____

$$\frac{x}{10} + \frac{y}{7} + \frac{z}{9} = 1.$$

A) $\bar{x} = 5, \bar{y} = \frac{7}{2}, \bar{z} = \frac{9}{2}$

B) $\bar{x} = \frac{5}{2}, \bar{y} = \frac{7}{4}, \bar{z} = \frac{9}{4}$

C) $\bar{x} = \frac{10}{3}, \bar{y} = \frac{7}{3}, \bar{z} = 3$

D) $\bar{x} = \frac{20}{3}, \bar{y} = \frac{14}{3}, \bar{z} = 6$

Use a CAS integration utility to evaluate the triple integral of the given function over the specified solid region.

172) $F(x, y, z) = x + y + z$ over the tetrahedron bounded by the coordinate planes and the plane 172) _____

$$\frac{x}{6} + \frac{y}{10} + \frac{z}{4} = 1$$

A) $\frac{800}{3}$

B) $\frac{1600}{9}$

C) $\frac{400}{3}$

D) 200

Find the volume of the indicated region.

173) The region that lies under the plane $z = 7x + 2y$ and over the triangle bounded by the lines $y = x$, 173) _____

$$y = 2x, \text{ and } x + y = 6$$

A) 49

B) 63

C) 45

D) 59

Solve the problem.

174) Find the average height of the paraboloid $z = 8x^2 + 9y^2$ above the annular region $4 \leq x^2 + y^2 \leq 25$ in the xy -plane. 174) _____

A) $\frac{725}{4}$

B) $\frac{493}{6}$

C) $\frac{377}{2}$

D) $\frac{493}{4}$

Use the given transformation to evaluate the integral.

175) $u = 2x + y - z, v = -x + y + z, w = -x + y + 2z;$ 175) _____

$$\int \int \int_R (2x + y - z)(z - x + y) dx dy dz,$$

where R is the parallelepiped bounded by the planes $2x + y - z = 5, 2x + y - z = 10, -x + y + z = 5,$

$-x + y + z = 10, -x + y + 2z = 8, -x + y + 2z = 9$

A) $\frac{1875}{2}$

B) $\frac{16875}{2}$

C) $\frac{16875}{4}$

D) $\frac{1875}{4}$

Solve the problem.

176) Let D be the smaller cap cut from a solid ball of radius 7 units by a plane 6 units from the center of the sphere. Set up the triple integral for the volume of D in spherical coordinates. 176) _____

A) $\int_0^{2\pi} \int_0^{\tan^{-1}(\sqrt{13}/6)} \int_0^7 \rho^2 \sin \phi d\rho d\phi d\theta$

B) $\int_0^{2\pi} \int_0^{\tan^{-1}(\sqrt{13}/7)} \int_0^7 \rho^2 \sin \phi d\rho d\phi d\theta$

C) $\int_0^{2\pi} \int_0^{\tan^{-1}(\sqrt{13}/7)} \int_0^7 \rho^2 \sin \phi d\rho d\phi d\theta$

D) $\int_0^{2\pi} \int_0^{\tan^{-1}(\sqrt{13}/6)} \int_0^7 \rho^2 \sin \phi d\rho d\phi d\theta$

Find the volume of the indicated region.

177) The region bounded below by the xy -plane, laterally by the cylinder $r = 4 \cos \theta$, and above by the plane $z = 3$ 177) _____

A) 12π B) 9π C) 3π D) 36π

Evaluate the spherical coordinate integral.

178) $\int_0^\pi \int_0^\pi \int_0^{2 \sin \phi} \rho^2 \sin \phi d\rho d\phi d\theta$ 178) _____

A) $\frac{8}{3}\pi^2$ B) $\frac{4}{3}\pi^2$ C) $1\pi^2$ D) $\frac{8}{9}\pi^2$

Solve the problem.

179) Rewrite the integral 179) _____

$$\int_0^8 \int_0^5 (1-x/8) \int_0^{4(1-x/8-y/5)} dz dy dx$$

in the order $dx dy dz$.

A) $\int_0^4 \int_0^5 (1-z/4) \int_0^{8(1-y/5-z/4)} dx dy dz$

B) $\int_0^4 \int_0^8 (1-z/4) \int_0^{5(1-y/5-z/4)} dx dy dz$

C) $\int_0^8 \int_0^4 (1-x/8) \int_0^{5(1-x/8-y/5)} dx dy dz$

D) $\int_0^8 \int_0^5 (1-x/8) \int_0^{4(1-x/8-y/5)} dx dy dz$

- 180) Find the centroid of the rectangular solid defined by $0 \leq x \leq 8$, $0 \leq y \leq 7$, $0 \leq z \leq 4$.

- A) $\bar{x} = \frac{8}{3}$, $\bar{y} = \frac{7}{3}$, $\bar{z} = \frac{4}{3}$
 B) $\bar{x} = 8$, $\bar{y} = 7$, $\bar{z} = 4$
 C) $\bar{x} = 4$, $\bar{y} = \frac{7}{2}$, $\bar{z} = 2$
 D) $\bar{x} = 2$, $\bar{y} = \frac{7}{4}$, $\bar{z} = 1$

180) _____

Use the given transformation to evaluate the integral.

181) $x = 9u$, $y = 2v$, $z = 8w$;

$$\int_R \int \int x^2 y^2 z^2 dx dy dz,$$

where R is the interior of the ellipsoid $\frac{x^2}{81} + \frac{y^2}{4} + \frac{z^2}{64} = 1$

- A) $\frac{32}{105}\pi$
 B) $\frac{192}{35}\pi$
 C) $\frac{64}{105}\pi$
 D) $\frac{96}{35}\pi$

181) _____

Write an equivalent double integral with the order of integration reversed.

182) $\int_0^{10} \int_0^{3y/10} dx dy$

A) $\int_0^3 \int_0^{3x/10} dy dx$

C) $\int_0^3 \int_{10x/3}^{10} dy dx$

B) $\int_0^x \int_0^{10/3} dy dx$

D) $\int_0^3 \int_0^{x/10} dy dx$

182) _____

Solve the problem.

- 183) Find the centroid of a hemispherical shell having outer radius 10 and inner radius 7 if the density varies as the square of the distance from the base.

- A) $\bar{x} = 0$, $\bar{y} = 0$, $\bar{z} = \frac{1470585}{221848}$
 B) $\bar{x} = 0$, $\bar{y} = 0$, $\bar{z} = \frac{196078}{27731}$
 C) $\bar{x} = 0$, $\bar{y} = 0$, $\bar{z} = \frac{882351}{221848}$
 D) $\bar{x} = 0$, $\bar{y} = 0$, $\bar{z} = \frac{882351}{110924}$

183) _____

Find the area of the region specified by the integral(s).

184) $\int_{-2}^7 \int_{y^2}^{5y+14} dx dy$

A) $\frac{243}{2}$

B) $\frac{2048}{3}$

C) 972

D) $\frac{1331}{6}$

184) _____

Solve the problem.

- 185) Find the moment of inertia of a sphere of radius 5 and constant density δ about a diameter.

- A) $\frac{6250}{3}\delta\pi$
 B) $1875\delta\pi$
 C) $\frac{5000}{3}\delta\pi$
 D) $\frac{21875}{12}\delta\pi$

185) _____

Use a spherical coordinate integral to find the volume of the given solid.

- 186) The solid between the sphere $\rho = 1$ and the cardioid of revolution $\rho = 9 + 7 \cos \phi$

- A) $\frac{4676}{3}\pi$
 B) $\frac{5845}{3}\pi$
 C) $\frac{5845}{6}\pi$
 D) $\frac{2338}{3}\pi$

186) _____

Solve the problem.

- 187) Let A, B, and C be the squares $\{(x, y) | 0 \leq x, y \leq 2\}$, $\{(x, y) | 2 \leq x, y \leq 4\}$, and $\{(x, y) | 4 \leq x, y \leq 8\}$.
 respectively. Use Pappus's formula to find the centroid of $A \cup B \cup C$.

- A) $\bar{x} = \frac{14}{3}$, $\bar{y} = \frac{14}{3}$
 B) $\bar{x} = 5$, $\bar{y} = 5$
 C) $\bar{x} = \frac{9}{2}$, $\bar{y} = \frac{9}{2}$
 D) $\bar{x} = \frac{16}{3}$, $\bar{y} = \frac{16}{3}$

187) _____

Use a CAS integration utility to evaluate the triple integral of the given function over the specified solid region.

188) $F(x, y, z) = \int_{(x^2+14)(y^2+14)(z^2+14)}^{1} dx dy dz$ over the rectangular cube $0 \leq x, y, z \leq 1$

- A) $\frac{\sqrt{15}}{9,261,000}$
 B) $\frac{\sqrt{15}}{44,100}$
 C) $\frac{\sqrt{15}}{661,500}$
 D) $\frac{\sqrt{15}}{617,400}$

188) _____

Solve the problem.

- 189) Rewrite the integral

$$\int_0^5 \int_0^{2(1-z/5)} \int_0^{10(1-y/2-z/5)} dx dy dz$$

in the order $dz dy dx$.

A) $\int_0^5 \int_0^{2(1-x/10)} \int_0^{10(1-x/10-y/2)} dz dy dx$

B) $\int_0^{10} \int_0^{2(1-z/5)} \int_0^{5(1-y/2-z/5)} dz dy dx$

C) $\int_0^5 \int_0^{2(1-z/5)} \int_0^{10(1-y/2-z/5)} dz dy dx$

D) $\int_0^{10} \int_0^{2(1-x/10)} \int_0^{5(1-x/10-y/2)} dz dy dx$

189) _____

Find the volume of the indicated region.

- 190) The region bounded by the paraboloid $z = 1 - \frac{x^2}{100} - \frac{y^2}{36}$ and the xy-plane

- A) 180π
 B) 300π
 C) 30π
 D) 20π

190) _____

Evaluate the integral.

191) $\int_2^9 \int_3^{10} \int_0^{5r} r dz dr d\theta$

- A) $\frac{25235}{6}$
 B) $\frac{25235}{3}$
 C) $\frac{25235}{4}$
 D) $\frac{25235}{2}$

191) _____

Find the volume of the indicated region.

- 192) The region enclosed by the cylinder $x^2 + y^2 = 81$ and the planes $z = 0$ and $x + y + z = 18$
 A) 729π B) 1458π C) 2187π D) 2916π

192) _____

Solve the problem.

- 193) Let D be the region bounded below by the xy-plane, above by the sphere $x^2 + y^2 + z^2 = 100$, and on the sides by the cylinder $x^2 + y^2 = 49$. Set up the triple integral in cylindrical coordinates that gives the volume of D using the order of integration $dr \, dz \, d\theta$.

$$\begin{aligned} & A) \int_0^{2\pi} \int_0^{\sqrt{51}} \int_0^{10} r \, dr \, dz \, d\theta + \int_0^{2\pi} \int_{\sqrt{51}}^7 \int_{\sqrt{49-z^2}}^{\sqrt{100-z^2}} r \, dr \, dz \, d\theta \\ & B) \int_0^{2\pi} \int_0^{\sqrt{51}} \int_0^7 r \, dr \, dz \, d\theta + \int_0^{2\pi} \int_{\sqrt{51}}^{10} \int_{\sqrt{49-z^2}}^{\sqrt{100-z^2}} r \, dr \, dz \, d\theta \\ & C) \int_0^{2\pi} \int_0^{\sqrt{51}} \int_0^{10} r \, dr \, dz \, d\theta + \int_0^{2\pi} \int_0^7 \int_{\sqrt{51}}^{\sqrt{100-z^2}} r \, dr \, dz \, d\theta \\ & D) \int_0^{2\pi} \int_0^{\sqrt{51}} \int_0^7 r \, dr \, dz \, d\theta + \int_0^{2\pi} \int_{\sqrt{51}}^{10} \int_{\sqrt{49-z^2}}^{\sqrt{100-z^2}} r \, dr \, dz \, d\theta \end{aligned}$$

193) _____

- 194) Find the mass of a thin triangular plate bounded by the coordinate axes and the line $x + y = 6$ if $\delta(x, y) = x + y$.
 A) 288 B) 144 C) 72 D) 36

194) _____

- 195) The northern third of Indiana is a rectangle measuring 96 miles by 132 miles. Thus, let $D = [0, 96] \times [0, 132]$. Assuming that the total annual snowfall (in inches), $S(x, y)$, at $(x, y) \in D$ is given by the function

$$S(x, y) = 60e^{-0.001(2x+y)}, (x, y) \in D,$$

find the average snowfall on D.

- A) 51.14 inches B) 52.06 inches C) 52.44 inches D) 51.78 inches

195) _____

- 196) A cubical box in the first octant is bounded by the coordinate planes and the planes $x = 2$, $y = 4$, $z = 10$. The box is filled with a liquid of density $\delta(x, y, z) = x + y + z + 3$. Find the work done by (constant) gravity g in moving the liquid in the container down to the the xy-plane.

$$A) \frac{12160}{3}g \quad B) \frac{30400}{9}g \quad C) \frac{15200}{3}g \quad D) 3800g$$

196) _____

- 197) Find the center of mass of the region of constant density bounded from above by the sphere $x^2 + y^2 + z^2 = 4$ and from below by the cone $z = \sqrt{x^2 + y^2}$.

$$\begin{aligned} & A) x = 0, y = 0, z = \frac{5}{8}(2 + \sqrt{2}) \quad B) x = 0, y = 0, z = \frac{3}{8}(2 + \sqrt{2}) \\ & C) x = 0, y = 0, z = \frac{3}{4}(2 + \sqrt{2}) \quad D) x = 0, y = 0, z = \frac{7}{8}(2 + \sqrt{2}) \end{aligned}$$

197) _____

- 198) Find the center of mass of a thin triangular plate bounded by the coordinate axes and the line $x + y = 5$ if $\delta(x, y) = x + y$.

$$\begin{aligned} & A) x = \frac{10}{3}, y = \frac{10}{3} \quad B) x = \frac{15}{8}, y = \frac{15}{8} \quad C) x = \frac{25}{12}, y = \frac{25}{12} \quad D) x = \frac{5}{3}, y = \frac{5}{3} \end{aligned}$$

198) _____

Find the area of the region specified in polar coordinates.

- 199) The region enclosed by the curve $r = 10 \cos \theta$

$$\begin{aligned} & A) 50\pi \quad B) \frac{100}{3}\pi \quad C) 100\pi \quad D) 25\pi \end{aligned}$$

199) _____

Solve the problem.

- 200) Write

$$\int_0^6 \int_0^y f(x, y) \, dx \, dy + \int_6^{12} \int_0^{12-y} f(x, y) \, dx \, dy + \int_0^6 \int_y^6 f(x, y) \, dx \, dy$$

as a single iterated integral.

$$\begin{array}{ll} A) \int_0^6 \int_6^{12-x} f(x, y) \, dy \, dx & B) \int_0^6 \int_0^{12-x} f(x, y) \, dy \, dx \\ C) \int_{-6}^6 \int_x^{12-x} f(x, y) \, dy \, dx & D) \int_{-6}^6 \int_{-6}^{12-x} f(x, y) \, dy \, dx \end{array}$$

200) _____

Determine the order of integration and then evaluate the integral.

$$201) \int_0^{60} \int_{y/6}^{10} \tan^{-1} x^2 \, dx \, dy$$

$$\begin{array}{ll} A) 300 \tan^{-1} 100 - \frac{3}{2} \ln 10,001 & B) 300 \tan^{-1} 100 - 3 \ln 10,001 \\ C) 150 \tan^{-1} 100 - 3 \ln 10,001 & D) 150 \tan^{-1} 100 - \frac{3}{2} \ln 10,001 \end{array}$$

201) _____

Use a spherical coordinate integral to find the volume of the given solid.

- 202) The solid enclosed by the cardioid of revolution $\rho = 8 + 7 \cos \phi$, $z \geq 0$

$$\begin{aligned} & A) \frac{6647}{3}\pi \quad B) \frac{6647}{4}\pi \quad C) \frac{6647}{6}\pi \quad D) \frac{6647}{2}\pi \end{aligned}$$

202) _____

Solve the problem.

- 203) Let D be the region bounded below by the xy-plane, above by the sphere $x^2 + y^2 + z^2 = 64$, and on the sides by the cylinder $x^2 + y^2 = 4$. Set up the triple integral in cylindrical coordinates that gives the volume of D using the order of integration $dz \, d\theta \, dr$.

$$\begin{array}{ll} A) \int_0^8 \int_0^{2\pi} \int_0^{\sqrt{4-r^2}} r \, dz \, d\theta \, dr & B) \int_0^8 \int_0^{2\pi} \int_0^{\sqrt{4-r^2}} dz \, d\theta \, dr \\ C) \int_0^2 \int_0^{2\pi} \int_0^{\sqrt{64-r^2}} r \, dz \, d\theta \, dr & D) \int_0^2 \int_0^{2\pi} \int_0^{\sqrt{64-r^2}} dz \, d\theta \, dr \end{array}$$

203) _____

Evaluate the spherical coordinate integral.

$$204) \int_0^{\pi/2} \int_0^{\pi/2} \int_0^2 \cos \phi \rho^2 \sin \phi d\rho d\phi d\theta$$

A) $\frac{1}{3}\pi$

B) $\frac{1}{3}\pi^2$

C) $\frac{4}{9}\pi^2$

D) $\frac{4}{9}\pi$

204) _____

Solve the problem.

205) Find the average distance from a point $P(r, \theta)$ in the region bounded by $r = 3 + 7 \cos \theta$ to the origin. 205) _____

A) $\frac{495}{134}$

B) $\frac{330}{67}$

C) $\frac{165}{67}$

D) $\frac{1485}{268}$

206) Find the radius of gyration R_X of the rectangular solid defined by $0 \leq x \leq 8, 0 \leq y \leq 3, 0 \leq z \leq 7$. 206) _____

A) $\sqrt{\frac{58}{3}}$

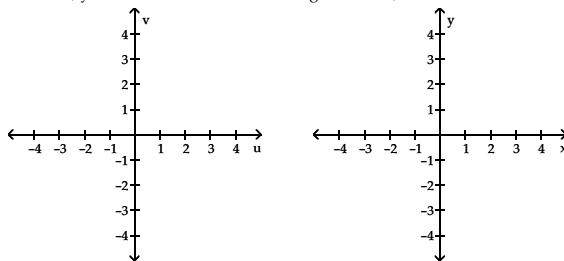
B) $\sqrt{\frac{113}{3}}$

C) $\sqrt{\frac{122}{3}}$

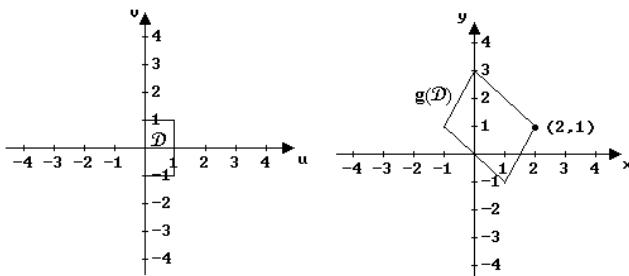
D) $\sqrt{\frac{73}{3}}$

Sketch D and $g(D)$ from the description of D and change of variables $(x, y) = g(u, v)$.

207) $x = u + v, y = 2u - v$ where D is the rectangle $0 \leq u \leq 1, -1 \leq v \leq 1$ 207)

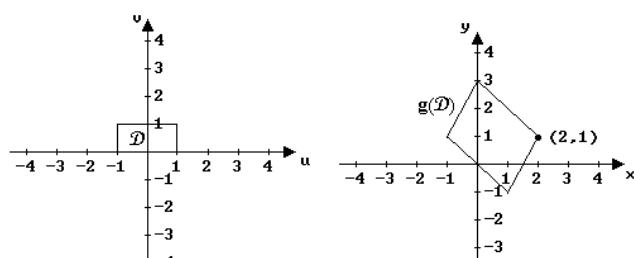


A)

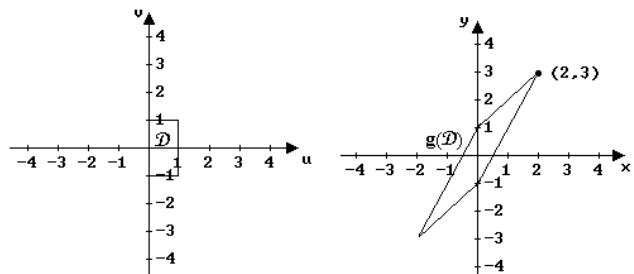


35

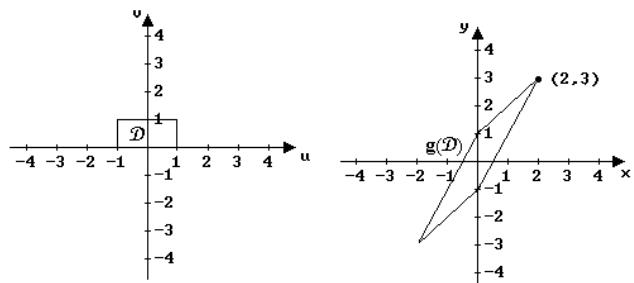
B)



C)

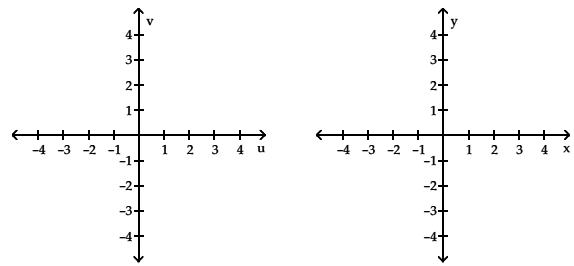


D)

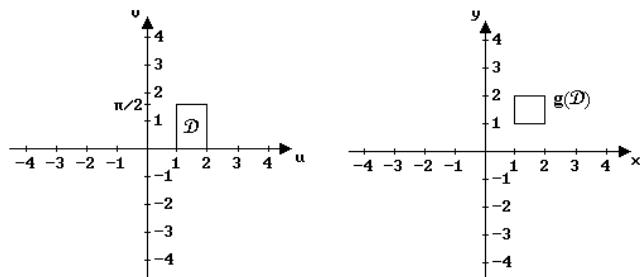


36

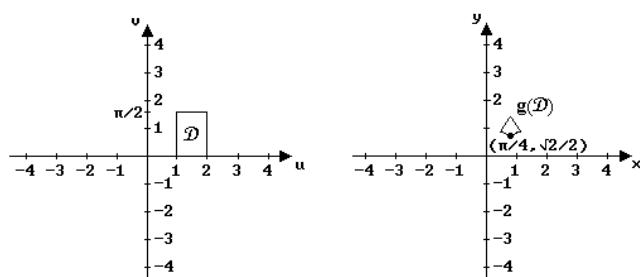
208) $x = u \cos v$, $y = u \sin v$ where D is the rectangle $1 \leq u \leq 2$, $0 \leq v \leq \frac{\pi}{2}$



A)

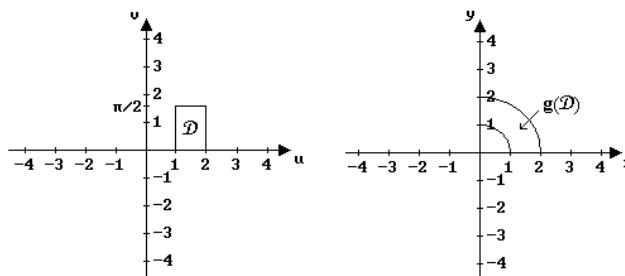


B)

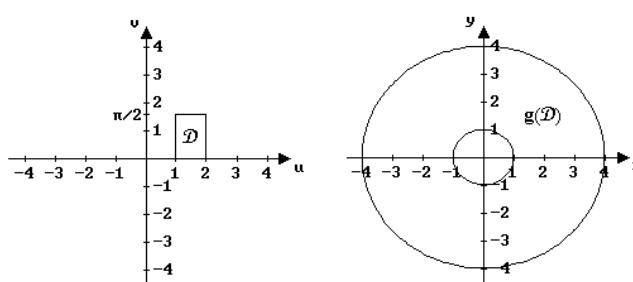


208)

C)



D)



Evaluate the improper integral.

$$209) \int_0^8 \int_0^9 \ln xy \, dx \, dy$$

A) $72(\ln 72 - 1)$

B) $\frac{\ln 72 - 1}{72}$

C) $72(\ln 72 - 2)$

D) $\frac{\ln 72 - 2}{72}$

209) _____

Solve the problem.

210) Write an iterated triple integral in the order $dz \, dy \, dx$ for the volume of the region enclosed by the paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$.

210) _____

A) $\int_{-2}^2 \int_{-\sqrt{8-x^2}}^{\sqrt{8-x^2}} \int_{x^2+y^2}^{4-x^2-y^2} dz \, dy \, dx$

B) $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^{8-x^2-y^2} dz \, dy \, dx$

C) $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^{4-x^2-y^2} dz \, dy \, dx$

D) $\int_{-2}^2 \int_{-\sqrt{8-x^2}}^{\sqrt{8-x^2}} \int_{x^2+y^2}^{8-x^2-y^2} dz \, dy \, dx$

Integrate the function f over the given region.

211) $f(x, y) = \sin 3x$ over the rectangle $0 \leq x \leq \frac{\pi}{3}, 0 \leq y \leq \pi$

A) $\frac{\pi}{3}$

B) π

C) $\frac{2\pi}{3}$

D) $\frac{\pi}{6}$

211) _____

Use a CAS integration utility to evaluate the triple integral of the given function over the specified solid region.

212) $F(x, y, z) = \frac{1}{(x^2 + 13)^{3/2} (y^2 + 13)^{5/2} (z^2 + 13)^{7/2}}$ over the rectangular cube $0 \leq x, y, z \leq 1$

A) $\frac{114923}{9.085908033e+13}\sqrt{14}$

B) $\frac{114923}{6.230336937e+13}\sqrt{14}$

C) $\frac{114923}{1.246067387e+14}\sqrt{14}$

D) $\frac{114923}{1.168188176e+14}\sqrt{14}$

212) _____

Solve the problem.

213) Find the moment of inertia about the x -axis of a thin plane of constant density $\delta = 7$ bounded by the coordinate axes and the line $\frac{x}{9} + \frac{y}{6} = 1$.

A) 756

B) 1701

C) 1134

D) $\frac{5103}{2}$

213) _____

Change the Cartesian integral to an equivalent polar integral, and then evaluate.

214) $\int_{-8}^8 \int_{-\sqrt{64-y^2}}^{\sqrt{64-y^2}} \ln(x^2 + y^2 + 1) dx dy$

A) $\pi(65 \ln 65 - 64)$

B) $\pi(64 \ln 65 + 64)$

C) $\pi(64 \ln 65 - 64)$

D) $\pi(65 \ln 65 + 64)$

214) _____

Find the volume of the indicated region.

215) The region bounded by the cylinders $r = 4$, $r = 10$ and the planes $z = 9$, $z = 10$

A) 84π

B) 252π

C) 336π

D) 168π

215) _____

Express the area of the region bounded by the given line(s) and/or curve(s) as an iterated double integral.

216) The lines $\frac{x}{2} + \frac{y}{10} = 1$, $\frac{x}{10} + \frac{y}{2} = 1$, and $y = 0$

A) $\int_0^{12} \int_{10(1-y/2)}^{2(1-y/2)} dx dy$

B) $\int_0^{1.67} \int_{2(1-y/10)}^{10(1-y/2)} dx dy$

C) $\int_0^{1.67} \int_{10(1-y/2)}^{2(1-y/10)} dx dy$

D) $\int_0^{12} \int_{2(1-y/10)}^{10(1-y/2)} dx dy$

216) _____

Solve the problem.

217) Find the average distance from a point $P(x, y)$ in the annular region $81 \leq x^2 + y^2 \leq 49$ to the origin.

A) $\frac{193}{24}$

B) $\frac{579}{64}$

C) $\frac{957}{64}$

D) $\frac{319}{24}$

217) _____

218) Find the moment of inertia about the y -axis of a thin disk bounded by the circle $r = 10 \cos \theta$ if the disk's density is given by $\delta(x, y) = \frac{1}{r^2}$

A) $\frac{75}{4}\pi$

B) $\frac{125}{4}\pi$

C) $\frac{125}{2}\pi$

D) $\frac{75}{2}\pi$

218) _____

Change the Cartesian integral to an equivalent polar integral, and then evaluate.

219) $\int_0^{\ln 4} \int_0^{\sqrt{(\ln 4)^2 - y^2}} e^{\sqrt{x^2 + y^2}} dx dy$

A) $\frac{\pi(4 \ln 4 + 3)}{2}$

B) $\frac{\pi(4 \ln 4 - 3)}{4}$

C) $\frac{\pi(4 \ln 4 - 3)}{2}$

D) $\frac{\pi(4 \ln 4 + 3)}{4}$

219) _____

Evaluate the integral.

220) $\int_{-1}^1 \int_0^1 \int_0^3 (x^2 + y^2 + z^2) dx dy dz$

A) 23.2

B) 22

C) 28

D) -26

220) _____

Find the volume of the indicated region.

221) The region bounded by the paraboloid $z = 36 - x^2 - y^2$ and the xy -plane

A) 432π

B) 648π

C) 324π

D) 216π

221) _____

Solve the problem.

222) Evaluate

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{(1+x^2+y^2)^3} dx dy.$$

A) $\frac{\pi}{2}$

B) $\frac{\pi}{4}$

C) $\frac{\pi}{3}$

D) $\frac{\pi}{6}$

222) _____

Find the average value of $F(x, y, z)$ over the given region.

223) $F(x, y, z) = 3x + 6y + 5z$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 8, y = 3, z = 6$

A) 45

B) 54

C) 36

D) 24

223) _____

Use the given transformation to evaluate the integral.

224) $u = -4x + y, v = 2x + y;$

$$\int_R (2x+y) dx dy,$$

R

where R is the parallelogram bounded by the lines $y = 4x + 4, y = 4x + 7, y = -2x + 2, y = -2x + 8$

A) 15

B) 1080

C) 540

D) 30

224) _____

Solve the problem.

225) A solid of constant density is bounded below by the plane $z = 0$, above by the cone $z = r$, and on the sides by the cylinder $r = 9$. Find the center of mass.

A) $(0, 0, 3)$

B) $(0, 0, \frac{45}{8})$

C) $(0, 0, \frac{27}{8})$

D) $(0, 0, 6)$

225) _____

- 226) Find the center of mass of the rectangular solid of density $\delta(x, y, z) = xyz$ defined by $0 \leq x \leq 10$, $0 \leq y \leq 5$, $0 \leq z \leq 2$.

A) $\bar{x} = \frac{5}{2}, \bar{y} = \frac{5}{4}, \bar{z} = \frac{1}{2}$

C) $\bar{x} = \frac{10}{3}, \bar{y} = \frac{5}{3}, \bar{z} = \frac{2}{3}$

B) $\bar{x} = \frac{20}{3}, \bar{y} = \frac{10}{3}, \bar{z} = \frac{4}{3}$

D) $\bar{x} = 5, \bar{y} = \frac{5}{2}, \bar{z} = 1$

226) _____

Find the volume of the indicated region.

- 227) The region bounded by the coordinate planes and the planes $z = x + y$, $z = 7$

A) $\frac{343}{4}$

B) $\frac{343}{3}$

C) $\frac{343}{2}$

D) $\frac{343}{6}$

227) _____

Solve the problem.

- 228) A solid right circular cylinder is bounded by the cylinder $r = 1$ and the planes $z = 0$ and $z = 4$. Find the moment of inertia about the z -axis if the density is $\delta = 3z$.

A) 16π

B) 4π

C) 24π

D) 12π

228) _____

Evaluate the integral.

- 229) $\int_0^{\pi} \int_0^{\pi} \int_0^{\pi} \sin(8u+2) du dv dw$

A) $\frac{1}{8}\pi \cos 2$

B) $\frac{1}{8}\pi^2 \cos 2$

C) $\frac{1}{4}\pi \cos 2$

D) $\frac{1}{4}\pi^2 \cos 2$

229) _____

Find the volume of the indicated region.

- 230) The tetrahedron cut off from the first octant by the plane $\frac{x}{9} + \frac{y}{4} + \frac{z}{7} = 1$

A) 84

B) 63

C) 42

D) 126

230) _____

Solve the problem.

- 231) Find the centroid of the region cut from the fourth quadrant by the circle $x^2 + y^2 = 49$.

A) $\bar{x} = \frac{28}{3\pi}, \bar{y} = -\frac{28}{3\pi}$

B) $\bar{x} = \frac{7}{3\pi}, \bar{y} = -\frac{7}{3\pi}$

C) $\bar{x} = -\frac{3\pi}{28}, \bar{y} = \frac{3\pi}{28}$

D) $\bar{x} = \frac{3\pi}{28}, \bar{y} = -\frac{3\pi}{28}$

231) _____

Evaluate the integral by changing the order of integration in an appropriate way.

- 232) $\int_0^{16} \int_{x/2}^8 \int_0^{\infty} ze^{-(y^2+z^2)} dz dy dx$

A) $1(1 - e^{-128})$

B) $1(1 - e^{-64})$

C) $\frac{1}{2}(1 - e^{-128})$

D) $\frac{1}{2}(1 - e^{-64})$

232) _____

Express the area of the region bounded by the given line(s) and/or curve(s) as an iterated double integral.

- 233) The lines $\frac{x}{5} + \frac{y}{10} = 1$, $\frac{x}{10} + \frac{y}{5} = 1$, and $x = 0$

A) $\int_0^{3.3} \int_{5(1-x/10)}^{10(1-x/5)} dy dx$

C) $\int_0^{3.3} \int_{10(1-x/5)}^{5(1-x/10)} dy dx$

B) $\int_0^{15} \int_{10(1-x/5)}^{5(1-x/10)} dy dx$

D) $\int_0^{15} \int_{5(1-x/10)}^{10(1-x/5)} dy dx$

233) _____

Solve the problem.

234) Evaluate

$$\int_{-6}^6 \int_{-\sqrt{36-y^2}}^{\sqrt{36-y^2}} \int_0^{\sqrt{49-x^2-y^2}} dz dx dy$$

by transforming to cylindrical or spherical coordinates.

A) $\frac{4\pi}{3}(49 - (13)^3/2)$

C) $\frac{2\pi}{3}(49 - (13)^3/2)$

B) $\frac{2\pi}{3}(343 - (13)^3/2)$

D) $\frac{4\pi}{3}(343 - (13)^3/2)$

234) _____

Write an equivalent double integral with the order of integration reversed.

235) $\int_4^{13} \int_{17-x}^{13} dy dx$

A) $\int_4^{13} \int_{13-y}^{9} dx dy$

C) $\int_4^9 \int_{17-y}^{13} dx dy$

B) $\int_4^{13} \int_{17-y}^{13} dx dy$

D) $\int_4^9 \int_{13-y}^9 dx dy$

235) _____

Find the volume of the indicated region.

- 236) The region in the first octant bounded by the coordinate planes and the planes $x + z = 4$, $y + 5z = 20$

A) $\frac{80}{3}$

B) $\frac{64}{3}$

C) $\frac{320}{3}$

D) $\frac{1280}{3}$

236) _____

Evaluate the integral.

237) $\int_8^{10} \int_{6\pi}^{8\pi} \int_0^z \frac{r}{z} dr dz d\theta$

A) $14\pi^2$

B) $\frac{56}{9}\pi^2$

C) $7\pi^2$

D) $\frac{28}{3}\pi^2$

237) _____

Solve the problem.

- 238) A rectangular plate of constant density $\delta(x, y) = 1$ occupies the region bounded by the lines $x = 7$ and $y = 10$ in the first quadrant. The moment of inertia I_a of the rectangle about the line $y = a$ is given by

$$I_a = \int_0^7 \int_0^{10} (y - a)^2 dy dx.$$

Find the value of a that minimizes I_a .

A) 1

B) 5

C) $\frac{7}{2}$

D) $\frac{1}{2}$

238) _____

Use the given transformation to evaluate the integral.

239) $u = 2x + y - z, v = -x + y + z, w = -x + y + 2z;$

$$\int \int \int_R (2x + y - z) dx dy dz,$$

where R is the parallelepiped bounded by the planes $2x + y - z = 2, 2x + y - z = 4, -x + y + z = 4, -x + y + z = 5, -x + y + 2z = 7, -x + y + 2z = 9$.

- A) 6 B) 4 C) 36 D) 16

Solve the problem.

240) Let A and B be the squares

$$\{(x, y) \mid 0 \leq x, y \leq 2\} \text{ and } \{(x, y) \mid 3 \leq x, y \leq 8\},$$

respectively. Use Pappus's formula to find the centroid of $A \cup B$.

A) $\bar{x} = \frac{283}{58}, \bar{y} = \frac{283}{58}$ B) $\bar{x} = \frac{299}{58}, \bar{y} = \frac{299}{58}$ C) $\bar{x} = \frac{291}{58}, \bar{y} = \frac{291}{58}$ D) $\bar{x} = \frac{307}{58}, \bar{y} = \frac{307}{58}$

Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ or $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ (as appropriate) using the given equations.

241) $x = 8u \cos 7v, y = 8u \sin 7v$
 A) 392u B) 448v C) 448u D) 392v

Evaluate the integral.

242) $\int_0^6 \int_0^{36-x^2} x dy dx$

- A) 432 B) 324 C) 72 D) 54

243) $\int_0^{\pi/2} \int_3^8 \int_0^{\pi/3} \rho^4 \sin^2 \phi \cos \phi d\phi d\rho d\theta$
 A) $\frac{32525}{81}\pi\sqrt{3}$ B) $\frac{32525}{81}\pi\sqrt{2}$ C) $\frac{6505}{16}\pi\sqrt{2}$ D) $\frac{6505}{16}\pi\sqrt{3}$

239) _____

240) _____

241) _____

242) _____

243) _____

Set up the iterated integral for evaluating

$$\int \int \int_D f(r, \theta, z) dz r dr d\theta$$

over the given region D.

244) D is the right circular cylinder whose base is the circle $r = 4 \cos \theta$ in the xy-plane and whose top lies in the plane $z = 10 - x - y$.
 244) _____

A) $\int_0^\pi \int_0^{4 \sin \theta} \int_0^{10 - \sin \theta - \cos \theta} f(r, \theta, z) dz r dr d\theta$

B) $\int_0^{2\pi} \int_0^{4 \cos \theta} \int_0^{10 - r(\cos \theta + \sin \theta)} f(r, \theta, z) dz r dr d\theta$

C) $\int_0^\pi \int_0^{4 \cos \theta} \int_0^{10 - r(\cos \theta + \sin \theta)} f(r, \theta, z) dz r dr d\theta$

D) $\int_0^{2\pi} \int_0^{4 \sin \theta} \int_0^{10 - \sin \theta - \cos \theta} f(r, \theta, z) dz r dr d\theta$

Solve the problem.

245) A cubical box in the first octant is bounded by the coordinate planes and the planes $x = 1, y = 1, z = 1$. The box is filled with a liquid of density $\delta(x, y, z) = x + y + z + 4$. Find the work done by (constant) gravity g in moving the liquid in the container down to the the xy-plane.
 245) _____

A) $\frac{17}{2}g$ B) $\frac{17}{3}g$ C) $\frac{17}{4}g$ D) $\frac{17}{6}g$

246) If $f(x, y) = (3000e^y)/(1 + |x|/2)$ represents the population density of a planar region on Earth, where x and y are measured in miles, find the number of people within the rectangle $-9 \leq x \leq 9$ and $-2 \leq y \leq 0$.
 246) _____

A) $6000(1 - e^{-2}) \ln \frac{11}{2} \approx 8844$ B) $12,000(1 - e^{-2}) \ln \frac{11}{2} \approx 17,688$
 C) $6000(1 - e^{-2}) \ln 5 \approx 12,440$ D) $3000(1 - e^{-2}) \ln \frac{11}{2} \approx 4422$

Evaluate the integral by changing the order of integration in an appropriate way.

247) $\int_0^1 \int_0^8 \int_y^8 \frac{x \sin z}{z} dz dy dx$
 247) _____

A) $1 + \cos 8$ B) $\frac{1 + \sin 8}{2}$ C) $1 - \sin 8$ D) $\frac{1 - \cos 8}{2}$

Evaluate the improper integral.

248) $\int_0^\infty \int_0^\infty e^{-(10x + 9y)} dy dx$
 248) _____

A) $\frac{1}{90}$ B) $\frac{\pi}{180}$ C) $\frac{1}{180}$ D) $\frac{\pi}{90}$

Evaluate the integral.

$$249) \int_0^{\ln 3} \int_{e^y}^3 e^y \, dx \, dy$$

A) 1

B) 8

C) 4

D) 2

249) _____

$$250) \int_0^{10\pi} \int_0^{13\pi} (\sin x + \cos y) \, dx \, dy$$

A) 20π

B) 11π

C) 10π

D) 21π

250) _____

Set up the iterated integral for evaluating

$$\int_D \int \int f(r, \theta, z) \, dz \, r \, dr \, d\theta$$

over the given region D.

251) D is the right circular cylinder whose base is the circle $r = 2 \sin \theta$ in the xy-plane and whose top lies in the plane $z = 7 - r$. 251) _____

- A) $\int_0^{2\pi} \int_0^{2 \sin \theta} \int_0^{7 - r \sin \theta} f(r, \theta, z) \, dz \, r \, dr \, d\theta$
- B) $\int_0^{2\pi} \int_0^{2 \sin \theta} \int_0^{7 - \sin \theta} f(r, \theta, z) \, dz \, r \, dr \, d\theta$
- C) $\int_0^\pi \int_0^{2 \sin \theta} \int_0^{7 - r \sin \theta} f(r, \theta, z) \, dz \, r \, dr \, d\theta$
- D) $\int_0^\pi \int_0^{2 \sin \theta} \int_0^{7 - \sin \theta} f(r, \theta, z) \, dz \, r \, dr \, d\theta$

252) D is the rectangular solid whose base is the triangle with vertices at $(0, 0)$, $(0, 5)$, and $(5, 5)$, and whose top lies in the plane $z = 8$. 252) _____

- A) $\int_0^{\pi/2} \int_0^{5 \csc \theta} \int_0^8 f(r, \theta, z) \, dz \, r \, dr \, d\theta$
- B) $\int_{\pi/4}^{\pi/2} \int_0^{5 \sec \theta} \int_0^8 f(r, \theta, z) \, dz \, r \, dr \, d\theta$
- C) $\int_{\pi/4}^{\pi/2} \int_0^{5 \csc \theta} \int_0^8 f(r, \theta, z) \, dz \, r \, dr \, d\theta$
- D) $\int_0^{\pi/2} \int_0^{5 \sec \theta} \int_0^8 f(r, \theta, z) \, dz \, r \, dr \, d\theta$

Solve the problem.

253) Evaluate

$$\int_0^{\infty} \frac{e^{-2x^2} - e^{-5x^2}}{x} \, dx$$

by writing the integrand as an integral.

A) $\frac{1}{2} \ln \frac{2}{5}$

B) $\frac{1}{2} \ln \frac{5}{2}$

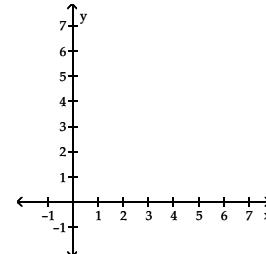
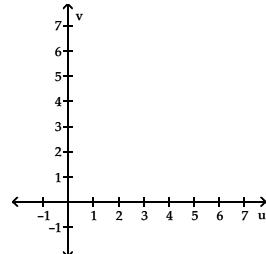
C) $\ln \frac{5}{2}$

D) $\ln \frac{2}{5}$

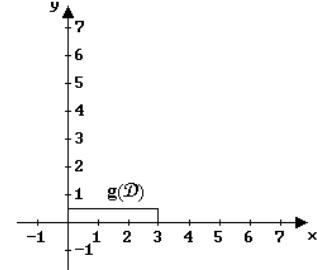
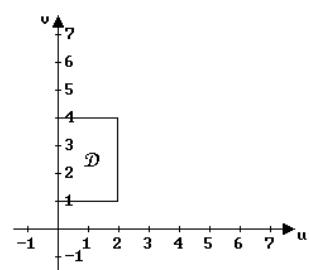
253) _____

Sketch D and $g(D)$ from the description of D and change of variables $(x, y) = g(u, v)$.

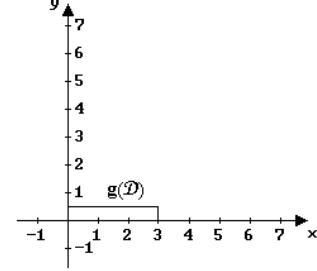
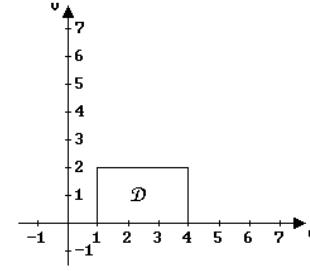
254) $x = 3u$, $y = \frac{1}{2}v$ where D is the rectangle $0 \leq u \leq 2$, $1 \leq v \leq 4$ 254)



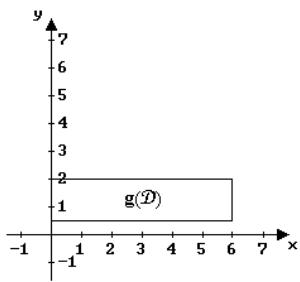
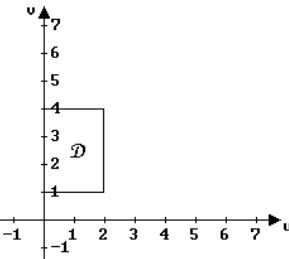
A)



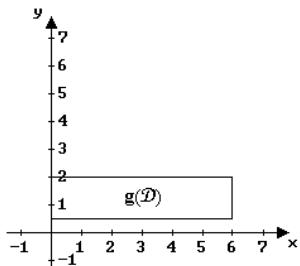
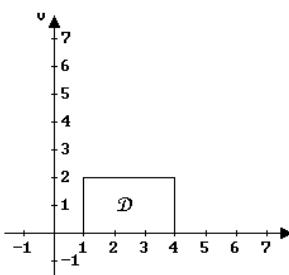
B)



C)



D)



Evaluate the integral.

255) $\int_0^1 \int_0^1 \int_0^1 (8x + 7y + 10z) dz dy dx$

A) 103

B) $\frac{25}{2}$ C) $\frac{25}{6}$ D) $\frac{25}{3}$

255) _____

Evaluate the spherical coordinate integral.

256) $\int_0^{2\pi} \int_0^{\pi/4} \int_0^4 (\rho^5 \cos \phi) \rho^2 \sin \phi d\rho d\phi d\theta$

A) $\frac{1.6e+09}{3}\pi$ B) 4096π C) 13436928π D) $\frac{2097152}{3}\pi$

256) _____

Solve the problem.

257) For what value of a is the volume of the tetrahedron formed by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{5} + \frac{z}{9} = 1$ equal to 8?

$$\frac{x}{a} + \frac{y}{5} + \frac{z}{9} = 1 \text{ equal to } 8?$$

A) $a = \frac{16}{15}$ B) $a = \frac{32}{45}$ C) $a = \frac{64}{45}$ D) $a = \frac{8}{15}$

47

258) Find the radius of gyration about the y -axis of a thin triangular plate bounded by the coordinate axes and the line $x + y = 10$ if $\delta(x, y) = x + y$.

258) _____

A) $\frac{10}{\sqrt{2}}$ B) $\frac{10}{\sqrt{3}}$ C) $\frac{10}{\sqrt{5}}$ D) $\frac{10}{\sqrt{6}}$ 259) What region in the xy -plane maximizes the value of

259) _____

$$\int \int (576 - 36x^2 - 16y^2) dA$$

A) The ellipse $36x^2 + 16y^2 = 1$ B) The ellipse $\frac{x^2}{16} + \frac{y^2}{36} = 1$ C) The ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$ D) The ellipse $16x^2 + 36y^2 = 576$ 260) Find the centroid of the infinite region in the first quadrant bounded by the coordinate axes and the curve $y = e^{-5x}$.

260) _____

A) $\bar{x} = \frac{1}{5}, \bar{y} = \frac{1}{2}$ B) $\bar{x} = \frac{1}{10}, \bar{y} = \frac{1}{2}$ C) $\bar{x} = \frac{1}{5}, \bar{y} = \frac{1}{4}$ D) $\bar{x} = \frac{1}{10}, \bar{y} = \frac{1}{4}$

Express the area of the region bounded by the given line(s) and/or curve(s) as an iterated double integral.

261) _____

261) The curves $y = x(x - 6)$ and $y = x(x - 6)(x - 10)$

$$A) \int_6^{10} \int_{x(x-6)}^{x(x-10)} dy dx$$

$$B) \int_6^{10} \int_{x(x-6)}^{x(x-10)} dy dx$$

$$C) \int_0^6 \int_{x(x-6)}^{x(x-10)} dy dx$$

$$D) \int_0^6 \int_{x(x-6)}^{x(x-10)} dy dx$$

262) The coordinate axes and the line $x + y = 5$

262) _____

$$A) \int_{-5}^5 \int_x^5 dy dx$$

$$B) \int_0^5 \int_0^{5-x} dy dx$$

$$C) \int_0^5 \int_5^x dy dx$$

$$D) \int_{-5}^5 \int_0^{5-x} dy dx$$

Solve the problem.

263) _____

263) Let D be the smaller cap cut from a solid ball of radius 8 units by a plane 7 units from the center of the sphere. Set up the triple integral for the volume of D in cylindrical coordinates.

$$A) \int_0^{2\pi} \int_0^{\sqrt{113}} \int_7^{\sqrt{64-r^2}} r dz dr d\theta$$

$$B) \int_0^{2\pi} \int_0^{\sqrt{15}} \int_8^{\sqrt{49-r^2}} r dz dr d\theta$$

$$C) \int_0^{2\pi} \int_0^{\sqrt{15}} \int_7^{\sqrt{64-r^2}} r dz dr d\theta$$

$$D) \int_0^{2\pi} \int_0^{\sqrt{113}} \int_8^{\sqrt{49-r^2}} r dz dr d\theta$$

48

- 264) Find the radius of gyration about the z-axis of a thick-walled right circular cylinder bounded on the inside by the cylinder $r = 1$, on the outside by the cylinder $r = 6$, and on the top and bottom by the planes $z = 7$ and $z = 11$.

A) $\frac{1295}{71}$ B) $\sqrt{\frac{1295}{71}}$ C) 2590π D) $\sqrt{\frac{37}{2}}$

Evaluate the integral by changing the order of integration in an appropriate way.

265) $\int_0^4 \int_y^4 \int_0^\pi \frac{\sin z \sin x}{x} dz dx dy$
 A) $2(1 + \sin 4)$ B) $2 \cos 4$ C) $2(1 - \cos 4)$ D) $2 \sin 4$

Solve the problem.

- 266) Set up the triple integral for the volume of the sphere $\rho = 2$ in spherical coordinates.

A) $\int_0^{2\pi} \int_0^\pi \int_0^2 \rho^2 \sin \phi d\rho d\phi d\theta$
 B) $\int_0^{2\pi} \int_0^{\pi/2} \int_0^2 d\rho d\phi d\theta$
 C) $\int_0^{2\pi} \int_0^\pi \int_0^2 d\rho d\phi d\theta$
 D) $\int_0^{2\pi} \int_0^{\pi/2} \int_0^2 \rho^2 \sin \phi d\rho d\phi d\theta$

- 267) Find the radius of gyration R_X of the region of constant density bounded by the paraboloid $z = 100 - x^2 - y^2$ and the xy-plane.

A) $\frac{10}{3}\sqrt{606}$ B) $\frac{5}{2}\sqrt{606}$ C) $\frac{10}{9}\sqrt{606}$ D) $\frac{5}{3}\sqrt{606}$

Evaluate the integral.

268) $\int_9^{10} \int_0^{\pi/2} \int_0^{\pi/2} \rho \sin \phi d\theta d\phi d\rho$
 A) $\frac{19}{6}\pi$ B) $\frac{19}{6}$ C) $\frac{19}{4}\pi$ D) $\frac{19}{4}$

Find the area of the region specified by the integral(s).

269) $\int_0^8 \int_0^{10(1-x/8)} dy dx$
 A) 5 B) 4 C) $\frac{160}{3}$ D) 40

Use a spherical coordinate integral to find the volume of the given solid.

- 270) The solid bounded below by the hemisphere $\rho = 1$, $z \geq 0$, and above by the cardioid of revolution $\rho = 8 + 7 \cos \phi$

A) $\frac{6643}{9}\pi$ B) $\frac{6643}{8}\pi$ C) $\frac{6643}{10}\pi$ D) $\frac{6643}{6}\pi$

264) _____

Evaluate the improper integral.

271) $\int_2^6 \int_1^{101} \frac{dx dy}{\sqrt{(x-1)(y-2)}}$
 A) 100 B) 60 C) 80 D) 120

271) _____

Evaluate the integral.

272) $\int_0^{10} \int_0^2 (x+y) dx dy$
 A) 6 B) 960 C) 120 D) 48

272) _____

Evaluate the integral by changing the order of integration in an appropriate way.

273) $\int_0^{216} \int_{\sqrt[3]{z}}^6 \int_1^{1297} \frac{1}{x(y^4+1)} dx dy dz$
 A) $\frac{\ln^2 1297}{4}$ B) $\frac{\ln 1297}{2}$ C) $\frac{\ln 1297}{4}$ D) $\frac{\ln^2 1297}{2}$

273) _____

Solve the problem.

274) Evaluate

$$\iiint_R (5x+y-z)(z-x+6y)(x-y+10z) dV$$
 where R is the parallelepiped enclosed by the planes $5x+y-z=1$, $5x+y-z=3$, $-x+6y+z=-2$, $-x+6y+z=5$, $x-y+10z=-1$, $x-y+10z=6$.
 A) $\frac{243}{107}$ B) $\frac{676}{321}$ C) $\frac{240}{107}$ D) $\frac{245}{107}$

274) _____

Change the Cartesian integral to an equivalent polar integral, and then evaluate.

275) $\int_0^{11} \int_0^{\sqrt{121-y^2}} (x^2+y^2) dx dy$
 A) $\frac{14,641\pi}{8}$ B) $\frac{121\pi}{8}$ C) $\frac{1331\pi}{4}$ D) $\frac{1331\pi}{8}$

275) _____

Find the volume of the indicated region.

- 276) The region bounded by the cylinder $x^2 + y^2 = 16$ and the planes $z = 0$ and $x + z = 5$

A) 400π B) 80π C) 100π D) 20π

276) _____

Evaluate the integral.

277) $\int_5^8 \int_5^{10} \int_0^{6r} r dr d\theta dz$
 A) $\frac{5805}{4}$ B) 3870 C) 1935 D) $\frac{5805}{2}$

277) _____

Set up the iterated integral for evaluating

$$\int \int \int_D f(r, \theta, z) dz r dr d\theta$$

over the given region D.

- 278) D is the solid right cylinder whose base is the region in the xy-plane that lies inside the cardioid $r = 10 + 3 \cos \theta$ and outside the circle $r = 7$, and whose top lies in the plane $z = 9$. 278)

$$\begin{array}{ll} A) \int_0^{\pi} \int_0^{10+3 \cos \theta} \int_0^9 f(r, \theta, z) dz r dr d\theta \\ B) \int_0^{2\pi} \int_7^{10+3 \cos \theta} \int_0^9 f(r, \theta, z) dz r dr d\theta \\ C) \int_0^{\pi} \int_7^{10+3 \cos \theta} \int_0^9 f(r, \theta, z) dz r dr d\theta \\ D) \int_0^{2\pi} \int_0^{10+3 \cos \theta} \int_0^9 f(r, \theta, z) dz r dr d\theta \end{array}$$

Use the given transformation to evaluate the integral.

279) $x = 10u, y = 5v, z = 6w;$

$$\int \int \int_R (x^2 + y^2 + z^2) dx dy dz,$$

where R is the interior of the ellipsoid $\frac{x^2}{100} + \frac{y^2}{25} + \frac{z^2}{36} = 1$

- A) 120π B) 180π C) 200π D) 240π 279)

Evaluate the integral.

280) $\int_0^{\pi/4} \int_0^3 \int_0^{2\pi} (\rho^3 \cos \phi) \rho^2 \sin \phi d\theta d\rho d\phi$ 280)

- A) $\frac{243}{4}\pi$ B) 2592π C) $\frac{1265625}{4}\pi$ D) $\frac{177147}{8}\pi$

Solve the problem.

- 281) Find the centroid of the region enclosed by the curve $r = 7 + 2 \cos \theta$.

A) $\bar{x} = \frac{100}{51}, \bar{y} = 0$ B) $\bar{x} = \frac{50}{51}, \bar{y} = 0$ C) $\bar{x} = \frac{52}{51}, \bar{y} = 0$ D) $\bar{x} = \frac{104}{51}, \bar{y} = 0$ 281)

Find the average value of $F(x, y, z)$ over the given region.

- 282) $F(x, y, z) = x^9 y^7 z^8$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 1, y = 1, z = 1$ 282)

A) $\frac{1}{191}$ B) $\frac{1}{720}$ C) $\frac{1}{504}$ D) $\frac{1}{695}$

Solve the problem.

- 283) Find the mass of the solid in the first octant between the spheres $x^2 + y^2 + z^2 = 36$ and $x^2 + y^2 + z^2 = 64$ if the density at any point is inversely proportional to its distance from the origin. 283)

A) $\frac{28}{3}k\pi$ B) $7k\pi$ C) $\frac{14}{3}k\pi$ D) $56k\pi$

- 284) Find the center of mass of the upper half of a solid ball of radius 5 centered at the origin if the density at any point is proportional to the distance from that point to the z-axis. 284)

A) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{5}{8}\pi\right)$
 B) $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 2)$
 C) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{5}{8}(1 + \sqrt{2})\right)$
 D) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{16}{3}\frac{1}{\pi}\right)$

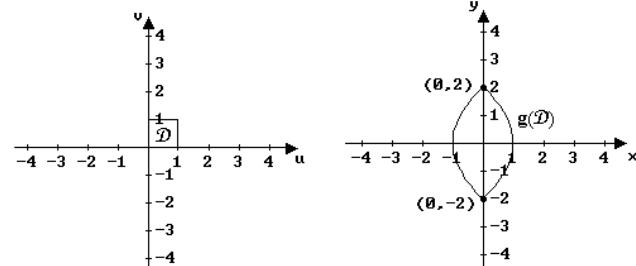
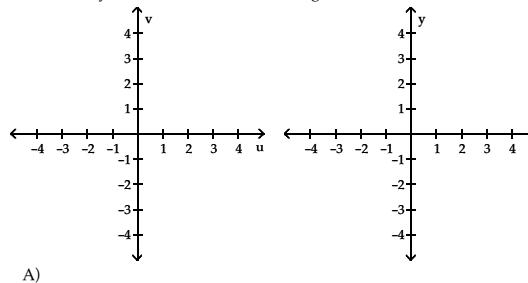
Find the volume of the indicated region.

- 285) The region bounded below by the xy-plane, laterally by the cylinder $r = 5 \sin \theta$, and above by the plane $z = 9 - x$ 285)

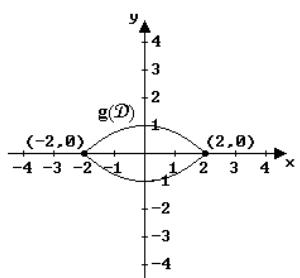
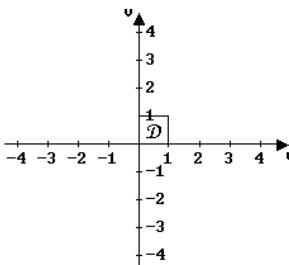
A) $\frac{45}{4}\pi$ B) $\frac{2025}{4}\pi$ C) $\frac{225}{4}\pi$ D) $\frac{405}{4}\pi$

Sketch D and $g(D)$ from the description of D and change of variables $(x, y) = g(u, v)$.

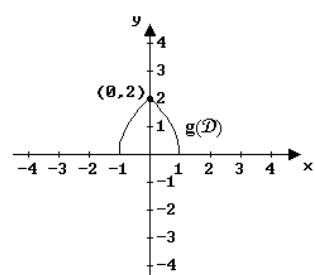
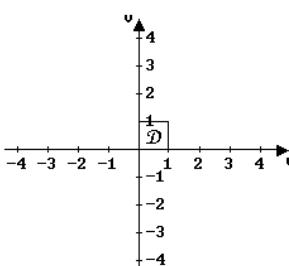
- 286) $x = u^2 - v^2, y = 2uv$ where D is the rectangle $0 \leq u \leq 1, 0 \leq v \leq 1$ 286)



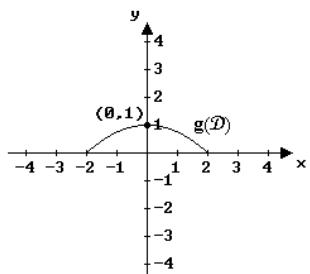
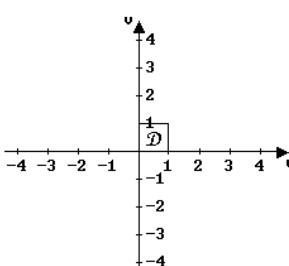
B)



C)



D)



Evaluate the integral by changing the order of integration in an appropriate way.

$$287) \int_1^7 \int_0^{400} \int_{\sqrt{y/4}}^{10} \frac{\sin x^2}{xz} dx dy dz$$

- A) $2 \ln 7 (1 - \cos 100)$
C) $2 \ln 7 (1 - \sin 100)$

- B) $1 \ln 7 (1 - \sin 100)$
D) $1 \ln 7 (1 - \cos 100)$

287) _____

Evaluate the improper integral.

$$288) \int_0^9 \int_0^{81} \frac{dx dy}{\sqrt{xy}}$$

- A) 36 B) 12 C) 108 D) 162

288) _____

Find the area of the region specified by the integral(s).

$$289) \int_0^2 \int_0^{2-x} dy dx$$

- A) $\frac{2}{3}$ B) 2 C) $\frac{4}{3}$ D) 1

289) _____

Find the area of the region specified in polar coordinates.

290) The region enclosed by the curve $r = 3 \sin 2\theta$

- A) $\frac{3}{2}\pi$ B) $\frac{9}{2}\pi$ C) $\frac{9}{4}\pi$ D) $\frac{9}{8}\pi$

290) _____

Solve the problem.

291) Find the moment of inertia about the origin of a thin disk bounded by the circle $r = 5 \sin \theta$ if the disk's density is given by $\delta(x, y) = r^2$.

- A) $\frac{78125}{48}\pi$ B) $\frac{15625}{32}\pi$ C) $\frac{15625}{16}\pi$ D) $\frac{78125}{96}\pi$

291) _____

Express the area of the region bounded by the given line(s) and/or curve(s) as an iterated double integral.

292) The curve $y = \ln x$ and the lines $x = 6$ and $y = 0$

- A) $\int_0^6 \int_0^{\ln x} dy dx$
B) $\int_0^6 \int_{-6}^{\ln y} dy dx$
C) $\int_0^6 \int_{-6}^{\ln x} dy dx$
D) $\int_0^6 \int_0^{\ln x} dy dx$

292) _____

293) The curve $y = e^x$ and the lines $x + y = 8$ and $x = 8$

- A) $\int_0^8 \int_8^{e^x - x} dy dx$
B) $\int_0^8 \int_{8-x}^{e^x} dy dx$
C) $\int_0^8 \int_x^{e^x - x} dy dx$
D) $\int_0^8 \int_{x-8}^{e^x} dy dx$

293) _____

Evaluate the integral.

$$294) \int_3^9 \int_{-10}^6 3x \, dy \, dx$$

A) -2304

B) -864

C) 1728

D) -576

294) _____

Express the area of the region bounded by the given line(s) and/or curve(s) as an iterated double integral.

$$295) \text{ The parabola } x = y^2 \text{ and the line } y = \frac{x}{3} - \frac{70}{3}$$

295) _____

A) $\int_7^{10} \int_{y^2}^{3y+70} dx \, dy$

B) $\int_{-7}^{10} \int_0^{3y+70} dx \, dy$

C) $\int_7^{10} \int_0^{3y+70} dx \, dy$

D) $\int_{-7}^{10} \int_{y^2}^{3y+70} dx \, dy$

295) _____

Solve the problem.

296) Find the center of mass of the hemisphere $z = 4 - x^2 - y^2$ if the density is proportional to the distance from the center.

A) $\left(0, 0, \frac{16}{15}\right)$

B) $\left(0, 0, \frac{4}{5}\right)$

C) $\left(0, 0, \frac{6}{5}\right)$

D) $\left(0, 0, \frac{14}{15}\right)$

296) _____

297) Find the average distance from a point $P(x, y)$ in the first two quadrants of the disk $x^2 + y^2 \leq 81$ to the origin.

A) $\frac{9}{4}$

B) 6

C) $\frac{9}{2}$

D) 3

297) _____

298) Find the moment of inertia about the y-axis of the thin semicircular region of constant density $\delta = 5$ bounded by the x-axis and the curve $y = \sqrt{16 - x^2}$.

A) $\frac{1280\pi}{3}$

B) 160π

C) $\frac{640\pi}{3}$

D) 320π

298) _____

299) Let R be a thin triangular region cut off from the first quadrant by the line $x + y = 10$. If the density of R is given by $\delta(x, y) = ax + \frac{y}{a}$ where $a > 0$, for what value of a will the moment of inertia of R about the origin be a minimum?

A) $\frac{1}{10}$

B) 1

C) 10

D) $\frac{2}{3}$

299) _____

Provide an appropriate response.

300) What does the graph of the equation $\phi = \frac{\pi}{2}$ look like?

300) _____

A) The xz-plane

B) The z-axis

C) The xy-plane

D) The y-axis

Evaluate the cylindrical coordinate integral.

$$301) \int_0^{\pi} \int_0^4 \int_0^{6/r} \sin \theta \, dz \, r \, dr \, d\theta$$

301) _____

A) 72π

B) 24π

C) 72

D) 48

55

Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ or $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ (as appropriate) using the given equations.

302) $u = -4x - 3, v = -2y + 3, w = 3z + 1$

A) $-\frac{1}{9}$

B) -9

C) $\frac{1}{24}$

D) 24

302) _____

Solve the problem.

303) Find the moment of inertia about the x-axis of the region enclosed by the curve $r = 10 + 8 \sin \theta$ if $\delta(x, y) = 7$.

A) 74172π

B) 72380π

C) 49980π

D) 66780π

303) _____

Find the average value of the function f over the region R.

304) $f(x, y) = \sin \frac{3}{2}(x + y)$

304) _____

R: $0 \leq x \leq \frac{\pi}{3}, 0 \leq y \leq \frac{\pi}{3}$

A) $\frac{6}{\pi^2}$

B) π^2

C) $4\pi^2$

D) $\frac{8}{\pi^2}$

Find the average value of $F(x, y, z)$ over the given region.

305) $F(x, y, z) = 8x$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 7, y = 7, z = 7$

305) _____

A) 1568

B) 28

C) 224

D) 196

Find the average value of the function f over the region R.

306) $f(x, y) = 9x + 6y$

306) _____

R: $0 \leq x \leq 1, 0 \leq y \leq 1$

A) 15

B) $\frac{21}{2}$

C) 12

D) $\frac{15}{2}$

307) $f(x, y) = e^{10x}$

307) _____

R: $0 \leq x \leq \frac{1}{10}, 0 \leq y \leq \frac{1}{10}$

A) $2e - 1$

B) $\frac{2e - 1}{100}$

C) $\frac{e - 1}{10}$

D) $e - 1$

Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ or $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ (as appropriate) using the given equations.

308) $x = 6u \cos 3v, y = 6u \sin 3v, z = 2w$

308) _____

A) 108v

B) 216v

C) 216u

D) 108u

56

Solve the problem.

309) Evaluate

$$\int_0^{\infty} \frac{\sin ax}{x} dx, a > 0$$

by integrating

$$\int_0^{\infty} \int_0^{\infty} e^{-xy} \sin ax dy dx$$

- A) $\frac{\pi a}{3}$ B) $\frac{\pi a}{6}$ C) $\frac{\pi a}{4}$ D) $\frac{\pi}{2}$

Find the volume of the indicated region.

310) The region bounded by the coordinate planes, the parabolic cylinder $z = 16 - x^2$, and the plane $y = 6$

- A) 768 B) 288 C) 256 D) 576

Evaluate the integral by changing the order of integration in an appropriate way.

$$311) \int_0^{\pi/4} \int_0^{35} \int_{z/7}^5 \frac{\tan x \tan y}{y} dy dz dx$$

- A) $-\frac{7}{2} \ln 2 \ln \cos 5$ B) $7 \ln 2 \ln \cos 5$ C) $-7 \ln 2 \ln \cos 5$ D) $\frac{7}{2} \ln 2 \ln \cos 5$

Find the average value of the function f over the region R .

$$312) f(x, y) = 7x + 10y$$

R is the region bounded by the coordinate axes and the lines $x + y = 2$ and $x + y = 10$.

- A) $\frac{527}{12}$ B) $\frac{527}{24}$ C) $\frac{527}{18}$ D) $\frac{527}{9}$

Evaluate the integral.

$$313) \int_2^8 \int_8^9 \int_0^{2r} r d\theta dr dz$$

- A) $\frac{6076}{3}$ B) 4340 C) 868 D) $\frac{9548}{3}$

Use a CAS integration utility to evaluate the triple integral of the given function over the specified solid region.

314) $F(x, y, z) = 5x - 3y + 2z$ over the rectangular solid $0 \leq x \leq 6, 0 \leq y \leq 4, 0 \leq z \leq 7$

- A) 8064 B) 2688 C) 1344 D) 1792

Evaluate the integral.

$$315) \int_0^8 \int_0^4 \int_0^{4(1-z/8)} 2(1-y/4-z/8) dy dz dx$$

- A) 8 B) $\frac{32}{3}$ C) $\frac{64}{3}$ D) 16

309) _____

Use the given transformation to evaluate the integral.

$$316) u = y - x, v = y + x; \int_R \int \cosh\left(\frac{y-x}{y+x}\right) dx dy,$$

where R is the trapezoid with vertices at $(6, 0), (7, 0), (0, 6), (0, 7)$

- A) $\frac{13(e^2 - 1)}{2e}$ B) $\frac{13(e^2 - 1)}{3e}$ C) $\frac{13(e^2 - 1)}{4e}$ D) $\frac{13(e^2 - 1)}{6e}$

316) _____

Find the average value of the function f over the region R .

$$317) f(x, y) = 9x + 4y$$

R: $0 \leq x \leq 7, 0 \leq y \leq 6$

- A) $\frac{87}{2}$ B) $\frac{63}{2}$ C) $\frac{67}{2}$ D) $\frac{69}{2}$

317) _____

Solve the problem.

318) Write an iterated triple integral in the order $dz dy dx$ for the volume of the rectangular solid in the first octant bounded by the planes $x = 6, y = 2$, and $z = 7$.

- A) $\int_0^6 \int_0^2 \int_0^7 dz dy dx$
B) $\int_0^6 \int_2^7 \int_0^{7-y-x} dz dy dx$
C) $\int_0^7 \int_0^2 \int_0^6 dz dy dx$
D) $\int_0^7 \int_0^6 \int_0^{6-y-x} dz dy dx$

318) _____

Find the volume of the indicated region.

319) The region enclosed by the sphere $x^2 + y^2 + z^2 = 64$ and the cylinder $(x - 4)^2 + y^2 = 16$

- A) $\frac{1024}{9}(3\pi - 4)$ B) $\frac{256}{3}(3\pi - 4)$ C) $\frac{1024}{3}(3\pi - 4)$ D) $\frac{320}{3}(3\pi - 4)$

319) _____

Evaluate the integral.

$$320) \int_{-4}^7 \int_{-1}^2 2y dx dy$$

- A) 33 B) 99 C) -363 D) -99

320) _____

Find the volume of the indicated region.

321) The region under the surface $z = x^2 + y^4$, and bounded by the planes $x = 0$ and $y = 100$ and the cylinder $y = x^2$

- A) $\frac{6.000044e+10}{33}$ B) $\frac{6.00044e+09}{33}$ C) $\frac{6.0000044e+11}{33}$ D) $\frac{600440000}{33}$

321) _____

Provide an appropriate response.

322) What form do planes perpendicular to the y -axis have in cylindrical coordinates?

- A) $r = a \csc \theta$ B) $r = a \sec \theta$ C) $r = a \sin \theta$ D) $r = a \cos \theta$

322) _____

Find the area of the region specified in polar coordinates.

323) The region enclosed by the curve $r = 10 - 7 \cos \theta$

- A) $\frac{149}{7}\pi$ B) $\frac{198}{7}\pi$ C) $\frac{249}{7}\pi$ D) 149π

323) _____

Solve the problem.

- 324) Find the moment of inertia about the x-axis of a thin triangular plate bounded by the coordinate axes and the line $x + y = 10$ if $\delta(x, y) = x + y$. 324) _____

A) 10000 B) $\frac{20000}{3}$ C) $\frac{100000}{9}$ D) 20000

- 325) Integrate $f(x, y) = \sin(x^2 + y^2)$ over the region $0 \leq x^2 + y^2 \leq 4$. 325) _____

A) $\pi(1 - \cos 4)$ B) $2\pi(1 - \cos^2 2)$ C) $\pi(1 - \cos^2 2)$ D) $2\pi(1 - \cos 4)$

Evaluate the spherical coordinate integral.

- 326) $\int_0^{\pi/2} \int_0^{\pi/3} \int_{4 \sec \phi}^{5 \sec \phi} \rho^3 \sin \phi \cos \phi \, d\rho \, d\phi \, d\theta$ 326) _____

A) $\frac{1845}{8}\pi$ B) $\frac{1107}{8}\pi$ C) $\frac{1845}{16}\pi$ D) $\frac{1107}{16}\pi$

Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ or $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ (as appropriate) using the given equations.

- 327) $x = 5u + 3, y = -4v - 5, z = -3w + 1$ 327) _____

A) -900 B) 60 C) -15 D) 15

Solve the problem.

- 328) Find the center of mass of the region of density $\delta(x, y, z) = \frac{1}{64 - x^2 - y^2}$ bounded by the paraboloid 328) _____

$z = 64 - x^2 - y^2$ and the xy-plane.

A) $\bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{1}{2}$ B) $\bar{x} = 0, \bar{y} = 0, \bar{z} = 8$
 C) $\bar{x} = 0, \bar{y} = 0, \bar{z} = 4$ D) $\bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{8}{3}$

- 329) Find the moment of inertia I_y of the region of constant density $\delta(x, y, z) = 1$ bounded by the 329) _____

paraboloid $z = 9 - x^2 - y^2$ and the xy-plane.

A) $\frac{135}{2}\pi$ B) 810π C) $\frac{1215}{2}\pi$ D) $\frac{405}{2}\pi$

- 330) Find the average height of the part of the paraboloid $z = 36 - x^2 - y^2$ that lies above the xy-plane. 330) _____

A) 54 B) 12 C) 18 D) 90

Find the volume of the indicated region.

- 331) The region inside the solid sphere $\rho \leq 9$ that lies between the cones $\phi = \frac{\pi}{3}$ and $\phi = \frac{\pi}{2}$ 331) _____

A) $\frac{729}{2}\pi$ B) $\frac{243}{2}\pi$ C) 243π D) $\frac{729}{4}\pi$

Evaluate the integral by changing the order of integration in an appropriate way.

- 332) $\int_0^\infty \int_0^{45} \int_{y/5}^9 e^{-(x^2 + z)} \, dx \, dy \, dz$ 332) _____

A) $\frac{5}{4}(1 - e^{-90})$ B) $\frac{5}{2}(1 - e^{-90})$ C) $\frac{5}{4}(1 - e^{-81})$ D) $\frac{5}{2}(1 - e^{-81})$

Solve the problem.

- 333) Let D be the region bounded below by the xy-plane, above by the sphere $x^2 + y^2 + z^2 = 36$, and on the sides by the cylinder $x^2 + y^2 = 16$. Set up the triple integral in cylindrical coordinates that gives the volume of D using the order of integration $dz \, dr \, d\theta$. 333) _____

A) $\int_0^{2\pi} \int_0^4 \int_{\sqrt{36 - z^2}}^r r \, dz \, dr \, d\theta$ B) $\int_0^{2\pi} \int_0^4 \int_{\sqrt{36 - r^2}}^r r \, dz \, dr \, d\theta$
 C) $\int_0^{2\pi} \int_0^6 \int_{\sqrt{16 - z^2}}^r r \, dz \, dr \, d\theta$ D) $\int_0^{2\pi} \int_0^6 \int_{\sqrt{16 - r^2}}^r r \, dz \, dr \, d\theta$

Write an equivalent double integral with the order of integration reversed.

- 334) $\int_0^{\pi/2} \int_0^{\sin x} (10x + 8y) \, dy \, dx$ 334) _____

A) $\int_0^{\pi/2} \int_{\sin y}^1 (10x + 8y) \, dx \, dy$ B) $\int_0^1 \int_{\pi/2}^{\sin^{-1} y} (10x + 8y) \, dx \, dy$
 C) $\int_0^1 \int_{\sin^{-1} y}^{\pi/2} (10x + 8y) \, dx \, dy$ D) $\int_0^1 \int_0^{\sin^{-1} y} (10x + 8y) \, dx \, dy$

Solve the problem.

- 335) Find the moment of inertia about the y-axis of a thin infinite region in the first quadrant bounded by the coordinate axes and the curve $y = e^{-8x}$ if $\delta(x, y) = xy$. 335) _____

A) $\frac{3}{20480}$ B) $\frac{3}{65536}$ C) $\frac{5}{65536}$ D) $\frac{3}{8192}$

Evaluate the cylindrical coordinate integral.

- 336) $\int_0^{\pi/2} \int_0^{\cos^2 \theta} \int_{6r^4}^{10r^4} z \sin \theta \, dz \, r \, dr \, d\theta$ 336) _____

A) $\frac{22}{35}$ B) $\frac{41}{105}$ C) $\frac{16}{105}$ D) $\frac{1}{9}$

Solve the problem.

- 337) Write an iterated triple integral in the order $dx\ dy\ dz$ for the volume of the tetrahedron cut from the first octant by the plane $\frac{x}{3} + \frac{y}{5} + \frac{z}{4} = 1$. 337) _____

A) $\int_0^4 \int_0^{3(1-y/5)} \int_0^{3(1-y/5-z/4)} dx\ dy\ dz$
 B) $\int_0^4 \int_0^{1-y/5} \int_0^{1-y/5-z/4} dx\ dy\ dz$
 C) $\int_0^4 \int_0^{5(1-z/4)} \int_0^{3(1-y/5-z/4)} dx\ dy\ dz$
 D) $\int_0^4 \int_0^{1-z/4} \int_0^{1-y/5-z/4} dx\ dy\ dz$

Evaluate the integral.

- 338) $\int_{-5}^0 \int_{-2}^0 (9x - 6y) dy\ dx$ 338) _____
 A) $-\frac{33}{2}$ B) -165 C) -33 D) $-\frac{165}{2}$

- 339) $\int_0^2 \int_{-1}^y \int_5^z yz\ dx\ dz\ dy$ 339) _____
 A) $-\frac{61}{3}$ B) $-\frac{128}{3}$ C) $\frac{68}{3}$ D) $\frac{23}{3}$

Evaluate the integral by changing the order of integration in an appropriate way.

- 340) $\int_0^2 \int_1^2 \int_{\sqrt{x/2}}^1 \frac{ey^3}{z} dy\ dz\ dx$ 340) _____
 A) $\frac{2}{3} \ln 3 (e-1)$ B) $1 \ln 2 (e-1)$ C) $1 \ln 3 (e-1)$ D) $\frac{2}{3} \ln 2 (e-1)$

Evaluate the improper integral.

- 341) $\int_0^{81} \int_0^{81} \frac{dy\ dx}{\sqrt{x+y}}$ 341) _____
 A) $108(1 - \ln 2)$ B) $216(1 - \ln 2)$ C) $972(1 - \ln 2)$ D) $1944(1 - \ln 2)$

Find the volume of the indicated region.

- 342) The region bounded by the paraboloid $z = 16 - x^2 - y^2$ and the xy-plane 342) _____
 A) 32π B) 128π C) $\frac{64}{3}\pi$ D) $\frac{256}{3}\pi$

Evaluate the improper integral.

- 343) $\int_1^{10} \int_0^\infty \frac{1}{x(y^2+1)} dy\ dx$ 343) _____
 A) $\frac{\ln 10}{2}$ B) $\frac{\ln 10}{3}$ C) $\frac{(\ln 10)\pi}{2}$ D) $\frac{(\ln 10)\pi}{3}$

Change the Cartesian integral to an equivalent polar integral, and then evaluate.

- 344) $\int_{-10}^{10} \int_{-\sqrt{100-x^2}}^{\sqrt{100-x^2}} \frac{1}{(1+x^2+y^2)^2} dy\ dx$ 344) _____
 A) $\frac{100}{101}\pi$ B) $\frac{400}{101}\pi$ C) $\frac{100}{201}\pi$ D) $\frac{200}{101}\pi$

Solve the problem.

- 345) Evaluate 345) _____
 $\int_0^\infty \int_0^\infty \frac{1}{(1+x^2+y^2)^7} dx\ dy$.
 A) $\frac{\pi}{6}$ B) $\frac{\pi}{24}$ C) $\frac{\pi}{32}$ D) $\frac{\pi}{8}$

Find the average value of $F(x, y, z)$ over the given region.

- 346) $F(x, y, z) = xyz$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x=4, y=10, z=5$ 346) _____
 A) 25 B) $\frac{25}{2}$ C) $\frac{100}{9}$ D) $\frac{50}{3}$

Solve the problem.

- 347) Find the center of mass of a tetrahedron of density $\delta(x, y, z) = x + y + z$ bounded by the coordinate planes and the plane $\frac{x}{5} + \frac{y}{8} + \frac{z}{6} = 1$. 347) _____
 A) $\bar{x} = \frac{30}{19}, \bar{y} = \frac{54}{19}, \bar{z} = \frac{75}{38}$ B) $\bar{x} = \frac{40}{19}, \bar{y} = \frac{72}{19}, \bar{z} = \frac{50}{19}$
 C) $\bar{x} = \frac{24}{19}, \bar{y} = \frac{216}{95}, \bar{z} = \frac{30}{19}$ D) $\bar{x} = \frac{20}{19}, \bar{y} = \frac{36}{19}, \bar{z} = \frac{25}{19}$

Find the average value of the function over the region.

- 348) $f(\rho, \phi, \theta) = \rho$ over the "ice cream cone" cut from the solid sphere $\rho \leq 6$ by the cone $\phi = \pi/3$ 348) _____
 A) $\frac{2}{9}$ B) $\frac{1}{8}$ C) 324π D) $\frac{9}{2}$

Find the volume of the indicated region.

- 349) The region that lies under the plane $\frac{x}{2} + \frac{y}{6} + \frac{z}{7} = 1$ and above the square $0 \leq x, y \leq \frac{3}{2}$ 349) _____
 A) $\frac{63}{8}$ B) $\frac{147}{16}$ C) $\frac{441}{8}$ D) $\frac{441}{16}$

Use a spherical coordinate integral to find the volume of the given solid.

- 350) The solid between the spheres $\rho = 3 \cos \phi$ and $\rho = 5 \cos \phi$
 A) $\frac{49}{2}\pi$ B) $\frac{98}{3}\pi$ C) $\frac{49}{3}\pi$ D) 49π

350) _____

Evaluate the cylindrical coordinate integral.

- 351) $\int_0^{6\pi} \int_0^{10} \int_r^{2r} dz \, r \, dr \, d\theta$
 A) 2000π B) 200π C) 40000π D) 20000π

351) _____

Solve the problem.

- 352) Let R be a thin triangular region cut off from the first quadrant by the line $x + y = 12$. If the density of R is given by $\delta(x, y) = ax + \frac{y}{a}$ where $a > 0$, for what value of a will the moment of inertia of R about the x-axis be a minimum?
 A) $\frac{1}{\sqrt{3}}$ B) $\sqrt{3}$ C) $\frac{1}{\sqrt{12}}$ D) $\sqrt{12}$

352) _____

Use a CAS integration utility to evaluate the triple integral of the given function over the specified solid region.

- 353) $F(x, y, z) = x^2 y^4 z^2$ over the cylinder bounded by $x^2 + y^2 \leq 16$ and the planes $z = -6, z = -4$
 A) $\frac{622592}{13}\pi$ B) $\frac{2490368}{59}\pi$ C) $\frac{2490368}{49}\pi$ D) $\frac{155648}{3}\pi$

353) _____

Express the area of the region bounded by the given line(s) and/or curve(s) as an iterated double integral.

- 354) The coordinate axes and the line $\frac{x}{5} + \frac{y}{4} = 1$
 A) $\int_0^5 \int_{y/4}^5 dx \, dy$ B) $\int_0^5 \int_{y/5}^4 dx \, dy$
 C) $\int_0^5 \int_{x/5}^4 dy \, dx$ D) $\int_0^5 \int_0^{4(1-x/5)} dy \, dx$

354) _____

Solve the problem.

- 355) Find the mass of a tetrahedron of density $\delta(x, y, z) = x + y + z$ bounded by the coordinate planes and the plane $\frac{x}{9} + \frac{y}{7} + \frac{z}{2} = 1$.
 A) $\frac{756}{5}$ B) $\frac{567}{5}$ C) 126 D) $\frac{189}{2}$

355) _____

Find the volume of the indicated region.

- 356) The region bounded above by the sphere $x^2 + y^2 + z^2 = 36$ and below by the cone $z = \sqrt{x^2 + y^2}$
 A) $72\pi(2 - \sqrt{2})$ B) $72\pi(2 - \sqrt{3})$ C) $54\pi(2 - \sqrt{3})$ D) $54\pi(2 - \sqrt{2})$

356) _____

Use the given transformation to evaluate the integral.

357) $x = 7u, y = 6v, z = 4w;$
 $\int \int \int_R \left[\frac{x^2}{49} + \frac{y^2}{36} + \frac{z^2}{16} \right]^{\pi} dx \, dy \, dz,$

where R is the interior of the ellipsoid $\frac{x^2}{49} + \frac{y^2}{36} + \frac{z^2}{16} = 1$

- A) $\frac{336\pi}{\pi+1}$ B) $\frac{672\pi}{\pi+1}$ C) $\frac{672\pi}{\pi+2}$

- D) $\frac{672\pi}{\pi+3}$

357) _____

Evaluate the integral.

358) $\int_0^{\pi} \int_0^{(1-\cos\phi)/2} \int_0^{8\pi} p^2 \sin\phi \, dp \, d\theta \, d\phi$

- A) 4π B) $\frac{8}{3}\pi$ C) $\frac{4}{3}\pi$

- D) 2π

358) _____

Find the average value of the function f over the region R.

359) $f(x, y) = \frac{1}{(xy)^2}$

R: $1 \leq x \leq 7, 1 \leq y \leq 7$

- A) $\frac{1}{7}$ B) $\frac{\ln 7}{7}$ C) $\frac{1}{49}$

- D) $\frac{\ln 7}{49}$

359) _____

Evaluate the integral.

360) $\int_0^4 \int_0^{10} \int_0^5 xyz \, dx \, dy \, dz$

- A) $\frac{20000}{3}$ B) 10000 C) 5000

- D) $\frac{10000}{3}$

360) _____

Integrate the function f over the given region.

361) $f(x, y) = xy$ over the rectangle $4 \leq x \leq 6, 2 \leq y \leq 5$

- A) 140 B) 105 C) 210

- D) 70

361) _____

Evaluate the cylindrical coordinate integral.

362) $\int_{3\pi}^{8\pi} \int_3^{10} \int_{7/r}^{8/r} dz \, r \, dr \, d\theta$

- A) 70π B) 490π C) 245π

- D) 35π

362) _____

Evaluate the integral.

363) $\int_0^2 \int_0^{\sqrt{z}} \int_0^{5\pi} (r^2 \sin^2 \theta + z^2) r \, d\theta \, dr \, dz$

- A) $\frac{35}{6}$ B) $\frac{140}{3}\pi$ C) $\frac{35}{3}\pi$

- D) $\frac{7}{3}\pi$

363) _____

Find the volume of the indicated region.

- 364) The tetrahedron bounded by the coordinate planes and the plane $\frac{x}{3} + \frac{y}{5} + \frac{z}{4} = 1$ 364) _____
 A) 15 B) 30 C) 20 D) 10

Solve the problem.

- 365) Find the moment of inertia I_y of a tetrahedron of constant density $\delta(x, y, z) = x + y + z$ bounded by the coordinate planes and the plane $\frac{x}{2} + \frac{y}{5} + \frac{z}{3} = 1$. 365) _____
 A) $\frac{750}{47}$ B) $\frac{50}{3}$ C) $\frac{250}{9}$ D) $\frac{375}{16}$

- 366) Find the center of mass of the hemisphere of constant density bounded $z = \sqrt{49 - x^2 - y^2}$ and the xy -plane. 366) _____
 A) $\bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{7}{4}$ B) $\bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{7}{3}$
 C) $\bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{14}{3}$ D) $\bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{21}{8}$

- 367) Find the radius of gyration about the x -axis of a thin infinite region in the first quadrant bounded by the coordinate axes and the curve $y = e^{-2x}$ if $\delta(x, y) = xy$. 367) _____
 A) $\frac{\sqrt{3}}{4}$ B) $\frac{\sqrt{5}}{2}$ C) $\frac{\sqrt{5}}{4}$ D) $\frac{\sqrt{2}}{4}$

- 368) Find the volume of the parallelepiped enclosed by the planes $3x + y - z = 1, 3x + y - z = 3, -x + 3y + z = -2, -x + 3y + z = 5, x - y + 2z = -1, x - y + 2z = 6$. 368) _____
 A) $\frac{14}{13}$ B) $\frac{21}{26}$ C) $\frac{7}{13}$ D) $\frac{49}{13}$

Evaluate the integral.

- 369) $\int_{-6}^{10} \int_{-2}^8 dy dx$ 369) _____
 A) 68 B) 92 C) 1 D) 160

Solve the problem.

- 370) Find the centroid of the region cut from the first quadrant by the line $x + y = 9$. 370) _____
 A) $\bar{x} = \frac{9}{2}, \bar{y} = \frac{9}{2}$ B) $\bar{x} = 3, \bar{y} = 3$ C) $\bar{x} = 6, \bar{y} = 6$ D) $\bar{x} = \frac{9}{4}, \bar{y} = \frac{9}{4}$

Evaluate the cylindrical coordinate integral.

- 371) $\int_0^{\pi/2} \int_8^{16} \int_{1/r}^{1/r} z \cos \theta dz r dr d\theta$ 371) _____
 A) $\frac{512 \ln 2 - 3}{1024}$ B) $\frac{256 \ln 2 - 3}{1024}$ C) $\frac{256 \ln 2 - 6}{1024}$ D) $\frac{512 \ln 2 - 6}{1024}$

Use a CAS integration utility to evaluate the triple integral of the given function over the specified solid region.

- 372) $F(x, y, z) = (x + y + z)^2$ over the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{4} + \frac{y}{8} + \frac{z}{6} = 1$ 372) _____
 A) $\frac{1760}{3}$ B) $\frac{2816}{5}$ C) 660 D) 704

Solve the problem.

- 373) Given that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$,
 evaluate $\int_{-\infty}^\infty e^{-5x^2} dx$.
 A) $\frac{1}{2}\sqrt{5\pi}$ B) $\sqrt{\frac{\pi}{5}}$ C) $\sqrt{5\pi}$ D) $\frac{1}{2}\sqrt{\frac{\pi}{5}}$ 373) _____

- 374) Let D be the region bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $z = \sqrt{49 - x^2 - y^2}$. Set up the triple integral in cylindrical coordinates that gives the volume of using the order of integration $dz dr d\theta$. 374) _____

- A) $\int_0^{\pi/2} \int_0^7 \int_0^{\sqrt{49 - r^2}} r dz dr d\theta$ B) $\int_0^{\pi/2} \int_0^7 \int_0^{\sqrt{49 - r^2}} r dz dr d\theta$
 C) $\int_0^{2\pi} \int_0^7 \int_0^{\sqrt{49 - r^2}} r dz dr d\theta$ D) $\int_0^{2\pi} \int_0^7 \int_0^{\sqrt{49 - r^2}} r dz dr d\theta$

- 375) Integrate $f(x, y) = \frac{\ln(x^2 + y^2)}{x^2 + y^2}$ over the region $1 \leq x^2 + y^2 \leq 16$. 375) _____
 A) $2\pi \ln 4$ B) $\pi(\ln 4)^2$ C) $\pi \ln 4$ D) $2\pi(\ln 4)^2$

- 376) Evaluate $\int_0^\infty \int_0^\infty \frac{1}{(7 + x^2 + y^2)^4} dx dy$.
 A) $\frac{\pi}{1029}$ B) $\frac{\pi}{4116}$ C) $\frac{\pi}{1715}$ D) $\frac{\pi}{6860}$ 376) _____

- 377) Let A and B be the squares $\{(x, y) | 0 \leq x, y \leq 3\}$ and $\{(x, y) | 3 \leq x, y \leq 4\}$, respectively. Use Pappus's formula to find the centroid of $A \cup B$.
 A) $\bar{x} = \frac{17}{10}, \bar{y} = \frac{17}{10}$ B) $\bar{x} = \frac{13}{2}, \bar{y} = \frac{13}{2}$ C) $\bar{x} = \frac{101}{10}, \bar{y} = \frac{101}{10}$ D) $\bar{x} = \frac{53}{10}, \bar{y} = \frac{53}{10}$ 377) _____

Evaluate the integral.

$$378) \int_0^{\pi/2} \int_2^{10} \int_0^{\pi/3} \rho^3 \sin \phi \cos \phi d\phi d\rho d\theta$$

- A) 780π B) 936π C) 1560π D) 468π

378) _____

Find the average value of $F(x, y, z)$ over the given region.

$$379) F(x, y, z) = x^2 + y^2 + z^2 \text{ over the cube in the first octant bounded by the coordinate planes and the planes } x = 4, y = 4, z = 4$$

- A) 48 B) 32 C) 16 D) 4

379) _____

Evaluate the cylindrical coordinate integral.

$$380) \int_0^{7\pi} \int_0^4 \int_{-\sqrt{16-r^2}}^{\sqrt{16-r^2}} dz r dr d\theta$$

- A) $\frac{448}{3}\pi$ B) $\frac{896}{3}\pi$ C) $\frac{2240}{3}\pi$ D) 224π

380) _____

Express the area of the region bounded by the given line(s) and/or curve(s) as an iterated double integral.

$$381) \text{The lines } x = 0, y = 3x, \text{ and } y = 5$$

- A) $\int_0^3 \int_0^{x/5} dy dx$
B) $\int_0^3 \int_0^{y/5} dx dy$
C) $\int_0^5 \int_0^{y/3} dx dy$
D) $\int_0^5 \int_0^{y/3} dy dx$

381) _____

Solve the problem.

$$382) \text{For what value of } b \text{ is the volume of the ellipsoid } \frac{x^2}{100} + \frac{y^2}{b^2} + \frac{z^2}{36} = 1 \text{ equal to } 8\pi?$$

- A) $b = \frac{1}{10}$ B) $b = \frac{1}{15}$ C) $b = \frac{1}{5}$ D) $b = \frac{2}{15}$

382) _____

$$383) \text{Find the center of mass of the region of constant density bounded by the paraboloid } z = 25 - x^2 - y^2 \text{ and the } xy\text{-plane.}$$

- A) $\bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{5}{2}$
B) $\bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{25}{2}$
C) $\bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{5}{3}$
D) $\bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{25}{3}$

383) _____

Evaluate the integral.

$$384) \int_0^1 \int_0^1 (10x + 2y) dy dx$$

- A) 4 B) -4 C) 52 D) 6

384) _____

$$385) \int_2^9 \int_{5\pi}^{9\pi} \int_0^\theta rz dz d\theta dr$$

- A) $\frac{36089}{3}\pi^3$
B) $\frac{11627}{3}\pi^3$
C) $\frac{46508}{15}\pi^3$
D) $\frac{23858}{3}\pi^3$

385) _____

Evaluate the spherical coordinate integral.

$$386) \int_0^{\pi/2} \int_0^{\pi/2} \int_6^8 \rho \sin \phi d\rho d\phi d\theta$$

- A) 7π
B) 7
C) $\frac{14}{3}$
D) $\frac{14}{3}\pi$

386) _____

$$387) \int_0^{6\pi} \int_0^\pi \int_0^{(1-\cos\phi)/2} \rho^2 \sin \phi d\rho d\phi d\theta$$

- A) 2π
B) 3π
C) 1π
D) $\frac{3}{2}\pi$

387) _____

Find the area of the region specified by the integral(s).

$$388) \int_0^4 \int_{4-x}^{5-x} dy dx + \int_4^5 \int_0^{5-x} dy dx$$

- A) $\frac{9}{2}$
B) 8
C) $\frac{41}{2}$
D) $\frac{25}{2}$

388) _____

Solve the problem.

389) Evaluate

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{(9+x^2+y^2)^{10}} dx dy.$$

- A) $\frac{\pi}{4.261625379e+09}$
B) $\frac{\pi}{7.74840978e+09}$
C) $\frac{\pi}{3.486784401e+09}$
D) $\frac{\pi}{3.87420489e+09}$

389) _____

Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ or $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ (as appropriate) using the given equations.

$$390) x = -3u - 4v, y = 4u - 5v$$

- A) -31
B) -1
C) 31
D) 1

390) _____

Find the area of the region specified by the integral(s).

$$391) \int_0^4 \int_{x(x-4)}^{x(x-4)(x-8)} dy dx$$

- A) 128
B) $\frac{512}{3}$
C) $\frac{224}{3}$
D) $\frac{704}{3}$

391) _____

Evaluate the integral.

$$392) \int_0^{\pi/2} \int_0^{\pi/2} \int_0^9 \cos \phi \rho^2 \sin \phi d\rho d\theta d\phi$$

A) $\frac{243}{8}\pi$

B) $\frac{81}{2}\pi^2$

C) $\frac{243}{8}\pi^2$

D) $\frac{81}{2}\pi$

392) _____

Solve the problem.

393) Let D be the region that is bounded below by the cone $\phi = \frac{\pi}{4}$ and above by the sphere $\rho = 9$. Set up the triple integral for the volume of D in spherical coordinates.

A) $\int_0^{2\pi} \int_0^{\pi/2} \int_0^9 \rho^2 \sin \phi d\rho d\phi d\theta$

B) $\int_0^{2\pi} \int_0^{3\pi/4} \int_0^9 \rho^2 \sin \phi d\rho d\phi d\theta$

C) $\int_0^{2\pi} \int_0^{\pi/4} \int_0^9 \rho^2 \sin \phi d\rho d\phi d\theta$

D) $\int_0^{2\pi} \int_0^{\pi} \int_0^9 \rho^2 \sin \phi d\rho d\phi d\theta$

393) _____

Set up the iterated integral for evaluating

$$\int \int \int f(r, \theta, z) dz r dr d\theta$$

over the given region D.

394) D is the rectangular solid whose base is the triangle with vertices at (0, 0), (9, 0), and (9, 9), and whose top lies in the plane $z = 3$.

A) $\int_0^{\pi/4} \int_0^9 \csc \theta \int_0^3 f(r, \theta, z) dz r dr d\theta$

B) $\int_0^{\pi/2} \int_0^9 \csc \theta \int_0^3 f(r, \theta, z) dz r dr d\theta$

C) $\int_0^{\pi/4} \int_0^9 \sec \theta \int_0^3 f(r, \theta, z) dz r dr d\theta$

D) $\int_0^{\pi/2} \int_0^9 \sec \theta \int_0^3 f(r, \theta, z) dz r dr d\theta$

394) _____

Solve the problem.

395) Solve for a:

$$\int_0^{48a} \int_0^a \int_0^{x^2} dy dx dz = 10,000$$

A) $a = \frac{10}{3}$

B) $a = \frac{5}{3}$

C) $a = \frac{5}{2}$

D) $a = 5$

395) _____

Use the given transformation to evaluate the integral.

396) $u = y - x, v = y + x;$
$$\int \int_R \cos \left(\frac{y-x}{y+x} \right) dx dy,$$

where R is the trapezoid with vertices at (6, 0), (10, 0), (0, 6), (0, 10)

A) $16 \sin 1$

B) $16 \sin 2$

C) $32 \sin 1$

D) $32 \sin 2$

396) _____

Solve the problem.

397) Find the mass of a thin infinite region in the first quadrant bounded by the coordinate axes and the curve $y = e^{-5x}$ if $\delta(x, y) = xy$. 397) _____

A) $\frac{1}{100}$

B) $\frac{1}{75}$

C) $\frac{1}{200}$

D) $\frac{2}{75}$

Find the volume of the indicated region.

398) The region common to the interiors of the cylinders $x^2 + y^2 = 25$ and $x^2 + z^2 = 25$ 398) _____

A) $\frac{2000}{3}$

B) $\frac{250}{3}$

C) $\frac{500}{3}$

D) 500

Evaluate the integral.

399) $\int_0^{\pi/2} \int_4^8 \int_0^{\pi/2} \rho^3 \sin \phi d\rho d\phi d\theta$ 399) _____

A) 640π

B) 960π

C) $\frac{1280}{3}\pi$

D) 480π

400) $\int_1^{e^3} \int_1^{e^8} \int_1^{e^7} \frac{1}{xyz} dx dy dz$ 400) _____

A) 504

B) 168

C) 336

D) 56

401) $\int_0^9 \int_0^{\sqrt{9x}} x^2 dy dx$ 401) _____

A) $\frac{4374}{5}$

B) $\frac{13122}{7}$

C) $\frac{2187}{5}$

D) $\frac{6561}{7}$

Evaluate the spherical coordinate integral.

402) $\int_0^{\pi/2} \int_0^{\pi/3} \int_4^{9 \sec \phi} \rho^4 \sin^2 \phi \cos \phi d\rho d\phi d\theta$ 402) _____

A) $\frac{58025}{9}\pi\sqrt{2}$

B) $\frac{11605}{2}\pi\sqrt{2}$

C) $\frac{11605}{2}\pi\sqrt{3}$

D) $\frac{58025}{9}\pi\sqrt{3}$

Find the average value of the function f over the region R.

403) $f(x, y) = e^{x^2}$ 403) _____
R is the region bounded by $y = 6x, y = 7x, x = 0$, and $x = 1$.

A) $\frac{2e-1}{36}$

B) $2e-1$

C) $e-1$

D) $\frac{e-1}{6}$

Find the average value of the function over the region.

404) $f(\rho, \phi, \theta) = \rho$ over the solid ball $\rho \leq 5$ 404) _____

A) $\frac{4}{15}$

B) 5

C) $\frac{15}{4}$

D) 5π

Find the average value of $F(x, y, z)$ over the given region.

- 405) $F(x, y, z) = 2x$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 8, y = 6, z = 10$
 A) 128 B) 8 C) 16 D) 64 405) _____

Write an equivalent double integral with the order of integration reversed.

- 406) $\int_1^6 \int_0^{\ln x} 2x \, dy \, dx$ 406) _____
 A) $\int_0^{\ln 6} \int_{e^y}^6 2x \, dx \, dy$ B) $\int_0^{\ln 6} \int_{e^y}^6 12x \, dx \, dy$
 C) $\int_0^{\ln 6} \int_1^6 12x \, dx \, dy$ D) $\int_0^{\ln 6} \int_1^6 2x \, dx \, dy$

Evaluate the integral.

- 407) $\int_0^\infty \int_0^\infty \int_0^\infty e^{-(10x + 5y + 7z)} \, dz \, dx \, dy$ 407) _____
 A) $\frac{1}{350}$ B) $\frac{3}{350}$ C) $\frac{1}{700}$ D) $\frac{4}{175}$

Solve the problem.

- 408) A container has the shape of the region in the first octant that is bounded by the parabolic cylinder with equation $x = y^2$ and by the planes with equations $z = 0, x = 0$, and $z = 3 - y$. The container is filled with liquid of density δ . Calculate the work needed to pump the liquid to the top of the container (that is, to the level $z = 3$). Assume units of meters and kilograms.
 A) $81g\delta J$ B) $\frac{243}{8}g\delta J$ C) $\frac{81}{4}g\delta J$ D) $\frac{81}{5}g\delta J$

Find the area of the region specified by the integral(s).

- 409) $\int_0^8 \int_0^y dx \, dy + \int_0^8 \int_y^8 dx \, dy$ 409) _____
 A) 64 B) 128 C) 4 D) 8

Solve the problem.

- 410) Find the centroid of the region enclosed between the cone with equation $z = 3\sqrt{x^2 + y^2}$ and the plane with equation $z = 3$.
 A) $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, \frac{12}{5})$ B) $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, \frac{9}{4})$
 C) $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, \frac{15}{8})$ D) $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 2)$

Find the average value of $F(x, y, z)$ over the given region.

- 411) $F(x, y, z) = x^7 y^4 z^8$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 2, y = 1, z = 3$
 A) 17496 B) $\frac{69984}{19}$ C) $\frac{69984}{5}$ D) 69984 411) _____

Solve the problem.

- 412) Set up the triple integral for the volume of the sphere $\rho = 8$ in cylindrical coordinates. 412) _____

- A) $\int_0^{2\pi} \int_0^8 \int_{-\sqrt{64-r^2}}^{\sqrt{64-r^2}} r \, dz \, dr \, d\theta$
 B) $\int_0^{2\pi} \int_0^8 \int_0^{\sqrt{64-r^2}} dz \, dr \, d\theta$
 C) $\int_0^{2\pi} \int_0^8 \int_{-\sqrt{64-r^2}}^{\sqrt{64-r^2}} dz \, dr \, d\theta$
 D) $\int_0^{2\pi} \int_0^8 \int_0^{\sqrt{64-r^2}} r \, dz \, dr \, d\theta$

- 413) Write an iterated triple integral in the order $dz \, dy \, dx$ for the volume of the region in the first octant enclosed by the cylinder $x^2 + y^2 = 36$ and the plane $z = 2$. 413) _____

- A) $\int_{-6}^6 \int_{-\sqrt{36-y^2}}^{\sqrt{36-y^2}} \int_0^2 -y \, dz \, dy \, dx$
 B) $\int_{-6}^6 \int_{-\sqrt{36-x^2}}^{\sqrt{36-x^2}} \int_0^2 dz \, dy \, dx$
 C) $\int_{-6}^6 \int_{-\sqrt{36-y^2}}^{\sqrt{36-y^2}} \int_0^2 dz \, dy \, dx$
 D) $\int_{-6}^6 \int_{-\sqrt{36-x^2}}^{\sqrt{36-x^2}} \int_0^y dz \, dy \, dx$

Write an equivalent double integral with the order of integration reversed.

- 414) $\int_{\ln 2}^{\ln 5} \int_{e^y}^5 5y \, dx \, dy$ 414) _____
 A) $\int_{\ln 5}^5 \int_{\ln 2}^{\ln x} 5y \, dy \, dx$ B) $\int_2^5 \int_{\ln 2}^{\ln x} 5y \, dy \, dx$
 C) $\int_2^5 \int_{\ln x}^{\ln 5} 5y \, dy \, dx$ D) $\int_{\ln 2}^5 \int_{\ln 5}^{\ln x} 5y \, dy \, dx$

Solve the problem.

- 415) Find the mass of the region of density $\delta(x, y, z) = \frac{1}{9-x^2-y^2}$ bounded by the paraboloid $z = 9 - x^2 - y^2$ and the xy -plane. 415) _____
 A) 27π B) 3π C) 81π D) 9π

Find the volume of the indicated region.

- 416) The region that lies inside the sphere $x^2 + y^2 + z^2 = 81$ and outside the cylinder $x^2 + y^2 = 64$. 416) _____
 A) $\frac{5(729 - 17^3/2)\pi}{2}$ B) $\frac{4(729 - 17^3/2)\pi}{3}$ C) $\frac{2(729 - 17^3/2)\pi}{3}$ D) $\frac{3(729 - 17^3/2)\pi}{2}$

Solve the problem.

- 417) Evaluate 417) _____

$$\int_{-6}^6 \int_{-\sqrt{36-x^2}}^{\sqrt{36-x^2}} \int_0^8 (x^2 + y^2)z \, dz \, dy \, dx$$

by transforming to cylindrical or spherical coordinates.

- A) 576π B) 3456π C) 27648π D) 20736π

- 418) Find the moment of inertia about the origin of the region enclosed by the curve $r = 10 + 5 \sin \theta$ if $\delta(x, y) = \frac{1}{r^2}$.
- A) $\frac{425}{2}\pi$ B) $\frac{225}{2}\pi$ C) $\frac{325}{2}\pi$ D) $\frac{625}{2}\pi$

418) _____

Find the area of the region specified in polar coordinates.

- 419) One petal of the rose curve $r = 7 \sin 2\theta$
- A) $\frac{49}{6}\pi$ B) $\frac{49}{8}\pi$ C) $\frac{49}{4}\pi$ D) $\frac{49}{2}\pi$

419) _____

Set up the iterated integral for evaluating

$$\int_D \int \int f(r, \theta, z) dz r dr d\theta$$

over the given region D.

- 420) D is the right circular cylinder whose base is the circle $r = 5 \cos \theta$ in the xy-plane and whose top lies in the plane $z = 9 - x$.

420) _____

$$\begin{array}{ll} A) \int_0^\pi \int_0^{5 \cos \theta} \int_0^{9 - \sin \theta} f(r, \theta, z) dz r dr d\theta \\ B) \int_0^{2\pi} \int_0^{5 \sin \theta} \int_0^{9 - r \cos \theta} f(r, \theta, z) dz r dr d\theta \\ C) \int_0^{2\pi} \int_0^{5 \sin \theta} \int_0^{9 - \sin \theta} f(r, \theta, z) dz r dr d\theta \\ D) \int_0^\pi \int_0^{5 \cos \theta} \int_0^{9 - r \cos \theta} f(r, \theta, z) dz r dr d\theta \end{array}$$

Evaluate the integral.

- 421) $\int_7^8 \int_{5\pi}^{10\pi} \int_0^{\theta} rz dr d\theta dz$
- A) $\frac{14875}{4}\pi^3$ B) $\frac{219625}{12}\pi^3$ C) $\frac{132125}{12}\pi^3$ D) $\frac{102375}{4}\pi^3$

421) _____

Set up the iterated integral for evaluating

$$\int_D \int \int f(r, \theta, z) dz r dr d\theta$$

over the given region D.

- 422) D is the solid right cylinder whose base is the region between the circles $r = 3 \sin \theta$ and $r = 7 \sin \theta$, and whose top lies in the plane $z = 8 - x - y$.

$$\begin{array}{ll} A) \int_0^\pi \int_{3 \sin \theta}^{7 \sin \theta} \int_0^{8 - r(\cos \theta + \sin \theta)} f(r, \theta, z) dz r dr d\theta \\ B) \int_0^{2\pi} \int_{3 \sin \theta}^{7 \sin \theta} \int_0^{8 - r(\cos \theta - \sin \theta)} f(r, \theta, z) dz r dr d\theta \\ C) \int_0^{2\pi} \int_{3 \sin \theta}^{7 \sin \theta} \int_0^{8 - r(\cos \theta + \sin \theta)} f(r, \theta, z) dz r dr d\theta \\ D) \int_0^\pi \int_{3 \sin \theta}^{7 \sin \theta} \int_0^{8 - r(\cos \theta - \sin \theta)} f(r, \theta, z) dz r dr d\theta \end{array}$$

Integrate the function f over the given region.

- 423) $f(x, y) = \frac{1}{\ln x}$ over the region bounded by the x-axis, line $x = 10$, and curve $y = \ln x$
- A) 11 B) 10 C) 1 D) 9

Use the given transformation to evaluate the integral.

- 424) $u = y - x$, $v = y + x$;

$$\int_R e^{\frac{v-y}{y+x}} dx dy,$$

where R is the trapezoid with vertices at $(2, 0)$, $(3, 0)$, $(0, 2)$, $(0, 3)$

$$\begin{array}{ll} A) \frac{5(e^2 - 1)}{6e} & B) \frac{5(e^2 - 1)}{4e} \\ C) \frac{5(e^2 - 1)}{3e} & D) \frac{5(e^2 - 1)}{2e} \end{array}$$

Solve the problem.

425) Rewrite the integral

$$\int_0^{1/10} \int_0^{(1-10z)/9} \int_0^{(1-9y-10z)/2} dx dy dz$$

in the order $dz dy dx$.

A) $\int_0^{1/2} \int_0^{(1-2x)/9} \int_0^{(1-9x-2y)/10} dz dy dx$

B) $\int_0^{1/10} \int_0^{(1-2x)/9} \int_0^{(1-10x-9y)/2} dz dy dx$

C) $\int_0^{1/2} \int_0^{(1-2x)/9} \int_0^{(1-2x-9y)/10} dz dy dx$

D) $\int_0^{1/10} \int_0^{(1-10z)/9} \int_0^{(1-9y-10z)/2} dz dy dx$

Write an equivalent double integral with the order of integration reversed.

426) $\int_0^7 \int_{6x/7+4}^{10} dy dx$

A) $\int_4^{10} \int_0^{7y/(-16)} dx dy$

C) $\int_0^{10} \int_0^{7y/(-16)} dx dy$

B) $\int_4^7 \int_0^{7y/(-16)} dx dy$

D) $\int_7^{10} \int_0^{7y/(-16)} dx dy$

Solve the problem.

427) Find the moment of inertia I_Z of a tetrahedron of constant density $\delta(x, y, z) = 1$ bounded by the

coordinate planes and the plane $\frac{x}{9} + \frac{y}{7} + \frac{z}{2} = 1$.

A) $\frac{357}{2}$

B) $\frac{1113}{10}$

C) $\frac{1407}{5}$

D) 273

Evaluate the integral.

428) $\int_0^8 \int_0^{10} (7x - 3y) dx dy$

A) 184

B) 1840

C) 23

D) 230

Evaluate the cylindrical coordinate integral.

429) $\int_3^4 \int_0^{2\pi} \int_0^r \frac{3}{r^2} dr d\theta dz$

A) $\frac{21}{2}$

B) $\frac{7}{2}$

C) $\frac{21}{4}$

D) 7

425) _____

Find the area of the region specified in polar coordinates.

430) The region inside $r = 14 \sin \theta$ and outside $r = 7$

A) $49\left(\frac{\sqrt{3}}{2} + \frac{2\pi}{3}\right)$

B) $49\left(\frac{\sqrt{3}}{4} + \frac{\pi}{3}\right)$

C) $49\left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right)$

D) $49\left(\frac{\sqrt{3}}{4} + \frac{2\pi}{3}\right)$

430) _____

Solve the problem.

431) Let R be a thin triangular region cut off from the first quadrant by the line $x + y = 5$. If the density of R is given by $\delta(x, y) = ax + \frac{y}{a}$ where $a > 0$, for what value of a will the mass of R be a minimum?

A) $\frac{1}{5}$

B) $\frac{2}{3}$

C) 1

D) 5

Evaluate the integral.

432) $\int_0^1 \int_{3y}^1 dx dy$

A) $\frac{5}{2}$

B) 2

C) -1

D) $-\frac{1}{2}$

432) _____

Solve the problem.

433) Integrate $f(x, y) = \sin\sqrt{x^2 + y^2}$ over the region $0 \leq x^2 + y^2 \leq 36$.

433) _____

A) $2\pi(\sin 6 - 6 \cos 6)$

B) $\pi(\sin 6 - 6 \cos 6)$

C) $\pi(\sin 6 - \cos 6)$

D) $2\pi(\sin 6 - \cos 6)$

Evaluate the integral.

434) $\int_0^1 \int_9^{20} (s+t) dt ds$

A) 100

B) 150

C) 195

D) 145

434) _____

Determine the order of integration and then evaluate the integral.

435) $\int_0^{512} \int_{\sqrt{y/8}}^8 \frac{\sin x^2}{x} dx dy$

A) $4(1 - \cos 64)$

B) $2\left(1 - \frac{\cos 64}{8}\right)$

C) $4\left(1 - \frac{\cos 64}{8}\right)$

D) $2(1 - \cos 64)$

435) _____

Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ or $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ (as appropriate) using the given equations.

436) $u = -5x + 3y, v = 4x + 4y$

436) _____

A) -32

B) 32

C) $-\frac{1}{32}$

D) $\frac{1}{32}$

Solve the problem.

437) Find the moment of inertia about the origin of a thin plane of constant density $\delta = 8$ bounded by the coordinate axes and the line $x + y = 8$.

437) _____

A) $\frac{256}{3}$

B) 8192

C) $\frac{2048}{3}$

D) $\frac{16384}{3}$

Determine the order of integration and then evaluate the integral.

$$438) \int_0^6 \int_x^6 \frac{\sin y}{y} dy dx$$

- A) $1 - \cos 6$ B) $1 + \cos 6$ C) $-\cos 6$ D) $\cos 6$

438) _____

Evaluate the improper integral.

$$439) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dx dy}{(x^2 + 64)(y^2 + 16)}$$

- A) $\frac{\pi}{32}$ B) $\frac{\pi^2}{32}$ C) $\frac{\pi^2}{64}$ D) $\frac{\pi}{64}$

439) _____

Express the area of the region bounded by the given line(s) and/or curve(s) as an iterated double integral.

- 440) The parabola $y = x^2$ and the line $y = 2x + 35$

440) _____

- A) $\int_{-5}^7 \int_0^{2x+35-x^2} dx dy$
 B) $\int_0^7 \int_{35}^{2x+x^2} dy dx$
 C) $\int_0^7 \int_{x^2}^{2x+35} dy dx$
 D) $\int_{-5}^7 \int_{x^2}^{2x+35} dy dx$

Solve the problem.

- 441) Let D be the smaller cap cut from a solid ball of radius 8 units by a plane 2 units from the center of the sphere. Set up the triple integral for the volume of D in rectangular coordinates.

- A) $\int_{-\sqrt{68}}^{\sqrt{68}} \int_{-\sqrt{60-x^2}}^{\sqrt{60-x^2}} \int_{8}^{\sqrt{4-x^2-y^2}} dz dy dx$
 B) $\int_{-\sqrt{60}}^{\sqrt{60}} \int_{-\sqrt{60-x^2}}^{\sqrt{60-x^2}} \int_{8}^{\sqrt{4-x^2-y^2}} dz dy dx$
 C) $\int_{-\sqrt{68}}^{\sqrt{68}} \int_{-\sqrt{60-x^2}}^{\sqrt{60-x^2}} \int_{2}^{\sqrt{64-x^2-y^2}} dz dy dx$
 D) $\int_{-\sqrt{60}}^{\sqrt{60}} \int_{-\sqrt{60-x^2}}^{\sqrt{60-x^2}} \int_{2}^{\sqrt{64-x^2-y^2}} dz dy dx$

- 442) Find the moment of inertia I_L of the rectangular solid of density $\delta(x, y, z) = 1$ defined by $0 \leq x \leq 3$, $0 \leq y \leq 9$, $0 \leq z \leq 8$, where L is the line through the points $(4, 1, 0)$, $(4, 2, 0)$.

- A) 3528 B) 6120 C) 7848 D) 6984

442) _____

- 443) Solve for a:

$$\int_a^{10a} \int_a^{10a} \int_a^{10a} dx dz dy = 27$$

- A) $a = \frac{2}{3}$ B) $a = \frac{3}{11}$ C) $a = \frac{1}{3}$ D) $a = \frac{6}{11}$

443) _____

Find the average value of the function f over the region R.

$$444) f(x, y) = \frac{1}{xy}$$

444) _____

R is the region bounded by $y = \frac{1}{x}$, $y = \frac{2}{x}$, $x = 7$, and $x = 10$.

- A) $\frac{7}{10} \ln 2$ B) $70 \ln 2$ C) $\frac{10}{7} \ln 2$ D) $\ln 2$

Use a CAS integration utility to evaluate the triple integral of the given function over the specified solid region.

$$445) F(x, y, z) = x^2 + y^2 + z^2 \text{ over the tetrahedron bounded by the coordinate planes and the plane } \frac{x}{5} + \frac{y}{10} + \frac{z}{7} = 1$$

445) _____

- A) 1015 B) $\frac{5075}{6}$ C) 812 D) $\frac{15225}{16}$

Solve the problem.

- 446) Let A and B be the squares $\{(x, y) \mid 0 \leq x, y \leq 4\}$ and $\{(x, y) \mid 8 \leq x, y \leq 12\}$,

respectively. Use Pappus's formula to find the centroid of $A \cup B$.

- A) $\bar{x} = 5, \bar{y} = 5$ B) $\bar{x} = 6, \bar{y} = 6$ C) $\bar{x} = 7, \bar{y} = 7$ D) $\bar{x} = \frac{14}{3}, \bar{y} = \frac{14}{3}$

- 447) Find the average distance from a point $P(r, \theta)$ in the region bounded by $r = 5 + 8 \sin \theta$ to the origin.

447) _____

- A) $\frac{605}{114}$ B) $\frac{605}{76}$ C) $\frac{1210}{171}$ D) $\frac{605}{171}$

- 448) Find the moment of inertia about the z-axis of a region of constant density δ enclosed by the

448) _____

paraboloid $z = 36 - x^2 - y^2$ and the xy-plane.

- A) $7776\delta\pi$ B) $36\delta\pi$ C) $216\delta\pi$ D) $6\delta\pi$

Find the volume of the indicated region.

- 449) The region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 64$

449) _____

- A) $\frac{4096}{3}\pi$ B) $\frac{2048}{3}\pi$ C) 1024π D) 2048π

Integrate the function f over the given region.

- 450) $f(x, y) = xy$ over the triangular region with vertices $(0, 0)$, $(3, 0)$, and $(0, 6)$

450) _____

- A) $\frac{3}{4}$ B) $\frac{9}{4}$ C) $\frac{27}{2}$ D) $\frac{9}{2}$

Solve the problem.

451) Given that

$$\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2},$$

evaluate

$$\int_0^{\infty} e^{-3x^2} dx.$$

A) $\frac{1}{2}\sqrt{\frac{\pi}{3}}$

B) $\sqrt{3\pi}$

C) $\frac{1}{2}\sqrt{3\pi}$

D) $\sqrt{\frac{\pi}{3}}$

451) _____

Evaluate the spherical coordinate integral.

$$452) \int_0^{\pi/2} \int_0^{\pi/2} \int_9^{10} (\rho \sin \phi)^2 d\rho d\phi d\theta$$

A) $\frac{271}{24}\pi^2$

B) $\frac{271}{18}\pi$

C) $\frac{271}{18}\pi^2$

D) $\frac{271}{24}\pi$

452) _____

Solve the problem.

453) Find the mass of a thin circular plate bounded by $x^2 + y^2 = 64$ if $\delta(x, y) = x^2 + y^2$.

A) 32π

B) 2048π

C) 256π

D) 4π

453) _____

Evaluate the integral by changing the order of integration in an appropriate way.

$$454) \int_0^{125} \int_0^3 \int_{\sqrt[3]{x}}^5 \frac{z}{y^4 + 1} dy dz dx$$

A) $\frac{9}{4} \ln 126$

B) $\frac{9}{8} \ln 126$

C) $\frac{9}{8} \ln 626$

D) $\frac{9}{4} \ln 626$

454) _____

Solve the problem.

455) Find the mass of a thin disk bounded by the circle $r = 8 \sin \theta$ if the disk's density is given by

$\delta(x, y) = x^2 + y^2$.

A) 1280π

B) 384π

C) 768π

D) 640π

455) _____

Evaluate the integral by changing the order of integration in an appropriate way.

$$456) \int_1^2 \int_0^{21} \int_{y/3}^7 \frac{\tan x}{xz} dx dy dz$$

A) $7 \ln 4 \ln \cos 7$

B) $-7 \ln 4 \ln \cos 7$

C) $7 \ln 2 \ln \cos 7$

D) $-7 \ln 2 \ln \cos 7$

456) _____

Change the Cartesian integral to an equivalent polar integral, and then evaluate.

$$457) \int_{-2}^0 \int_{-\sqrt{4-x^2}}^0 \frac{1}{1+\sqrt{x^2+y^2}} dy dx$$

A) $\frac{\pi(2 - \ln 3)}{4}$

B) $\frac{\pi(2 + \ln 3)}{2}$

C) $\frac{\pi(2 + \ln 3)}{4}$

D) $\frac{\pi(2 - \ln 3)}{2}$

457) _____

Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ or $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ (as appropriate) using the given equations.

458) $x = 7u \cosh 8v, y = 7u \sinh 8v$

A) $392u$

B) $448v$

C) $392v$

D) $448u$

458) _____

TRUE/FALSE. Write 'T' if the statement is true and 'F' if the statement is false.

Answer the question.

459) True or false? Consider the double integral

$$\int_0^2 \int_3^6 (x^2 + y) dy dx.$$

The first step in calculating this integral involves integrating with respect to x.

460) True or false? Consider the double integral

$$\int_0^2 \int_3^6 (x^2 + y) dy dx.$$

The first step in calculating this integral involves holding y constant.

461) True or false? Consider the double integral

$$\int_0^2 \int_3^6 (x^2 + y) dx dy.$$

The first step in calculating this integral involves integrating with respect to x.

462) True or false? Consider the double integral

$$\int_0^2 \int_3^6 (x^2 + y) dx dy.$$

The first step in calculating this integral involves holding y constant.

Answer Key

Testname: TEST 6

- 1) Find the area of the region between the curves $y = x^2$ and $y = 2x$ for x between 0 and 2.
- 2) This integral represents the volume of the region enclosed between the paraboloid with equation $z = 3 - r^2$ (or $z = 3 - x^2 - y^2$) and the xy -plane.
- 3) Find the volume bounded above by $f(x,y) = y + x$ which lies over the region for which $4 \leq x \leq 6$ and $1 \leq y \leq 2$.
- 4) Find the area of the region bounded by the curve $y = x^2$ and the lines $y = 0$, $x = 0$, and $x = 5$.
- 5) Find the volume bounded above by $f(x,y) = y + x$ which lies over the region for which $1 \leq x \leq 2$ and $4 \leq y \leq 6$.
- 6) This integral represents the volume of a wedge cut from a sphere of radius 1 by two half-planes whose edges meet at a 45° angle along a diameter of the sphere. The region resembles a section of an orange and is $\frac{1}{8}$ of the entire sphere.

7) B

8) B

9) B

10) C

11) B

12) C

13) A

14) B

15) C

16) A

17) D

18) C

19) B

20) D

21) A

22) D

23) B

24) B

25) B

26) D

27) A

28) D

29) C

30) A

31) D

32) D

33) B

34) B

35) C

36) C

37) D

38) D

39) C

40) A

41) D

42) B

43) C

44) D

45) D

46) D

Answer Key

Testname: TEST 6

- 47) B
- 48) A
- 49) A
- 50) B
- 51) D
- 52) B
- 53) B
- 54) B
- 55) A
- 56) C
- 57) A
- 58) D
- 59) C
- 60) A
- 61) C
- 62) D
- 63) D
- 64) C
- 65) C
- 66) A
- 67) D
- 68) B
- 69) D
- 70) A
- 71) B
- 72) C
- 73) A
- 74) A
- 75) C
- 76) D
- 77) C
- 78) D
- 79) D
- 80) D
- 81) A
- 82) C
- 83) C
- 84) B
- 85) D
- 86) A
- 87) D
- 88) A
- 89) B
- 90) B
- 91) A
- 92) B
- 93) C
- 94) A
- 95) A
- 96) C

Answer Key
Testname: TEST 6

97) A
98) C
99) B
100) C
101) B
102) D
103) D
104) C
105) B
106) A
107) D
108) B
109) B
110) D
111) B
112) D
113) C
114) B
115) D
116) C
117) C
118) D
119) C
120) C
121) D
122) C
123) B
124) D
125) B
126) C
127) D
128) C
129) C
130) D
131) C
132) B
133) A
134) D
135) B
136) C
137) D
138) D
139) D
140) B
141) B
142) A
143) A
144) B
145) A
146) A

Answer Key
Testname: TEST 6

147) D
148) D
149) A
150) B
151) D
152) B
153) A
154) B
155) B
156) B
157) B
158) C
159) B
160) D
161) D
162) D
163) B
164) B
165) D
166) A
167) C
168) C
169) A
170) A
171) B
172) D
173) A
174) D
175) D
176) D
177) A
178) C
179) A
180) C
181) C
182) C
183) A
184) A
185) C
186) A
187) A
188) D
189) D
190) C
191) B
192) B
193) D
194) C
195) A
196) C

Answer Key
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197) B
198) B
199) D
200) B
201) A
202) C
203) C
204) A
205) B
206) A
207) A
208) C
209) C
210) B
211) C
212) D
213) C
214) A
215) A
216) B
217) A
218) A
219) C
220) B
221) B
222) A
223) C
224) A
225) C
226) B
227) D
228) D
229) D
230) C
231) A
232) D
233) A
234) B
235) B
236) C
237) A
238) B
239) B
240) A
241) C
242) B
243) D
244) C
245) D
246) B

Answer Key
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247) D
248) A
249) D
250) A
251) C
252) C
253) B
254) C
255) B
256) B
257) A
258) C
259) B
260) C
261) D
262) B
263) C
264) D
265) C
266) A
267) D
268) C
269) D
270) D
271) C
272) C
273) A
274) D
275) A
276) B
277) B
278) B
279) D
280) A
281) A
282) B
283) B
284) D
285) C
286) C
287) A
288) C
289) B
290) B
291) D
292) D
293) B
294) C
295) D
296) B

Answer Key
Testname: TEST 6

297) B
298) B
299) B
300) C
301) D
302) C
303) B
304) D
305) B
306) D
307) D
308) C
309) D
310) C
311) A
312) D
313) C
314) B
315) B
316) C
317) A
318) A
319) A
320) B
321) C
322) A
323) C
324) B
325) A
326) D
327) B
328) A
329) C
330) C
331) C
332) D
333) B
334) C
335) B
336) C
337) C
338) B
339) A
340) D
341) D
342) B
343) C
344) A
345) B
346) A

Answer Key
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347) C
348) D
349) A
350) C
351) A
352) B
353) D
354) D
355) D
356) A
357) D
358) C
359) C
360) C
361) B
362) D
363) C
364) D
365) B
366) D
367) D
368) D
369) D
370) B
371) A
372) D
373) B
374) C
375) D
376) B
377) A
378) D
379) C
380) B
381) C
382) A
383) D
384) D
385) B
386) A
387) C
388) A
389) C
390) C
391) C
392) A
393) C
394) C
395) D
396) C

Answer Key
Testname: TEST 6

397) C
398) A
399) D
400) B
401) B
402) C
403) C
404) C
405) B
406) A
407) A
408) D
409) A
410) B
411) C
412) A
413) B
414) B
415) D
416) B
417) D
418) B
419) B
420) D
421) A
422) A
423) D
424) B
425) C
426) A
427) D
428) B
429) A
430) C
431) C
432) D
433) A
434) D
435) A
436) C
437) D
438) A
439) B
440) D
441) D
442) B
443) C
444) D
445) A
446) B

Answer Key
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447) C
448) A
449) D
450) C
451) A
452) A
453) B
454) C
455) B
456) D
457) D
458) A
459) FALSE
460) FALSE
461) TRUE
462) TRUE