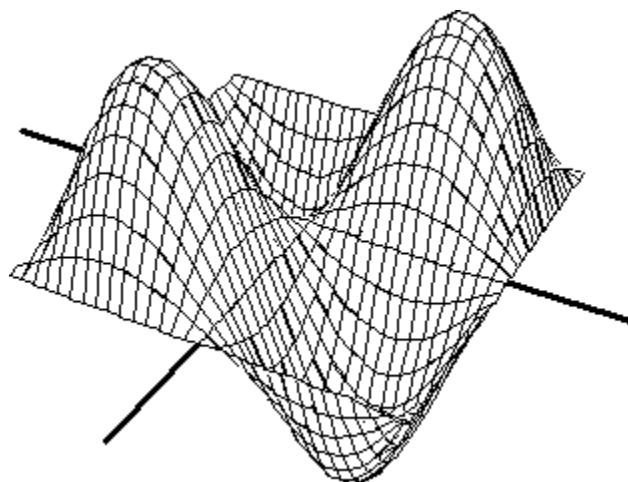


Name \_\_\_\_\_

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

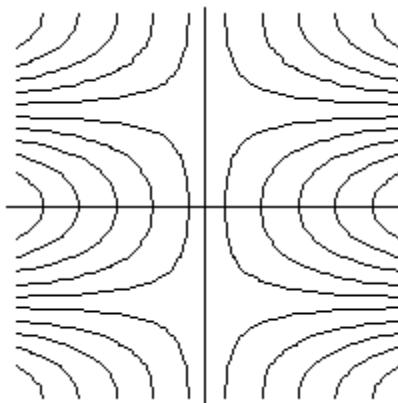
Match the surface show below to the graph of its level curves.

1)

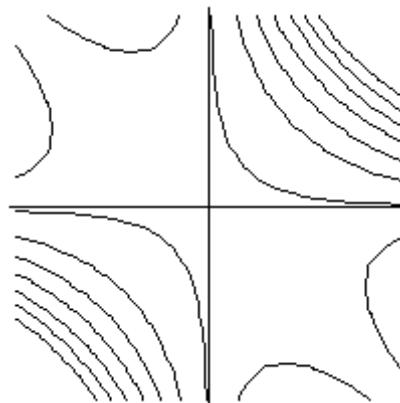


1) \_\_\_\_\_

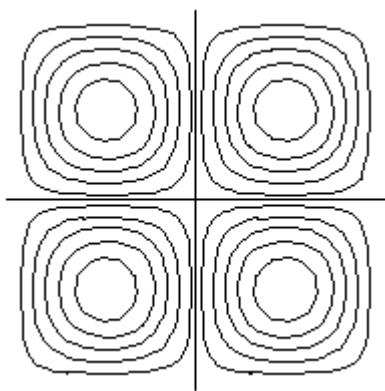
A)



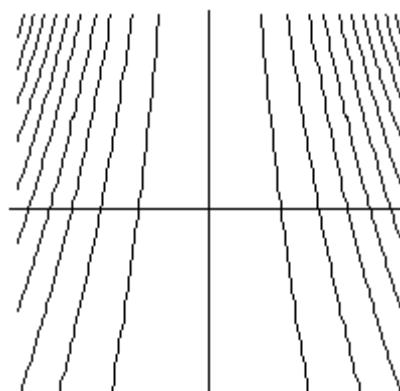
B)



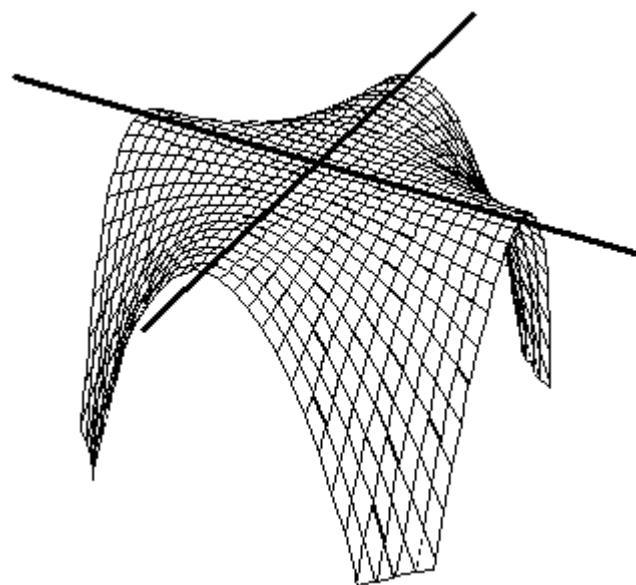
C)



D)

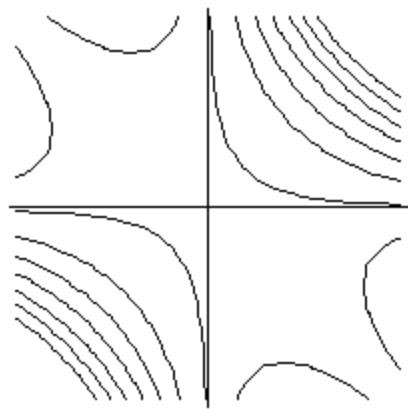


2)

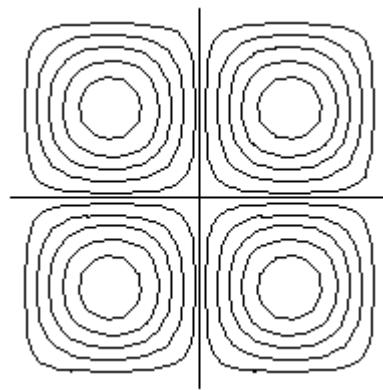


2) \_\_\_\_\_

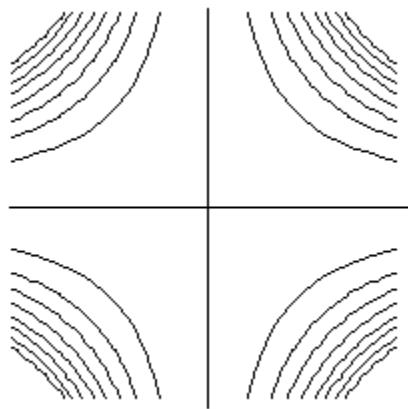
A)



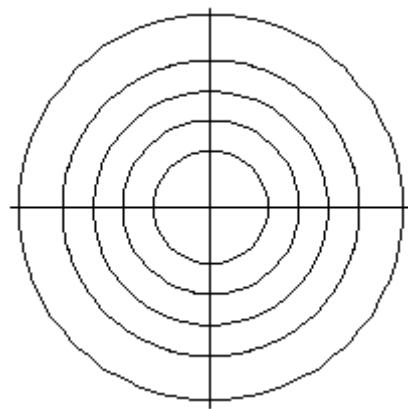
B)



C)

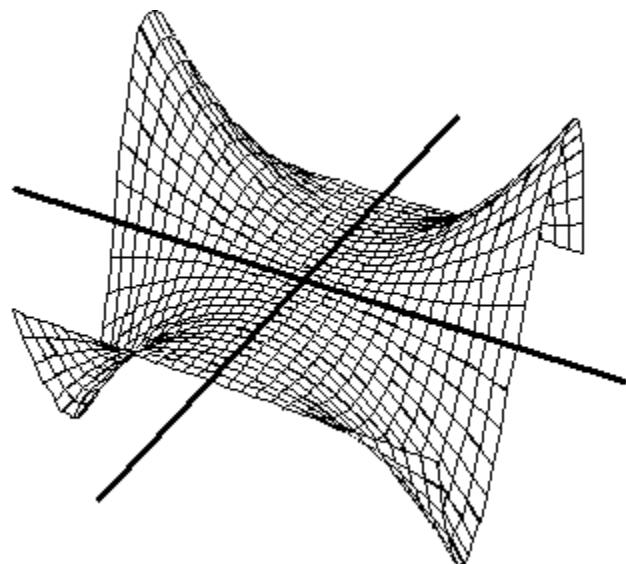


D)

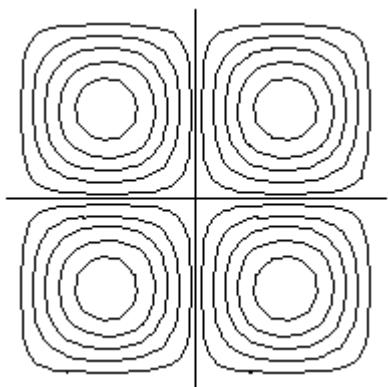


3)

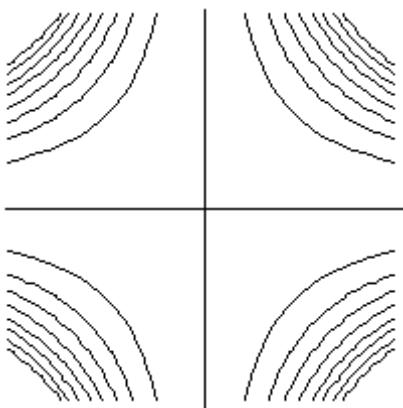
3) \_\_\_\_\_



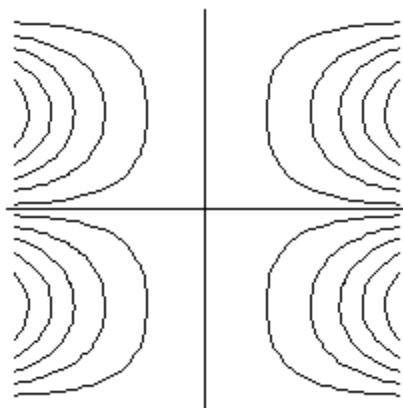
A)



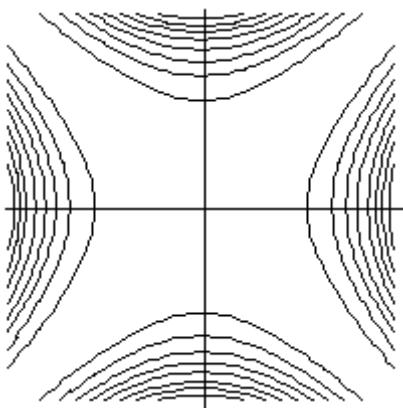
B)



C)

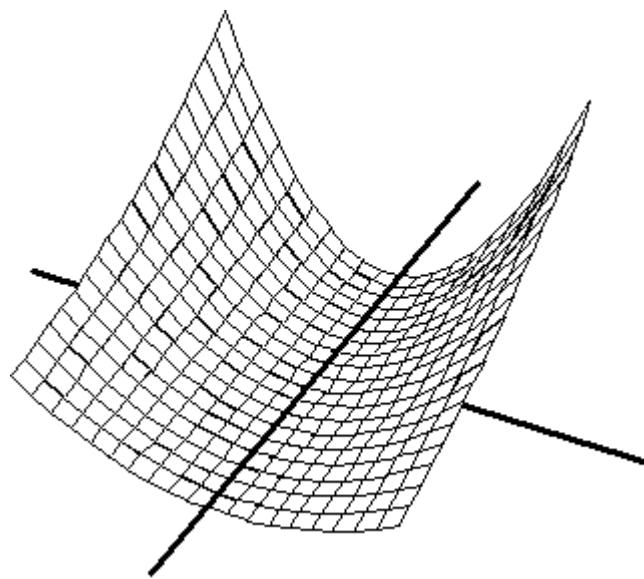


D)

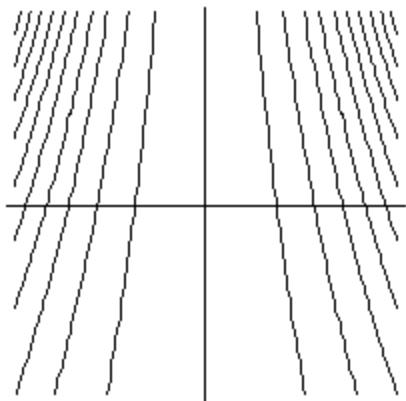


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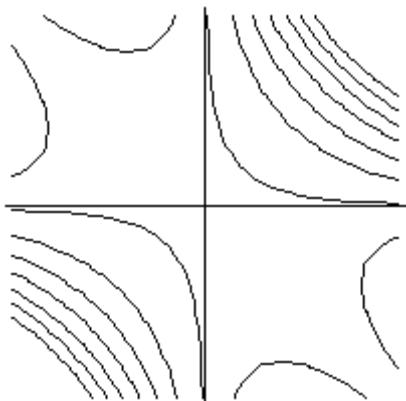
4) \_\_\_\_\_



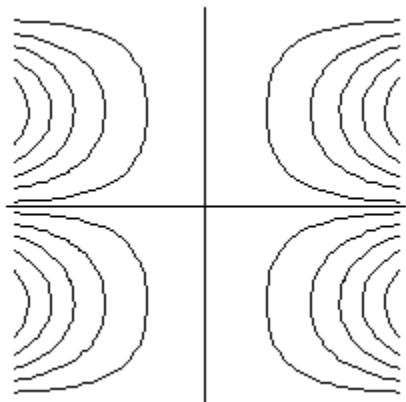
A)



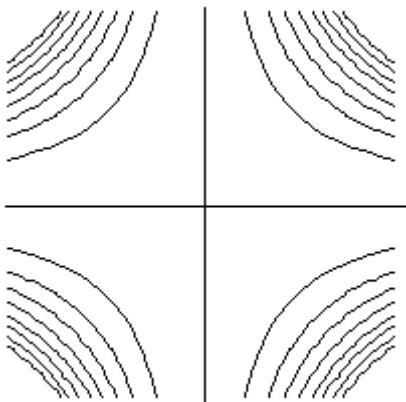
B)



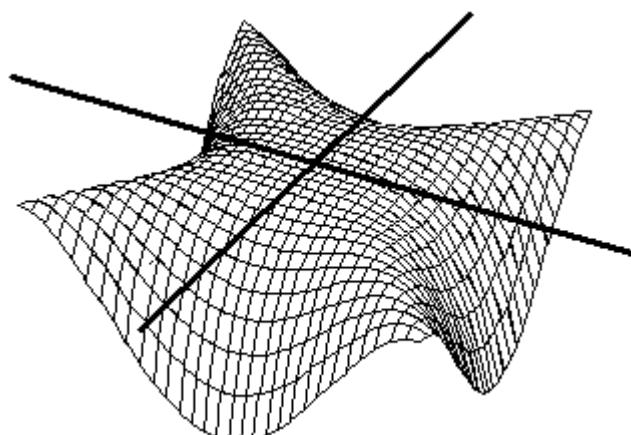
C)



D)

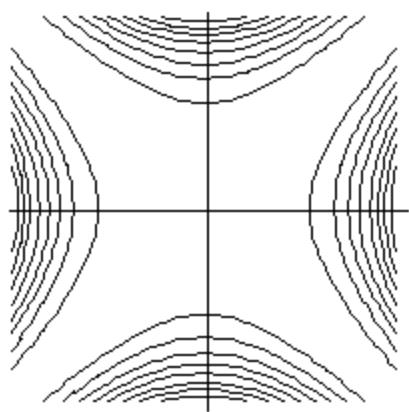


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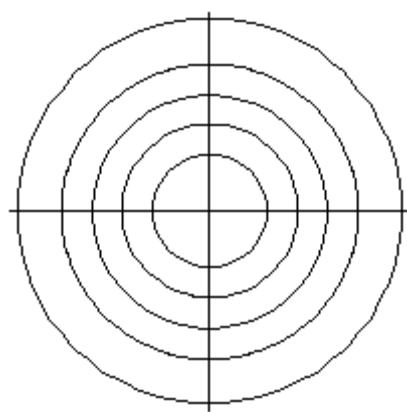


5) \_\_\_\_\_

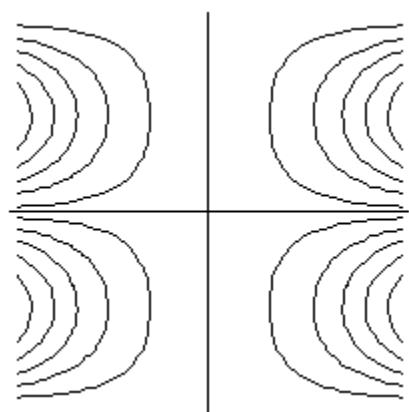
A)



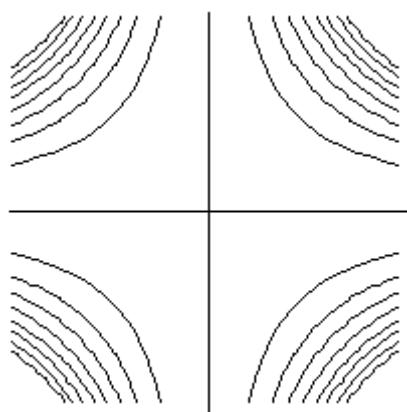
B)



C)

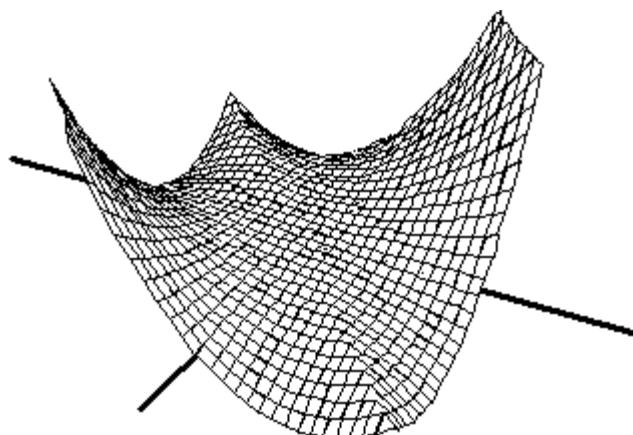


D)

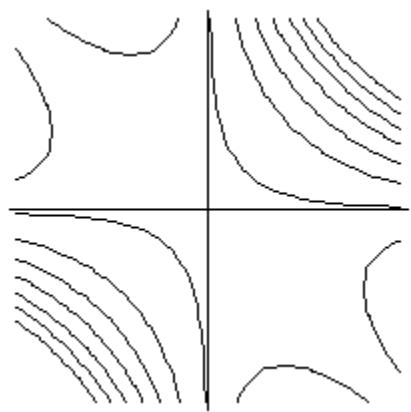


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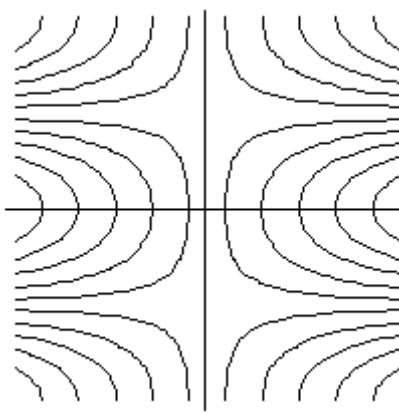
6) \_\_\_\_\_



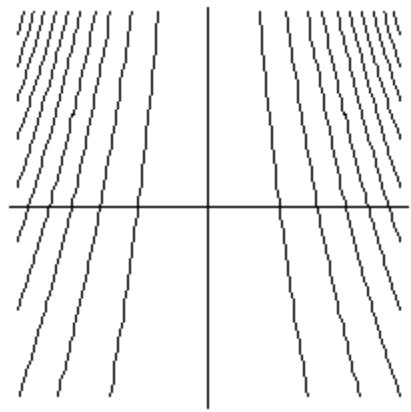
A)



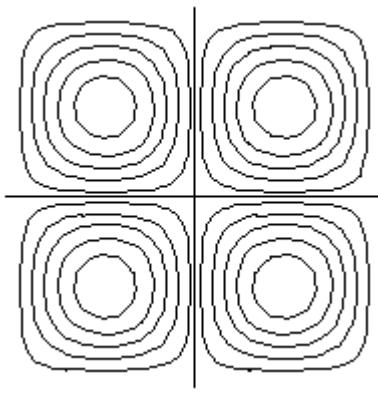
B)



C)



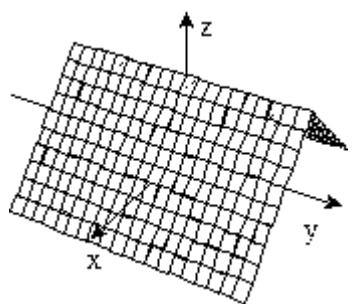
D)



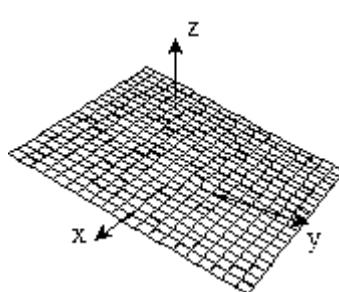
**Sketch the surface  $z = f(x,y)$ .**

7)  $f(x, y) = 1 - x - 2y$

A)

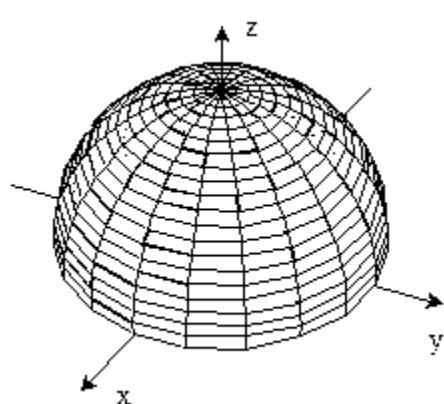


B)

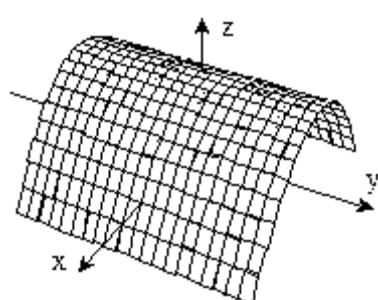


7) \_\_\_\_\_

C)

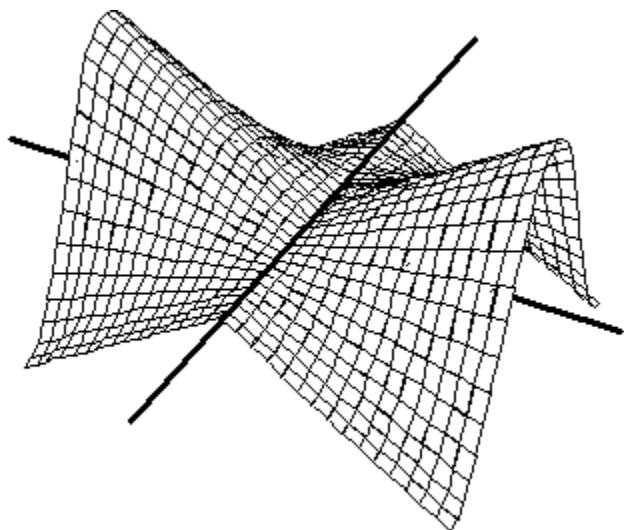


D)



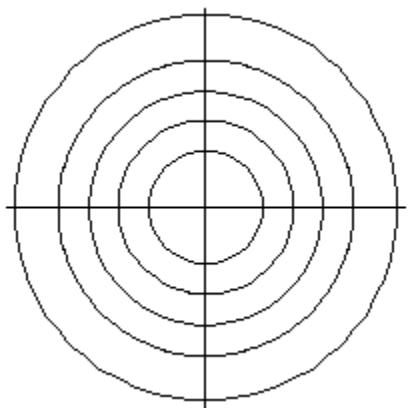
**Match the surface show below to the graph of its level curves.**

8)

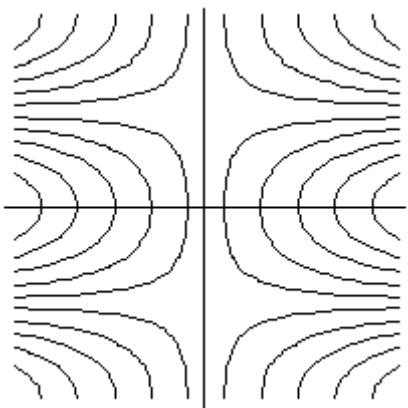


8) \_\_\_\_\_

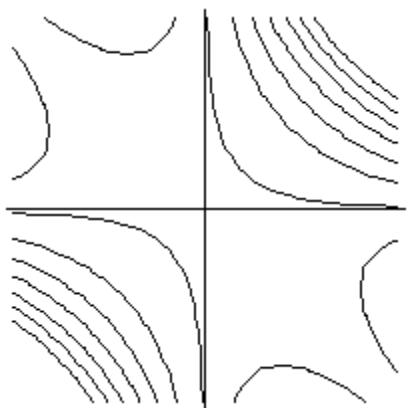
A)



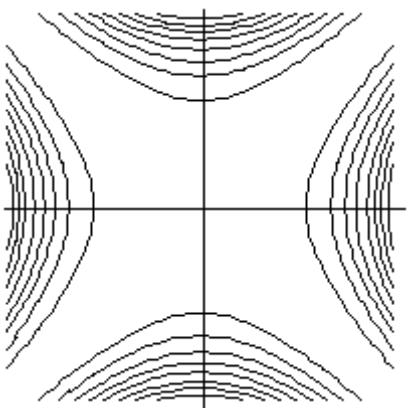
B)



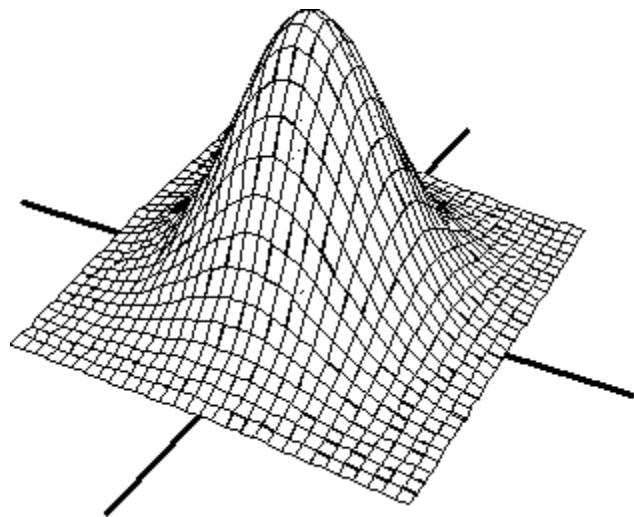
C)



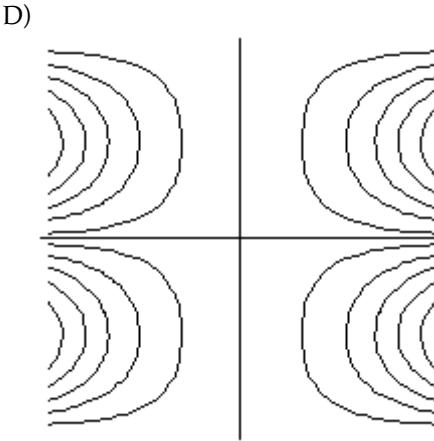
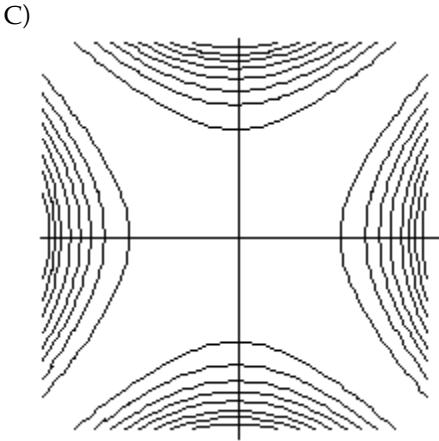
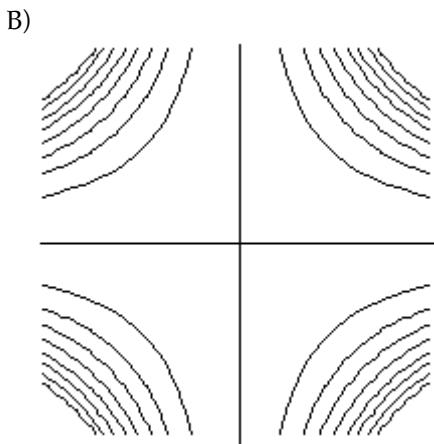
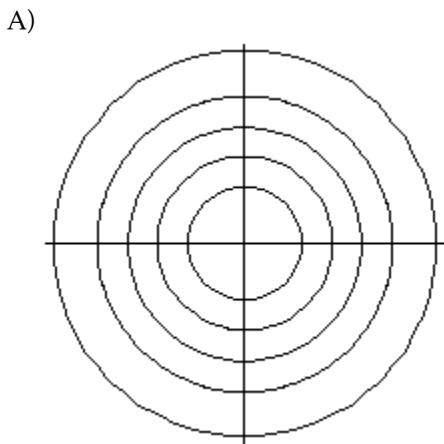
D)



9)



9) \_\_\_\_\_



**SHORT ANSWER.** Write the word or phrase that best completes each statement or answers the question.

**Provide an appropriate answer.**

10) A sample of matter has a temperature distribution given by  $T = f(x,y,z) = \frac{x^2}{5} + \frac{y^2}{3} + \frac{z^2}{2}$ .      10) \_\_\_\_\_

Describe the level surfaces corresponding to constant temperature and write an equation for the surface  $T = 300$ .

**Give an appropriate answer.**

11) Given the function  $f(x, y, z)$  and the positive number  $\epsilon$  as in the formal definition of a limit, find a positive number  $\delta$  as in the definition that insures  $|f(x, y, z) - f(0, 0, 0)| < \epsilon$ .      11) \_\_\_\_\_

$$f(x, y, z) = x + y + z ; \epsilon = 0.12$$

**Use the limit definition of the partial derivative to compute the indicated partial derivative of the function at the specified point.**

12) Find  $\frac{\partial f}{\partial y}$  at the point  $(-9, -5, 10)$ :  $f(x, y, z) = 4x^2y + 6y^2 + 3z$       12) \_\_\_\_\_

**Provide an appropriate response.**

- 13) We say that a function  $f(x, y, z)$  approaches the limit  $L$  as  $(x, y, z)$  approaches  $(x_0, y_0, z_0)$   
and write

$$\lim_{(x, y, z) \rightarrow (x_0, y_0, z_0)} f(x, y, z) = L$$

if for every number  $\epsilon > 0$ , there exists a corresponding number  $\delta > 0$  such that for all  
 $(x, y, z)$  in the domain of  $f$ ,  $0 < \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} < \delta \Rightarrow |f(x, y, z) - L| < \epsilon$ .

Show that the  $\delta$ - $\epsilon$  requirement in this definition is equivalent to

$$0 < |x - x_0| < \delta, 0 < |y - y_0| < \delta, \text{ and } 0 < |z - z_0| < \delta \Rightarrow |f(x, y, z) - L| < \epsilon.$$

- 14) Show that  $f(x, y, z) = x^3y^3z^3$  is continuous at every point  $(x_0, y_0, z_0)$ .

- 15) Does knowing that  $\left| \cos\left(\frac{1}{y}\right) \right| \leq 1$  tell you anything about  $\lim_{(x, y) \rightarrow (0, 0)} \sin(x) \cos\left(\frac{1}{y}\right)$ ?

Give reasons for your answer.

14) \_\_\_\_\_

15) \_\_\_\_\_

**Give an appropriate answer.**

- 16) Given the function  $f(x, y)$  and the positive number  $\epsilon$  as in the formal definition of a limit,  
find a positive number  $\delta$  as in the definition that insures  $|f(x, y) - f(0, 0)| < \epsilon$ .

$$f(x, y) = \frac{x + y}{x^2 + y^2 + 1}; \epsilon = 0.06$$

- 17) Given the function  $f(x, y)$  and the positive number  $\epsilon$  as in the formal definition of a limit,  
find a positive number  $\delta$  as in the definition that insures  $|f(x, y) - f(0, 0)| < \epsilon$ .

$$f(x, y) = x + y; \epsilon = 0.08$$

**Provide an appropriate response.**

- 18) Does knowing that  $4 - x^2y^3 < \frac{4 \tan^{-1}xy}{xy} < 4$  tell you anything about

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{4 \tan^{-1}xy}{xy}? \text{ Give reasons for your answer.}$$

16) \_\_\_\_\_

17) \_\_\_\_\_

**Find two paths of approach from which one can conclude that the function has no limit as  $(x, y)$  approaches  $(0, 0)$ .**

$$19) f(x, y) = \frac{xy}{x^2 + y^2}$$

18) \_\_\_\_\_

19) \_\_\_\_\_

**Use the limit definition of the partial derivative to compute the indicated partial derivative of the function at the specified point.**

- 20) Find  $\frac{\partial f}{\partial x}$  at the point  $(-5, 9)$ :  $f(x, y) = 5 - 3xy + 7xy^2$

20) \_\_\_\_\_

- 21) Find  $\frac{\partial f}{\partial y}$  at the point  $(-3, 7)$ :  $f(x, y) = 4 - 10xy + 2xy^2$

21) \_\_\_\_\_

**Find two paths of approach from which one can conclude that the function has no limit as  $(x, y)$  approaches  $(0, 0)$ .**

$$22) \quad f(x, y) = \frac{x^2}{x^4 + y^2}$$

22) \_\_\_\_\_

**Give an appropriate answer.**

- 23) Given the function  $f(x, y)$  and the positive number  $\epsilon$  as in the formal definition of a limit,  
find a positive number  $\delta$  as in the definition that insures  $|f(x, y) - f(0, 0)| < \epsilon$ .

23) \_\_\_\_\_

$$f(x, y) = \frac{2x + y}{x^2 y^2 + 1}; \epsilon = 0.09$$

**Use the limit definition of the partial derivative to compute the indicated partial derivative of the function at the specified point.**

24) Find  $\frac{\partial f}{\partial y}$  at the point  $(-7, -4)$ :  $f(x, y) = 9x^2 + 4xy + 6y^2$

24) \_\_\_\_\_

**Give an appropriate answer.**

- 25) Given the function  $f(x, y, z)$  and the positive number  $\epsilon$  as in the formal definition of a limit,  
find a positive number  $\delta$  as in the definition that insures  $|f(x, y, z) - f(0, 0, 0)| < \epsilon$ .

25) \_\_\_\_\_

$$f(x, y, z) = \sin^2 x + \sin^2 y + \sin^2 z; \epsilon = 0.12$$

**Find two paths of approach from which one can conclude that the function has no limit as  $(x, y)$  approaches  $(0, 0)$ .**

$$26) \quad f(x, y) = \frac{x^2 y}{x^4 + y^2}$$

26) \_\_\_\_\_

$$27) \quad f(x, y) = \frac{|xy|}{xy}$$

27) \_\_\_\_\_

$$28) \quad f(x, y) = \frac{y^2}{y^2 - x}$$

28) \_\_\_\_\_

**Use the limit definition of the partial derivative to compute the indicated partial derivative of the function at the specified point.**

29) Find  $\frac{\partial f}{\partial x}$  at the point  $(3, -7, 1)$ :  $f(x, y, z) = 5x^2 y + 4y^2 + 4z$

29) \_\_\_\_\_

**Find two paths of approach from which one can conclude that the function has no limit as  $(x, y)$  approaches  $(0, 0)$ .**

$$30) \quad f(x, y) = \frac{x^2 - y}{x^2 + y}$$

30) \_\_\_\_\_

**Provide an appropriate response.**

- 31) Does knowing that  $\left| \sin\left(\frac{1}{y}\right) \right| \leq 1$  tell you anything about  $\lim_{(x, y) \rightarrow (0, 0)} \sin(x) \sin\left(\frac{1}{y}\right)$ ? Give 31) \_\_\_\_\_  
reasons for your answer.

**Use the limit definition of the partial derivative to compute the indicated partial derivative of the function at the specified point.**

32) Find  $\frac{\partial f}{\partial y}$  at the point  $(-5, -5, -3)$ :  $f(x, y, z) = xyz - 4y^2 - 10z$  32) \_\_\_\_\_

**Provide an appropriate response.**

33) Show that  $f(x, y, z) = e^{x^2} + y^2 + z^2$  is continuous at the origin. 33) \_\_\_\_\_

**Find two paths of approach from which one can conclude that the function has no limit as  $(x, y)$  approaches  $(0, 0)$ .**

34)  $f(x, y) = \frac{3y}{\sqrt{3x^2 + 3y^2}}$  34) \_\_\_\_\_

**Solve the problem.**

35) If  $f(x, y, z)$  is differentiable,  $x = r - s$ ,  $y = s - t$ , and  $z = t - r$ , show that  
 $\frac{\partial f}{\partial r} + \frac{\partial f}{\partial s} + \frac{\partial f}{\partial t} = 0$ . 35) \_\_\_\_\_

**Use the limit definition of the partial derivative to compute the indicated partial derivative of the function at the specified point.**

36) Find  $\frac{\partial f}{\partial z}$  at the point  $(1, -3, 7)$ :  $f(x, y, z) = xyz - 3y^2 - 9z$  36) \_\_\_\_\_

**Give an appropriate answer.**

37) Given the function  $f(x, y, z)$  and the positive number  $\epsilon$  as in the formal definition of a limit, 37) \_\_\_\_\_  
find a positive number  $\delta$  as in the definition that insures  $|f(x, y, z) - f(0, 0, 0)| < \epsilon$ .

$$f(x, y, z) = \frac{\sqrt{x^2 + y^2 + z^2}}{x + 1}; \epsilon = 0.01$$

**Find two paths of approach from which one can conclude that the function has no limit as  $(x, y)$  approaches  $(0, 0)$ .**

38)  $f(x, y) = \frac{x^3 + y^6}{x^3}$  38) \_\_\_\_\_

**Provide an appropriate answer.**

39) The Ideal Gas Law states that the pressure  $P$  of a gas obeys the function  
 $P = f(V, n, T) = \frac{nkT}{V}$ , where  $n$  is the number of particles in the sample,  $T$  is the Kelvin  
temperature of the gas,  $V$  is the volume of the gas in liters, and  $k$  is the constant  $1.38 \times 10^{-23}$ . Does the pressure have a local minimum along the line  $V = 3t$ ,  $n = 3t$ ,  $T = 3t$ ? If so,  
what is the value of the minimum pressure? Give reasons for your answer.

**Give an appropriate answer.**

40) Given the function  $f(x, y)$  and the positive number  $\epsilon$  as in the formal definition of a limit, 40) \_\_\_\_\_  
find a positive number  $\delta$  as in the definition that insures  $|f(x, y) - f(0, 0)| < \epsilon$ .

$$f(x, y) = (1 + \cos x)(x + y); \epsilon = 0.08$$

- 41) Given the function  $f(x, y, z)$  and the positive number  $\epsilon$  as in the formal definition of a limit, find a positive number  $\delta$  as in the definition that insures  $|f(x, y, z) - f(0, 0, 0)| < \epsilon$ . 41) \_\_\_\_\_

$$f(x, y, z) = x + y - z ; \epsilon = 0.03$$

**Use the limit definition of the partial derivative to compute the indicated partial derivative of the function at the specified point.**

- 42) Find  $\frac{\partial f}{\partial x}$  at the point  $(4, 8)$ :  $f(x, y) = 7x^2 + 4xy + 5y^2$  42) \_\_\_\_\_

**Provide an appropriate response.**

- 43) If  $f(x_0, y_0) = -2$  and the limit of  $f(x, y)$  exists as  $(x, y)$  approaches  $(x_0, y_0)$ , what can you say about the continuity of  $f(x, y)$  at the point  $(x_0, y_0)$ ? Give reasons for your answer. 43) \_\_\_\_\_

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

**Solve the problem.**

- 44) A rectangular box with square base and no top is to have a volume of  $32 \text{ ft}^3$ . What is the least amount of material required? 44) \_\_\_\_\_

A)  $36 \text{ ft}^2$       B)  $48 \text{ ft}^2$       C)  $42 \text{ ft}^2$       D)  $40 \text{ ft}^2$

**Find all the second order partial derivatives of the given function.**

- 45)  $f(x, y) = x \ln(y - x)$  45) \_\_\_\_\_

A)  $\frac{\partial^2 f}{\partial x^2} = \frac{x - 2y}{(y - x)^2}; \frac{\partial^2 f}{\partial y^2} = -\frac{x}{(y - x)^2}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -\frac{y}{(y - x)^2}$

B)  $\frac{\partial^2 f}{\partial x^2} = \frac{x - 2y}{(y - x)^2}; \frac{\partial^2 f}{\partial y^2} = \frac{x}{(y - x)^2}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{y}{(y - x)^2}$

C)  $\frac{\partial^2 f}{\partial x^2} = \frac{2y - x}{(y - x)^2}; \frac{\partial^2 f}{\partial y^2} = -\frac{x}{(y - x)^2}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{y}{(y - x)^2}$

D)  $\frac{\partial^2 f}{\partial x^2} = \frac{x - 2y}{(y - x)^2}; \frac{\partial^2 f}{\partial y^2} = -\frac{x}{(y - x)^2}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{y}{(y - x)^2}$

**Find the extreme values of the function subject to the given constraint.**

- 46)  $f(x, y) = xy, x^2 + y^2 = 200$  46) \_\_\_\_\_

A) Maximum: 100 at  $(10, 10)$  and  $(-10, -10)$ ; minimum: -100 at  $(10, -10)$  and  $(-10, 10)$

B) Maximum: 100 at  $(10, 10)$ ; minimum: 0 at  $(0, 0)$

C) Maximum: 100 at  $(10, -10)$  and  $(-10, 10)$ ; minimum: -100 at  $(10, 10)$  and  $(-10, -10)$

D) Maximum: 100 at  $(10, 10)$ ; minimum: -100 at  $(-10, -10)$

**Solve the problem.**

- 47) Find the derivative of the function  $f(x, y, z) = \ln(xy + yz + zx)$  at the point  $(-5, -10, -15)$  in the direction in which the function decreases most rapidly. 47) \_\_\_\_\_

A)  $-\frac{1}{11}\sqrt{2}$

B)  $-\frac{1}{17}\sqrt{2}$

C)  $-\frac{1}{13}\sqrt{2}$

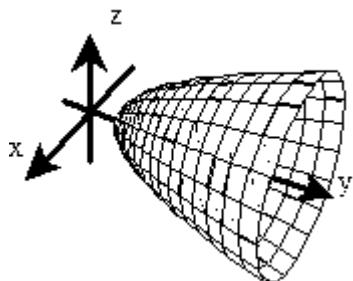
D)  $-\frac{1}{7}\sqrt{2}$

**Sketch a typical level surface for the function.**

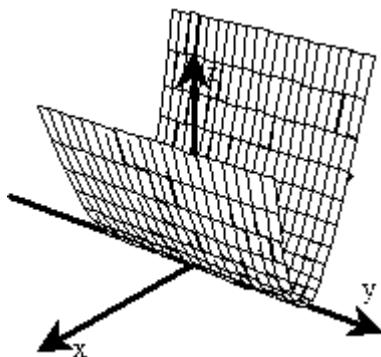
48)  $f(x, y, z) = \sqrt{y - x^2 - z^2}$

48) \_\_\_\_\_

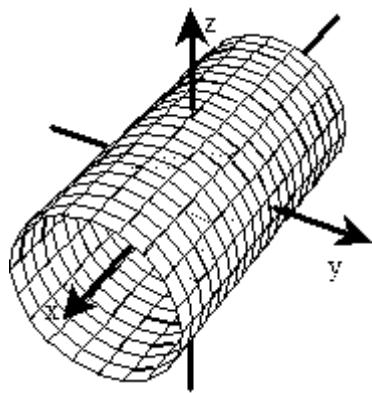
A)



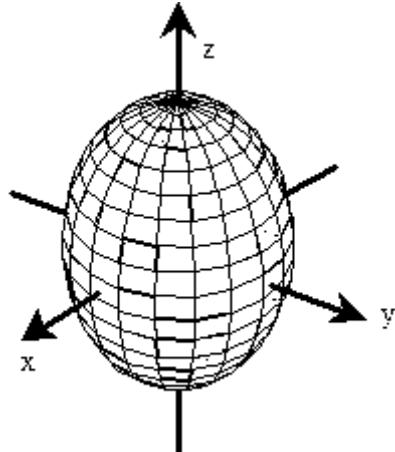
B)



C)



D)



**Find an upper bound for the magnitude  $|E|$  of the error in the linear approximation  $L$  to  $f$  at the given point over the given region.**

49)  $f(x, y, z) = \tan^{-1} xyz$  at  $(8, 8, 8)$

49) \_\_\_\_\_

R:  $|x - 8| \leq 0.2, |y - 8| \leq 0.2, |z - 8| \leq 0.2$

A)  $|E| \leq 0.00000007$

B)  $|E| \leq 0.00000008$

C)  $|E| \leq 0.00000011$

D)  $|E| \leq 0.00000014$

**Find the requested partial derivative.**

50)  $(\partial w / \partial z)_x$  at  $(x, y, z, w) = (1, 2, 9, 266)$  if  $w = x^2 + y^2 + z^2 + 10xyz$  and  $z = x^3 + y^3$

50) \_\_\_\_\_

A)  $\frac{275}{4}$

B)  $\frac{275}{6}$

C)  $\frac{275}{3}$

D)  $\frac{275}{2}$

**Find all the first order partial derivatives for the following function.**

51)  $f(x, y, z) = x^2y + y^2z + xz^2$

51) \_\_\_\_\_

A)  $\frac{\partial f}{\partial x} = 2y + z^2; \frac{\partial f}{\partial y} = x^2 + 2z; \frac{\partial f}{\partial z} = y^2 + 2x$

B)  $\frac{\partial f}{\partial x} = 2xy; \frac{\partial f}{\partial y} = x^2 + 2yz; \frac{\partial f}{\partial z} = y^2 + 2xz$

C)  $\frac{\partial f}{\partial x} = 2xy + z^2; \frac{\partial f}{\partial y} = x^2 + 2yz; \frac{\partial f}{\partial z} = y^2 + 2xz$

D)  $\frac{\partial f}{\partial x} = 2xy + z^2; \frac{\partial f}{\partial y} = x^2 + yz; \frac{\partial f}{\partial z} = y^2 + xz$

**Solve the problem.**

52) Evaluate  $\frac{\partial u}{\partial y}$  at  $(x, y, z) = (2, 2, 0)$  for the function  $u(p, q, r) = e^{pq} \cos(r)$ ;  $p = \frac{1}{x}$ ,  $q = x^2 \ln y$ ,  $r = z$ .

52) \_\_\_\_\_

A) 0

B) 8

C) 4

D) 1

53) Evaluate  $\frac{\partial w}{\partial u}$  at  $(u, v) = (1, 4)$  for the function  $w(x, y) = xy - y^2$ ;  $x = u - v$ ,  $y = uv$ .

53) \_\_\_\_\_

A) 1

B) 5

C) -3

D) -40

**Find the extreme values of the function subject to the given constraint.**

54)  $f(x, y) = 12x + 3y$ ,  $xy = 4$ ,  $x > 0$ ,  $y > 0$

54) \_\_\_\_\_

A) Maximum: 30 at  $(2, 2)$ ; minimum: 24 at  $(1, 4)$

B) Maximum: none; minimum: 24 at  $(1, 4)$

C) Maximum: none; minimum: 30 at  $(2, 2)$

D) Maximum: 51 at  $(4, 1)$ ; minimum: 30 at  $(2, 2)$

**Solve the problem.**

55) Write an equation for the tangent line to the curve  $x^2 - 2xy + y^2 = 4$  at the point  $(-1, 1)$ .

55) \_\_\_\_\_

A)  $y = x + 1$

B)  $x + y = 1$

C)  $x - y + 2 = 0$

D)  $y = x - 2$

56) Find the point of intersection of the three tangent planes to the paraboloid  $z = 5x^2 + 8y^2$  at the points  $(0, 0, 0)$ ,  $(1, 0, 5)$ , and  $(0, 1, 8)$ .

56) \_\_\_\_\_

A)  $\left(\frac{1}{4}, \frac{1}{4}, 0\right)$

B)  $\left(\frac{1}{3}, \frac{1}{3}, 0\right)$

C)  $\left(\frac{2}{3}, \frac{2}{3}, 0\right)$

D)  $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$

57) Evaluate  $\frac{dw}{dt}$  at  $t = 2$  for the function  $w(x, y, z) = e^{xyz^2}$ ;  $x = t$ ,  $y = t$ ,  $z = \frac{1}{t}$ .

57) \_\_\_\_\_

A) 1

B) 1e

C) 0

D) -1e

58) For the space curve  $x = t$ ,  $y = t^2$ ,  $z = t$ , find the points at which the function  $f(x, y)$  takes on extreme values if  $f_x = 36$ ,  $f_y = \frac{t}{2}$ , and  $f_z = 96t$ .

58) \_\_\_\_\_

A)  $t = -3$  and  $t = -4$

B)  $t = -6$  and  $t = -8$

C)  $t = 0$

D)  $t = -12$  and  $t = -16$

**Find the domain and range and describe the level curves for the function  $f(x,y)$ .**

59)  $f(x, y) = \sqrt{36 - x^2 - y^2}$

59) \_\_\_\_\_

- A) Domain: all points in the  $x-y$  plane satisfying  $x^2 + y^2 = 36$ ; range: real numbers  $0 \leq z \leq 6$ ; level curves: circles with centers at  $(0, 0)$  and radii  $r, 0 < r \leq 6$
- B) Domain: all points in the  $x-y$  plane; range: real numbers  $0 \leq z \leq 6$ ; level curves: circles with centers at  $(0, 0)$  and radii  $r, 0 < r \leq 6$
- C) Domain: all points in the  $x-y$  plane; range: all real numbers; level curves: circles with centers at  $(0, 0)$
- D) Domain: all points in the  $x-y$  plane satisfying  $x^2 + y^2 \leq 36$ ; range: real numbers  $0 \leq z \leq 6$ ; level curves: circles with centers at  $(0, 0)$  and radii  $r, 0 < r \leq 6$

**Solve the problem.**

60) Find the least squares line through the points  $(1, 10)$  and  $(2, -6)$ .

60) \_\_\_\_\_

- A)  $y = 4x + 26$       B)  $y = -16x + 14$       C)  $y = -16x + 26$       D)  $y = 4x + 14$

**Find all the second order partial derivatives of the given function.**

61)  $f(x, y) = \frac{x}{x+y}$

61) \_\_\_\_\_

- A)  $\frac{\partial^2 f}{\partial x^2} = -\frac{2y}{(x+y)^3}, \frac{\partial^2 f}{\partial y^2} = \frac{2x}{(x+y)^3}, \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{y-x}{(x+y)^3}$
- B)  $\frac{\partial^2 f}{\partial x^2} = -\frac{2y}{(x+y)^3}, \frac{\partial^2 f}{\partial y^2} = \frac{2x}{(x+y)^3}, \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{x-y}{(x+y)^3}$
- C)  $\frac{\partial^2 f}{\partial x^2} = -\frac{y}{(x+y)^3}, \frac{\partial^2 f}{\partial y^2} = \frac{x}{(x+y)^3}, \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{x-y}{(x+y)^3}$
- D)  $\frac{\partial^2 f}{\partial x^2} = \frac{2y}{(x+y)^3(x+y)^3}, \frac{\partial^2 f}{\partial y^2} = -\frac{2x}{(x+y)^3}, \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{x-y}{(x+y)^3}$

**Find the linearization of the function at the given point.**

62)  $f(x, y) = -8x^2 + 3y^2 - 3$  at  $(-7, -5)$

62) \_\_\_\_\_

- A)  $L(x, y) = 112x - 30y - 320$       B)  $L(x, y) = -16x + 6y - 320$   
 C)  $L(x, y) = -16x + 6y + 314$       D)  $L(x, y) = 112x - 30y + 314$

**Find the derivative of the function at the given point in the direction of A.**

63)  $f(x, y) = \ln(-10x - 9y), (4, -5), A = 6i + 8j$

63) \_\_\_\_\_

- A)  $-\frac{46}{25}$       B)  $-\frac{76}{25}$       C)  $-\frac{56}{25}$       D)  $-\frac{66}{25}$

**Use implicit differentiation to find the specified derivative at the given point.**

64) Find  $\frac{\partial x}{\partial y}$  at the point  $\left(1, \frac{\pi}{28}, 7\right)$  for  $e^{x^2} \cos(yz) = 0$ .

64) \_\_\_\_\_

- A)  $\frac{7}{2}$       B)  $-\frac{7}{2}$       C)  $-\frac{2}{7}$       D)  $\frac{2}{7}$

65) Find  $\frac{\partial y}{\partial x}$  at the point  $(1, 2, e^7)$  for  $\ln(xz)y + 6y^3 = 0$ .

65) \_\_\_\_\_

- A)  $\frac{79}{2}$       B)  $\frac{2}{79}$       C)  $-\frac{2}{79}$       D)  $-\frac{79}{2}$

**Find all local extreme values of the given function and identify each as a local maximum, local minimum, or saddle point.**

66)  $f(x, y) = x^3 + y^3 - 300x - 75y - 2$

66) \_\_\_\_\_

- A)  $f(-10, -5) = 2248$ , local maximum;  $f(10, 5) = -2252$ , local minimum
- B)  $f(10, 5) = -2252$ , local minimum;  $f(10, -5) = -1752$ , saddle point;  $f(-10, 5) = 1748$ , saddle point;  $f(-10, -5) = 2248$ , local maximum
- C)  $f(-10, -5) = 2248$ , local maximum
- D)  $f(10, -5) = -1752$ , saddle point;  $f(-10, 5) = 1748$ , saddle point

**Solve the problem.**

67) Find an equation for the level surface of the function  $f(x, y, z) = \sum_{n=0}^{\infty} \frac{(-1)^n (x+y)^{2n}}{(2n)! z^{2n}}$  that passes

67) \_\_\_\_\_

through the point  $(\pi, \pi, 1)$ .

A)  $\frac{x+y}{z} = 1$

B)  $\frac{x+y}{z} = 2\pi$

C)  $\cos\left(\frac{x+y}{z}\right) = 0$

D)  $\cos\left(\frac{x+y}{z}\right) = 2\pi$

**Find the limit.**

68)  $\lim_{(x, y) \rightarrow (-7, 3)} \frac{4x^3 - 3xy^2}{4x^2 - 3y^2}$

68) \_\_\_\_\_

A) -7

B) 0

C)  $\infty$

D) 3

**Find the extreme values of the function subject to the given constraint.**

69)  $f(x, y) = x^2 + 4y^3$ ,  $x^2 + 2y^2 = 2$

69) \_\_\_\_\_

- A) Maximum: 4 at  $(0, 1)$ ; minimum: -31 at  $(1, -2)$
- B) Maximum: 8 at  $(2, 1)$ ; minimum: -4 at  $(0, -1)$
- C) Maximum: 8 at  $(2, 1)$ ; minimum: -31 at  $(1, -2)$
- D) Maximum: 4 at  $(0, 1)$ ; minimum: -4 at  $(0, -1)$

**Find all the first order partial derivatives for the following function.**

70)  $f(x, y) = \frac{x}{x+y}$

70) \_\_\_\_\_

A)  $\frac{\partial f}{\partial x} = \frac{2x+y}{(x+y)^2}$ ;  $\frac{\partial f}{\partial y} = \frac{x}{(x+y)^2}$

B)  $\frac{\partial f}{\partial x} = -\frac{y}{(x+y)^2}$ ;  $\frac{\partial f}{\partial y} = -\frac{x}{(x+y)^2}$

C)  $\frac{\partial f}{\partial x} = \frac{2x+y}{(x+y)^2}$ ;  $\frac{\partial f}{\partial y} = -\frac{x}{(x+y)^2}$

D)  $\frac{\partial f}{\partial x} = \frac{y}{(x+y)^2}$ ;  $\frac{\partial f}{\partial y} = -\frac{x}{(x+y)^2}$

**Provide an appropriate answer.**

71) Which one of the following space regions is closed?

71) \_\_\_\_\_

- The hemispherical region centered at the origin with  $z > 0$  and radius  $r$  bounded by  $0 \leq r \leq r_0$
- The  $xy$ -plane
- The half-space  $x > 0$
- Space itself

A) ii and iv

B) ii only

C) iv only

D) ii, iii, and iv

**Find all the first order partial derivatives for the following function.**

72)  $f(x,y,z) = xe^{(x^2+y^2+z^2)}$

72) \_\_\_\_\_

A)  $\frac{\partial f}{\partial x} = (1+2x^2)e^{(x^2+y^2+z^2)}; \frac{\partial f}{\partial y} = xe^{(x^2+y^2+z^2)}; \frac{\partial f}{\partial z} = xe^{(x^2+y^2+z^2)}$

B)  $\frac{\partial f}{\partial x} = (1+2x^2)e^{(x^2+y^2+z^2)}; \frac{\partial f}{\partial y} = 2xye^{(x^2+y^2+z^2)}; \frac{\partial f}{\partial z} = 2xze^{(x^2+y^2+z^2)}$

C)  $\frac{\partial f}{\partial x} = 2x^2e^{(x^2+y^2+z^2)}; \frac{\partial f}{\partial y} = xye^{(x^2+y^2+z^2)}; \frac{\partial f}{\partial z} = 2xze^{(x^2+y^2+z^2)}$

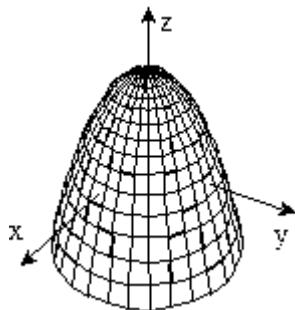
D)  $\frac{\partial f}{\partial x} = (1+2x^2)e^{(x^2+y^2+z^2)}; \frac{\partial f}{\partial y} = xy^2e^{(x^2+y^2+z^2)}; \frac{\partial f}{\partial z} = xz^2e^{(x^2+y^2+z^2)}$

**Sketch the surface  $z = f(x,y)$ .**

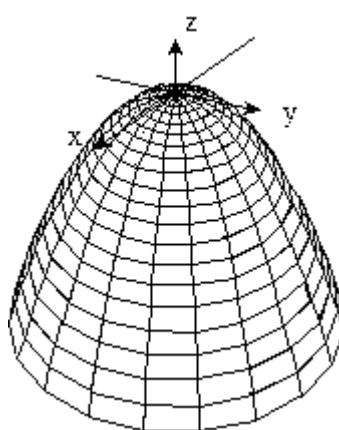
73)  $f(x, y) = -\sqrt{x^2 + y^2}$

73) \_\_\_\_\_

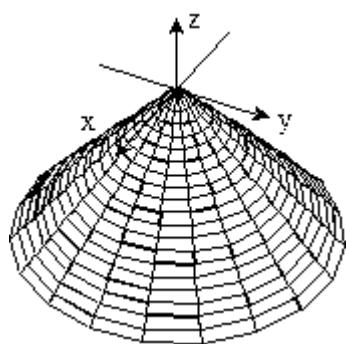
A)



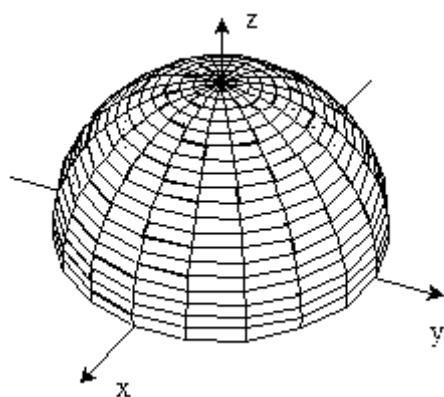
B)



C)



D)



**Find the absolute maxima and minima of the function on the given domain.**

74)  $f(x, y) = x^2 + y^2$  on the diamond-shaped region  $|x| + |y| \leq 4$

74) \_\_\_\_\_

A) Absolute maximum: 16 at  $(4, 0), (-4, 0), (0, 4)$ , and  $(0, -4)$ ; absolute minimum: 8 at  $(2, 2), (2, -2), (-2, 2)$ , and  $(-2, -2)$ .

B) Absolute maximum: 16 at  $(4, 0)$  and  $(0, 4)$ ; absolute minimum: 0 at  $(-4, 0)$  and  $(0, -4)$

C) Absolute maximum: 16 at  $(4, 0), (-4, 0), (0, 4)$ , and  $(0, -4)$ ; absolute minimum: 0 at  $(0, 0)$

D) Absolute maximum: 8 at  $(2, 2), (2, -2), (-2, 2)$ , and  $(-2, -2)$ ; absolute minimum: 0 at  $(0, 0)$

**Solve the problem.**

- 75) Find the distance from the point
- $(1, -1, 2)$
- to the plane
- $x + y - z = 3$
- .

75) \_\_\_\_\_

A)  $\frac{5}{\sqrt{2}}$

B)  $\frac{7}{\sqrt{3}}$

C)  $\frac{7}{\sqrt{2}}$

D)  $\frac{5}{\sqrt{3}}$

- 76) Find the extreme values of
- $f(x, y, z) = x^2 + y^2 + z^2$
- subject to
- $3x - y + z = 6$
- and
- $x + 2y + 2z = 2$
- .

76) \_\_\_\_\_

A) Maximum: none; minimum:  $\frac{148}{45}$  at  $\left(\frac{74}{45}, -\frac{20}{45}, \frac{28}{45}\right)$

B) Maximum: none; minimum:  $\frac{148}{45}$  at  $\left(-\frac{74}{45}, \frac{20}{45}, \frac{28}{45}\right)$

C) Maximum: none; minimum:  $\frac{148}{45}$  at  $\left(\frac{74}{45}, -\frac{20}{45}, -\frac{28}{45}\right)$

D) Maximum: none; minimum:  $\frac{148}{45}$  at  $\left(\frac{74}{45}, \frac{20}{45}, -\frac{28}{45}\right)$

- 77) Find an equation for the level curve of the function
- $f(x, y) = \sqrt{y^2 - 9}$
- that passes through the point
- $(0, 3)$
- .

77) \_\_\_\_\_

A)  $y = 3$

B)  $y = -9$

C)  $y = \pm 3$

D)  $y = 9$

- 78) Determine the point on the plane
- $8x + 9y + 4z = 19$
- that is closest to the point
- $(18, 17, 11)$
- .

78) \_\_\_\_\_

A)  $(2, -1, 3)$

B)  $(-2, -1, -3)$

C)  $(-2, 1, -3)$

D)  $(2, 1, -3)$

**Estimate the error in the quadratic approximation of the given function at the origin over the given region.**

- 79)
- $f(x, y) = \sin 5x \sin 2y$
- ,
- $-0.1 \leq x, y \leq 0.1$

79) \_\_\_\_\_

A)  $|E(x, y)| \leq 0.1667$

B)  $|E(x, y)| \leq 0.125$

C)  $|E(x, y)| \leq 0.25$

D)  $|E(x, y)| \leq 0.5$

**Find all the first order partial derivatives for the following function.**

- 80)
- $f(x, y) = xye^{-y}$

80) \_\_\_\_\_

A)  $\frac{\partial f}{\partial x} = ye^{-y}; \frac{\partial f}{\partial y} = -xye^{-y}$

B)  $\frac{\partial f}{\partial x} = ye^{-y}; \frac{\partial f}{\partial y} = xe^{-y}(1-y)$

C)  $\frac{\partial f}{\partial x} = ye^{-y}; \frac{\partial f}{\partial y} = xe^{-y}$

D)  $\frac{\partial f}{\partial x} = ye^{-y}; \frac{\partial f}{\partial y} = xe^{-y}(y-1)$

**Provide an appropriate answer.**

- 81) Find
- $\frac{\partial w}{\partial r}$
- when
- $r = -4$
- and
- $s = 1$
- if
- $w(x, y, z) = xz + y^2$
- ,
- $x = 3r + 1$
- ,
- $y = r + s$
- , and
- $z = r - s$
- .

81) \_\_\_\_\_

A)  $\frac{\partial w}{\partial r} = -15$

B)  $\frac{\partial w}{\partial r} = -14$

C)  $\frac{\partial w}{\partial r} = -32$

D)  $\frac{\partial w}{\partial r} = -24$

**Find the derivative of the function at the given point in the direction of A.**

- 82)
- $f(x, y, z) = 5xy^3z^2$
- ,
- $(5, 125, 25)$
- ,
- $A = -2i + j - 2k$

82) \_\_\_\_\_

A)  $-5.615234375e+09$

B)  $-\frac{1.66015625e+10}{3}$

C)  $-\frac{1.635742188e+10}{3}$

D)  $-5.37109375e+09$

**Provide an appropriate answer.**

83) Find  $\frac{\partial z}{\partial v}$  when  $u = 0$  and  $v = \frac{11\pi}{2}$  if  $z(x, y) = \sin x + \cos y$ ,  $x = u \cdot v$ , and  $y = u + v$ .

83) \_\_\_\_\_

A)  $\frac{\partial z}{\partial v} = 2$

B)  $\frac{\partial z}{\partial v} = 0$

C)  $\frac{\partial z}{\partial v} = 1$

D)  $\frac{\partial z}{\partial v} = -1$

**Find the limit.**

84)  $\lim_{\substack{(x, y) \rightarrow \left(\frac{9}{2}, \frac{9}{2}\right) \\ x+y \neq 9}} \frac{x+y-9}{\sqrt{x+y}-3}$

84) \_\_\_\_\_

A) 0

B) 6

C) 3

D) No limit

**Use Taylor's formula to find the requested approximation of  $f(x, y)$  near the origin.**

85) Quadratic approximation to  $f(x, y) = \frac{1}{1+10x+y}$

85) \_\_\_\_\_

A)  $1 - 10x - y + 100x^2 + 10xy + y^2$

B)  $1 + 10x + y + 100x^2 + 20xy + y^2$

C)  $1 - 10x - y + 100x^2 + 20xy + y^2$

D)  $1 + 10x + y + 100x^2 + 10xy + y^2$

**Determine whether the given function satisfies the wave equation.**

86)  $w(x, t) = \cos(5t - 5cx)$ ,  $c \neq 1$

86) \_\_\_\_\_

A) Yes

B) No

**Find the extreme values of the function subject to the given constraint.**

87)  $f(x, y, z) = (x-2)^2 + (y+1)^2 + (z-4)^2$ ,  $x-y+3z=7$

87) \_\_\_\_\_

A) Maximum: none; minimum:  $\frac{76}{11}$  at  $\left(\frac{14}{11}, \frac{3}{11}, \frac{20}{11}\right)$

B) Maximum: none; minimum:  $\frac{188}{11}$  at  $\left(-\frac{14}{11}, \frac{3}{11}, \frac{20}{11}\right)$

C) Maximum: none; minimum:  $\frac{64}{11}$  at  $\left(\frac{14}{11}, -\frac{3}{11}, \frac{20}{11}\right)$

D) Maximum: none; minimum: 36 at  $\left(\frac{14}{11}, \frac{3}{11}, -\frac{20}{11}\right)$

**Find the derivative of the function at the given point in the direction of A.**

88)  $f(x, y) = 10x + 6y$ ,  $(3, -5)$ ,  $A = 4\mathbf{i} - 3\mathbf{j}$

88) \_\_\_\_\_

A) 14

B)  $\frac{58}{5}$

C) 2

D)  $\frac{22}{5}$

**Find all local extreme values of the given function and identify each as a local maximum, local minimum, or saddle point.**

89)  $f(x, y) = (x^2 - 81)^2 + (y^2 - 9)^2$

89) \_\_\_\_\_

- A)  $f(0, 0) = 6642$ , local maximum;  $f(9, 3) = 0$ , local minimum;  $f(9, -3) = 0$ , local minimum;  
 $f(-9, 3) = 0$ , local minimum;  $f(-9, -3) = 0$ , local minimum
- B)  $f(0, 0) = 6642$ , local maximum;  $f(0, 3) = 6561$ , saddle point;  $f(0, -3) = 6561$ , saddle point;  
 $f(9, 0) = 6642$ , saddle point;  $f(9, 3) = 0$ , local minimum;  $f(9, -3) = 0$ , local minimum;  
 $f(-9, 0) = 81$ , saddle point;  $f(-9, 3) = 0$ , local minimum;  $f(-9, -3) = 0$ , local minimum
- C)  $f(0, 0) = 6642$ , local maximum;  $f(0, 3) = 6561$ , saddle point;  $f(9, 0) = 81$ , saddle point;  
 $f(9, 3) = 0$ , local minimum;  $f(-9, -3) = 0$ , local minimum
- D)  $f(0, 0) = 6642$ , local maximum;  $f(-9, -3) = 0$ , local minimum

**Find the requested partial derivative.**

90)  $(\partial w / \partial x)_y$  if  $w = x^3 + y^3 + z^3 + 6xyz$  and  $z = x^2 + y^2$

90) \_\_\_\_\_

- A)  $3x(3x + 2z^2) + 6y(2x^2 + 3z)$
- B)  $3x(3x + 2z^2) + 6y(2x^2 + z)$
- C)  $3x(x + 2z^2) + 6y(2x^2 + z)$
- D)  $3x(x + 2z^2) + 6y(2x^2 + 3z)$

**Find all the first order partial derivatives for the following function.**

91)  $f(x, y) = \ln\left(\frac{y^3}{x^5}\right)$

91) \_\_\_\_\_

- A)  $\frac{\partial f}{\partial x} = -\ln\left(\frac{5y^3}{x^6}\right); \frac{\partial f}{\partial y} = \ln\left(\frac{3y^2}{x^5}\right)$
- B)  $\frac{\partial f}{\partial x} = -\frac{5}{x}; \frac{\partial f}{\partial y} = \frac{3}{y}$
- C)  $\frac{\partial f}{\partial x} = \frac{3}{y}; \frac{\partial f}{\partial y} = \frac{5}{x}$
- D)  $\frac{\partial f}{\partial x} = -\ln\left(\frac{5}{x}\right); \frac{\partial f}{\partial y} = \ln\left(\frac{3}{y}\right)$

**Find the linearization of the function at the given point.**

92)  $f(x, y, z) = e^{10x} - 3y + 4z$  at  $(0, 0, 0)$

92) \_\_\_\_\_

- A)  $L(x, y, z) = 5x - \frac{3}{2}y + 2z$
- B)  $L(x, y, z) = 10x - 3y + 4z$
- C)  $L(x, y, z) = 5x - \frac{3}{2}y + 2z + 1$
- D)  $L(x, y, z) = 10x - 3y + 4z + 1$

**Solve the problem.**

93) Write parametric equations for the tangent line to the curve of intersection of the surfaces  $x + y + z = 4$  and  $x - y + 2z = 1$  at the point  $(1, 2, 1)$ .

93) \_\_\_\_\_

- A)  $x = 3t + 1, y = -t + 2, z = -2t + 1$
- B)  $x = 3t - 1, y = t + 2, z = -2t + 1$
- C)  $x = 3t + 1, y = t + 2, z = -2t + 1$
- D)  $x = 3t - 1, y = -t + 2, z = -2t + 1$

94) Find the extreme values of  $f(x, y, z) = x + 2y$  subject to  $x + y + z = 1$  and  $y^2 + z^2 = 4$ .

94) \_\_\_\_\_

- A) Maximum:  $1 + 2\sqrt{2}$  at  $(1, \sqrt{2}, \sqrt{2})$ ; minimum:  $1 - 2\sqrt{2}$  at  $(1, -\sqrt{2}, -\sqrt{2})$
- B) Maximum:  $1 + 2\sqrt{2}$  at  $(1, \sqrt{2}, -\sqrt{2})$ ; minimum:  $1 - 2\sqrt{2}$  at  $(1, -\sqrt{2}, \sqrt{2})$
- C) Maximum:  $1 + 2\sqrt{2}$  at  $(1, \sqrt{2}, -\sqrt{2})$ ; minimum:  $1 - 2\sqrt{2}$  at  $(1, -\sqrt{2}, \sqrt{2})$
- D) Maximum:  $1 + 2\sqrt{2}$  at  $(1, \sqrt{2}, \sqrt{2})$ ; minimum:  $1 - 2\sqrt{2}$  at  $(1, -\sqrt{2}, \sqrt{2})$

**Find the extreme values of the function subject to the given constraint.**

95)  $f(x, y) = x^2 + y^2, \quad xy^2 = 128$

95) \_\_\_\_\_

- A) Maximum: 48 at  $(4, 4\sqrt{2})$ ; minimum: -48 at  $(4, -4\sqrt{2})$
- B) Maximum: none; minimum: 0 at  $(0, 0)$
- C) Maximum: 48 at  $(4, \pm 4\sqrt{2})$ ; minimum: 0 at  $(0, 0)$
- D) Maximum: none; minimum: 48 at  $(4, \pm 4\sqrt{2})$

**Write a chain rule formula for the following derivative.**

96)  $\frac{\partial w}{\partial t}$  for  $w = f(p, q, r); p = g(t), q = h(t), r = k(t)$

96) \_\_\_\_\_

- A)  $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial p} + \frac{\partial w}{\partial q} + \frac{\partial w}{\partial r}$
- C)  $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial p} \frac{dt}{dp} + \frac{\partial w}{\partial q} \frac{dt}{dq} + \frac{\partial w}{\partial r} \frac{dt}{dr}$

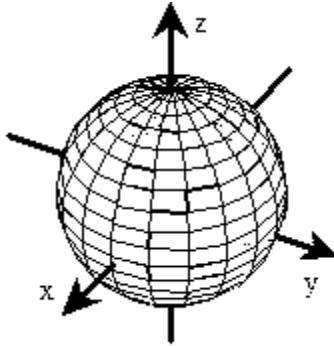
- B)  $\frac{\partial w}{\partial t} = \frac{dp}{dt} + \frac{dq}{dt} + \frac{dr}{dt}$
- D)  $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial p} \frac{dp}{dt} + \frac{\partial w}{\partial q} \frac{dq}{dt} + \frac{\partial w}{\partial r} \frac{dr}{dt}$

**Sketch a typical level surface for the function.**

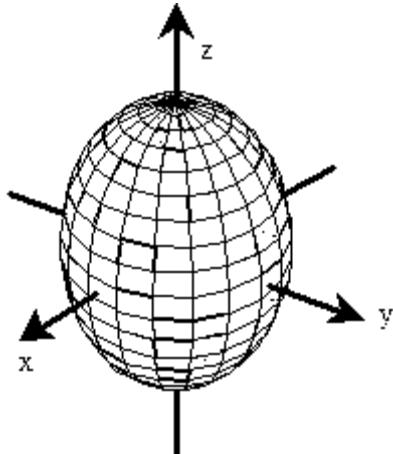
97)  $f(x, y, z) = e^{(x^2 + y^2 + z^2)}$

97) \_\_\_\_\_

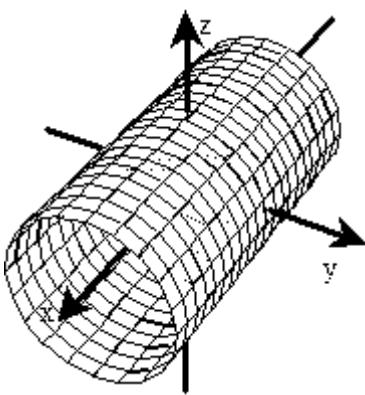
A)



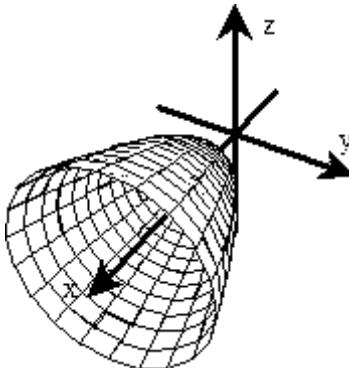
B)



C)



D)



**Use implicit differentiation to find the specified derivative at the given point.**

98) Find  $\frac{\partial y}{\partial z}$  at the point  $(5, 1, 3)$  for  $\frac{6}{x^2} + \frac{5}{y^2} + \frac{5}{z^2} = 0$ .

98) \_\_\_\_\_

A)  $-\frac{1}{27}$

B)  $\frac{1}{27}$

C) 27

D) -27

**Provide an appropriate answer.**

99) Find  $\frac{\partial w}{\partial u}$  when  $u = -3$  and  $v = 4$  if  $w(x, y, z) = \frac{xy^2}{z}$ ,  $x = \frac{u}{v}$ ,  $y = u + v$ , and  $z = u \cdot v$ .

99) \_\_\_\_\_

A)  $\frac{\partial w}{\partial u} = \frac{-8}{-27}$

B)  $\frac{\partial w}{\partial u} = \frac{3}{8}$

C)  $\frac{\partial w}{\partial u} = \frac{1}{16}$

D)  $\frac{\partial w}{\partial u} = \frac{1}{8}$

**Find all the first order partial derivatives for the following function.**

100)  $f(x, y, z) = (\sin xy)(\cos yz^2)$

100) \_\_\_\_\_

A)  $\frac{\partial f}{\partial x} = (y \cos xy)(\cos yz^2); \frac{\partial f}{\partial y} = (z^2 \sin xy)(\sin yz^2) - (x \cos xy)(\cos yz^2); \frac{\partial f}{\partial z} = 2(yz \sin xy)(\sin yz^2)$

B)  $\frac{\partial f}{\partial x} = (y \cos xy)(\cos yz^2); \frac{\partial f}{\partial y} = (x \cos xy)(\cos yz^2) - (z^2 \sin xy)(\sin yz^2); \frac{\partial f}{\partial z} = 2(yz \sin xy)(\sin yz^2)$

C)  $\frac{\partial f}{\partial x} = (y \cos xy)(\cos yz^2); \frac{\partial f}{\partial y} = (x \cos xy)(\cos yz^2) - (z^2 \sin xy)(\sin yz^2); \frac{\partial f}{\partial z} = -2(yz \sin xy)(\sin yz^2)$

D)  $\frac{\partial f}{\partial x} = (y \cos xy)(\cos yz^2); \frac{\partial f}{\partial y} = (x \cos xy)(\cos yz^2); \frac{\partial f}{\partial z} = -2(yz \sin xy)(\sin yz^2)$

**Solve the problem.**

101) Find parametric equations for the normal line to the surface  $-10x - 7y + 7z = 11$  at the point  $(1, -1, 2)$ .

101) \_\_\_\_\_

A)  $x = -t - 10, y = t - 7, z = -2t + 7$

B)  $x = t - 10, y = -t - 7, z = 2t + 7$

C)  $x = -10t + 1, y = -7t - 1, z = 7t + 2$

D)  $x = -10t - 1, y = -7t + 1, z = 7t - 2$

**Use polar coordinates to find the limit of the function as  $(x, y)$  approaches  $(0, 0)$ .**

102)  $f(x, y) = \frac{x^2y + xy^2}{x^2 + y^2}$

102) \_\_\_\_\_

A) 2

B) 0

C) 1

D) No limit

**Compute the gradient of the function at the given point.**

103)  $f(x, y, z) = \ln(x^2 + 3y^2 + 2z^2)$ ,  $(3, 3, 3)$

103) \_\_\_\_\_

A)  $\frac{1}{6}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{2}{9}\mathbf{k}$

B)  $\frac{1}{9}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{1}{2}\mathbf{k}$

C)  $\frac{1}{9}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{2}{9}\mathbf{k}$

D)  $\frac{1}{6}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{1}{2}\mathbf{k}$

**Find the linearization of the function at the given point.**

104)  $f(x, y, z) = 8xy + 5yz + 4zx$  at  $(1, 1, 1)$

104) \_\_\_\_\_

A)  $L(x, y, z) = 12xy + 13yz + 9z - 17$

B)  $L(x, y, z) = 12xy + 13yz + 9z - 34$

C)  $L(x, y, z) = 8xy + 5yz + 4z - 34$

D)  $L(x, y, z) = 8xy + 5yz + 4z - 17$

**Write a chain rule formula for the following derivative.**

105)  $\frac{\partial w}{\partial t}$  for  $w = f(x, y, z); x = g(r, s), y = h(t), z = k(r, s, t)$

105) \_\_\_\_\_

A)  $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial t}$

B)  $\frac{\partial w}{\partial t} = \frac{dy}{dt} + \frac{\partial z}{\partial t}$

C)  $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$

D)  $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial z} \frac{\partial z}{\partial t}$

**Solve the problem.**

- 106) Find the point on the line of intersection of the planes  $x + y + z = 1$  and  $3x + 2y + z = 6$  that is closest to the origin.

106) \_\_\_\_\_

A)  $\left(-\frac{7}{3}, \frac{1}{3}, \frac{5}{3}\right)$

B)  $\left(\frac{7}{3}, -\frac{1}{3}, -\frac{5}{3}\right)$

C)  $\left(\frac{7}{3}, \frac{1}{3}, -\frac{5}{3}\right)$

D)  $\left(\frac{7}{3}, -\frac{1}{3}, \frac{5}{3}\right)$

- 107) Find any local extrema (maxima, minima, or saddle points) of  $f(x, y)$  given that

107) \_\_\_\_\_

$f_x = 36x^2 - 36$  and  $f_y = 8y + 64$ .

- A) Local maximum at  $(1, -8)$ ; saddle point at  $(-1, -8)$   
B) Local minimum at  $(-1, -8)$ ; saddle point at  $(1, -8)$   
C) Local maximum at  $(-1, -8)$ ; saddle point at  $(1, -8)$   
D) Local minimum at  $(1, -8)$ ; saddle point at  $(-1, -8)$

**Write a chain rule formula for the following derivative.**

108)  $\frac{\partial z}{\partial t}$  for  $z = f(r, s); r = g(t), s = h(t)$

108) \_\_\_\_\_

A)  $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial r} \frac{dr}{dt} + \frac{\partial z}{\partial s} \frac{ds}{dt}$

B)  $\frac{\partial z}{\partial t} = \frac{dr}{dt} + \frac{ds}{dt}$

C)  $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial r} \frac{dr}{dt} + \frac{\partial z}{\partial s} \frac{ds}{dt}$

D)  $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial r} \frac{dr}{ds}$

**Solve the problem.**

- 109) Find the point on the parabola  $y = 2x^2$  that is closest to the line  $y = x - 3$ . Use the fact that the line connecting the closest points on each curve is normal to both curves.

109) \_\_\_\_\_

A)  $\left(\frac{1}{2}, \frac{1}{4}\right)$

B)  $\left(\frac{1}{4}, \frac{1}{8}\right)$

C)  $\left(\frac{1}{6}, \frac{1}{18}\right)$

D)  $\left(\frac{1}{2}, \frac{1}{2}\right)$

**Find the domain and range and describe the level curves for the function  $f(x, y)$ .**

110)  $f(x, y) = (6x - 4y)^5$

110) \_\_\_\_\_

- A) Domain: all points in the  $x$ - $y$  plane; range: all real numbers; level curves: lines  $6x - 4y = c$   
B) Domain: all points in the  $x$ - $y$  plane; range: all real numbers; level curves: lines  $6x - 4y = c, c \geq 0$   
C) Domain: all points in the  $x$ - $y$  plane; range: real numbers  $z \leq 0$ ; level curves: lines  $6x - 4y = c, c \leq 0$   
D) Domain: all points in the  $x$ - $y$  plane; range: real numbers  $z \geq 0$ ; level curves: lines  $6x - 4y = c$

**Solve the problem.**

- 111) Find the point on the curve of intersection of the paraboloid  $x^2 + y^2 + 2z = 4$  and the plane  $x - y + 2z = 0$  that is closest to the origin.

111) \_\_\_\_\_

A)  $(1, 1, -1)$

B)  $(1, -1, 1)$

C)  $(1, 1, 1)$

D)  $(-1, 1, 1)$

112) Find an equation for the level curve of the function  $f(x, y) = \sum_{n=0}^{\infty} \left(\frac{x}{y}\right)^n$  that passes through the point  $(1, 4)$ . 112) \_\_\_\_\_

A)  $\left| \frac{x}{y} \right| = 4$

B)  $\left| \frac{x}{y} \right| = \frac{1}{4}$

C)  $\left| \frac{x}{y} \right| \leq \frac{1}{4}$

D)  $x = 4y$

**Find an upper bound for the magnitude  $|E|$  of the error in the linear approximation  $L$  to  $f$  at the given point over the given region.**

113)  $f(x, y, z) = \ln(5x + 8y + 2z)$  at  $(1, 1, 1)$  113) \_\_\_\_\_

R:  $|x - 1| \leq 0.1, |y - 1| \leq 0.1, |z - 1| \leq 0.1$

A)  $|E| \leq 0.019$

B)  $|E| \leq 0.0158$

C)  $|E| \leq 0.0237$

D)  $|E| \leq 0.0178$

**Find all the first order partial derivatives for the following function.**

114)  $f(x, y, z) = \frac{\cos y}{xz^2}$  114) \_\_\_\_\_

A)  $\frac{\partial f}{\partial x} = \frac{\cos y}{z^2}, \frac{\partial f}{\partial y} = \frac{\sin y}{xz^2}, \frac{\partial f}{\partial z} = \frac{2 \cos y}{xz}$

B)  $\frac{\partial f}{\partial x} = -\frac{\cos y}{z^2}, \frac{\partial f}{\partial y} = -\frac{\sin y}{xz^2}, \frac{\partial f}{\partial z} = -\frac{2 \cos y}{xz}$

C)  $\frac{\partial f}{\partial x} = -\frac{\cos y}{x^2 z^2}, \frac{\partial f}{\partial y} = -\frac{\sin y}{xz^2}, \frac{\partial f}{\partial z} = -\frac{2 \cos y}{xz^3}$

D)  $\frac{\partial f}{\partial x} = \frac{\cos y}{x^2 z^2}, \frac{\partial f}{\partial y} = \frac{\sin y}{xz^2}, \frac{\partial f}{\partial z} = \frac{2 \cos y}{xz^3}$

**Provide an appropriate response.**

115) Define  $f(0, 0)$  in such a way that extends  $f(x, y) = \frac{9x^2 - x^2y + 9y^2}{x^2 + y^2}$  to be continuous at the origin. 115) \_\_\_\_\_

A)  $f(0, 0) = 18$

B)  $f(0, 0) = 2$

C)  $f(0, 0) = 9$

D)  $f(0, 0) = 0$

**Find the absolute maxima and minima of the function on the given domain.**

116)  $f(x, y) = x^2 + xy + y^2$  on the square  $-5 \leq x, y \leq 5$  116) \_\_\_\_\_

A) Absolute maximum: 75 at  $(5, 5)$  and  $(-5, -5)$ ; absolute minimum: 0 at  $(0, 0)$

B) Absolute maximum: 25 at  $(5, -5)$  and  $(-5, 5)$ ; absolute minimum:  $\frac{75}{4}$  at  $\left(-\frac{5}{2}, 5\right), \left(\frac{5}{2}, -5\right), \left(5, -\frac{5}{2}\right)$ , and  $\left(-5, \frac{5}{2}\right)$

C) Absolute maximum: 75 at  $(5, 5)$  and  $(-5, -5)$ ; absolute minimum: 25 at  $(5, -5)$  and  $(-5, 5)$

D) Absolute maximum: 25 at  $(5, -5)$  and  $(-5, 5)$ ; absolute minimum: 0 at  $(0, 0)$

**Find the domain and range and describe the level curves for the function  $f(x,y)$ .**

117)  $f(x, y) = \frac{1}{6x^2 + 9y^2}$

117) \_\_\_\_\_

- A) Domain: all points in the  $x-y$  plane; range: all real numbers; level curves: ellipses  $6x^2 + 9y^2 = c$
- B) Domain: all points in the  $x-y$  plane; range: real numbers  $> 0$ ; level curves: ellipses  $6x^2 + 9y^2 = c$
- C) Domain: all points in the  $x-y$  plane except  $(0, 0)$ ; range: real numbers  $> 0$ ; level curves: ellipses  $6x^2 + 9y^2 = c$
- D) Domain: all points in the  $x-y$  plane except  $(0, 0)$ ; range: all real numbers; level curves: ellipses  $6x^2 + 9y^2 = c$

**Answer the question.**

118) Find the direction in which the function is increasing or decreasing most rapidly at the point  $P_0$ .

118) \_\_\_\_\_

$$f(x, y, z) = x\sqrt{y^2 + z^2}, P_0(1, 2, 1)$$

A)  $\frac{1}{\sqrt{30}}\left(\frac{i}{6} + \frac{j}{15} + \frac{k}{30}\right)$

B)  $\frac{1}{\sqrt{30}}\left(\frac{i}{6} + \frac{j}{15} - \frac{k}{30}\right)$

C)  $\sqrt{30}\left(\frac{i}{6} + \frac{j}{15} - \frac{k}{30}\right)$

D)  $\sqrt{30}\left(\frac{i}{6} + \frac{j}{15} + \frac{k}{30}\right)$

**Solve the problem.**

119) Find the point on the paraboloid  $z = 2 - x^2 - y^2$  that is closest to the point  $(1, 1, 2)$ .

119) \_\_\_\_\_

A)  $\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\right)$

B)  $\left(-\frac{1}{2}, -\frac{1}{2}, \frac{3}{2}\right)$

C)  $\left(-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\right)$

D)  $\left(\frac{1}{2}, -\frac{1}{2}, \frac{3}{2}\right)$

**Write a chain rule formula for the following derivative.**

120)  $\frac{\partial w}{\partial x}$  for  $w = f(p, q); p = g(x, y), q = h(x, y)$

120) \_\_\_\_\_

A)  $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial p} \frac{\partial p}{\partial x}$

B)  $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial p} \frac{\partial p}{\partial x} + \frac{\partial w}{\partial q} \frac{\partial q}{\partial x}$

C)  $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial p} + \frac{\partial w}{\partial x} \frac{\partial x}{\partial q}$

D)  $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial p} + \frac{\partial w}{\partial q}$

**Solve the problem.**

121) Find the derivative of the function  $f(x, y) = \tan^{-1} \frac{y}{x}$  at the point  $(-10, 10)$  in the direction in which

121) \_\_\_\_\_

the function decreases most rapidly.

A)  $-\frac{\sqrt{2}}{20}$

B)  $-\frac{\sqrt{3}}{20}$

C)  $-\frac{\sqrt{3}}{30}$

D)  $-\frac{\sqrt{2}}{30}$

**Find the absolute maxima and minima of the function on the given domain.**

122)  $f(x, y) = 7x^2 + 10y^2$  on the closed triangular region bounded by the lines  $y = x$ ,  $y = 2x$ , and  $x + y = 6$

122) \_\_\_\_\_

- A) Absolute maximum: 188 at  $(2, 4)$ ; absolute minimum: 153 at  $(3, 3)$
- B) Absolute maximum: 153 at  $(3, 3)$ ; absolute minimum: 68 at  $(2, 2)$
- C) Absolute maximum: 153 at  $(3, 3)$ ; absolute minimum: 0 at  $(0, 0)$
- D) Absolute maximum: 188 at  $(2, 4)$ ; absolute minimum: 0 at  $(0, 0)$

Use Taylor's formula to find the requested approximation of  $f(x, y)$  near the origin.

123) Cubic approximation to  $f(x, y) = \frac{1}{(1 + 7x + y)^2}$

123) \_\_\_\_\_

- A)  $1 + 2x + 14y - 3x^2 - 56xy - 147y^2 + 4x^3 + 84x^2y + 588xy^2 + 1372y^3$
- B)  $1 - 2x - 14y + 3x^2 + 42xy + 147y^2 - 4x^3 - 84x^2y - 588xy^2 - 1372y^3$
- C)  $1 + 2x + 14y - 3x^2 - 42xy - 147y^2 + 4x^3 + 84x^2y + 588xy^2 + 1372y^3$
- D)  $1 - 2x - 14y + 3x^2 + 56xy + 147y^2 - 4x^3 - 84x^2y - 588xy^2 - 1372y^3$

Solve the problem.

124) Find any local extrema (maxima, minima, or saddle points) of  $f(x, y)$  given that  $f_x = 3x + 3y$  and  $f_y = -9x - 2y$ .

124) \_\_\_\_\_

- A) Local minimum at  $(0, 0)$
- B) Local minimum at  $\left(-1, -\frac{2}{9}\right)$
- C) Local maximum at  $\left(-1, -\frac{2}{9}\right)$
- D) Saddle point at  $(0, 0)$

125) Find an equation for the level surface of the function  $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$  that passes through the point  $(4, 3, 12)$ .

125) \_\_\_\_\_

- A)  $x^2 + y^2 + z^2 = 13$
- B)  $x + y + z = \pm 13$
- C)  $x + y + z = 13$
- D)  $x^2 + y^2 + z^2 = 169$

Provide an appropriate answer.

126) Find  $\frac{\partial z}{\partial v}$  when  $u = 3$  and  $v = 1$  if  $z(x) = \frac{x}{\sqrt{x+5}}$  and  $x = u \cdot v$ .

126) \_\_\_\_\_

- A)  $\frac{\partial z}{\partial v} = \frac{13}{2(8)^{3/2}}$
- B)  $\frac{\partial z}{\partial v} = 0$
- C)  $\frac{\partial z}{\partial v} = \frac{39}{2(8)^{3/2}}$
- D)  $\frac{\partial z}{\partial v} = \frac{39}{2\sqrt{8}}$

Find all the second order partial derivatives of the given function.

127)  $f(x, y) = \ln(x^2y - x)$

127) \_\_\_\_\_

- A)  $\frac{\partial^2 f}{\partial x^2} = \frac{2xy - 2x^2y^2 - 1}{(x^2y - x)^2}; \frac{\partial^2 f}{\partial y^2} = \frac{x^4}{(x^2y - x)^2}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{x^2}{(x^2y - x)^2}$
- B)  $\frac{\partial^2 f}{\partial x^2} = \frac{2xy - 2x^2y^2 - 1}{(x^2y - x)^2}; \frac{\partial^2 f}{\partial y^2} = -\frac{x^4}{(x^2y - x)^2}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -\frac{x^2}{(x^2y - x)^2}$
- C)  $\frac{\partial^2 f}{\partial x^2} = \frac{xy - x^2y^2 - 1}{(x^2y - x)^2}; \frac{\partial^2 f}{\partial y^2} = -\frac{x^4}{(x^2y - x)^2}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -\frac{x^2}{(x^2y - x)^2}$
- D)  $\frac{\partial^2 f}{\partial x^2} = \frac{2xy - 2x^2y^2 - 1}{(x^2y - x)^2}; \frac{\partial^2 f}{\partial y^2} = -\frac{x^2}{(x^2y - x)^2}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -\frac{x^2}{(x^2y - x)^2}$

Find the requested partial derivative.

128)  $(\partial w / \partial y)_x$  at  $(x, y, z, w) = (1, 1, 2, 16)$  if  $w = 2x + 4y + 5z$  and  $x + y = z$

128) \_\_\_\_\_

- A) 4
- B) 9
- C) 11
- D) 1

**Write a chain rule formula for the following derivative.**

129)  $\frac{\partial w}{\partial t}$  for  $w = f(x, y, z)$ ;  $x = g(s, t)$ ,  $y = h(s, t)$ ,  $z = k(s, t)$

129) \_\_\_\_\_

A)  $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial t} \frac{\partial x}{\partial t} + \frac{\partial w}{\partial t} \frac{\partial y}{\partial t}$

B)  $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial t}$

C)  $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial t}$

D)  $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y}$

**Solve the problem.**

130) Evaluate  $\frac{\partial u}{\partial z}$  at  $(x, y, z) = (5, 4, 5)$  for the function  $u(p, q, r) = p^2q^2 - r$ ;  $p = y - z$ ,  $q = x + z$ ,  $r = x + y$ .

130) \_\_\_\_\_

A) 110

B) -180

C) 440

D) 220

131) Find an equation for the level curve of the function  $f(x, y) = 64 - x^2 - y^2$  that passes through the point  $(\sqrt{3}, \sqrt{2})$ .

131) \_\_\_\_\_

A)  $x^2 + y^2 = 69$

B)  $x^2 + y^2 = -5$

C)  $x^2 - y^2 = 5$

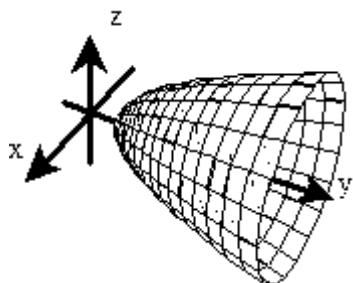
D)  $x^2 + y^2 = 5$

**Sketch a typical level surface for the function.**

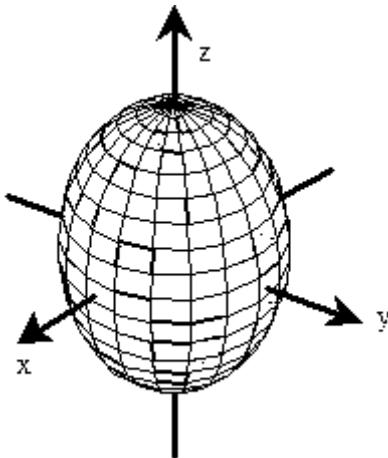
132)  $f(x, y, z) = \cos(x^2 + y^2 + z^2)$

132) \_\_\_\_\_

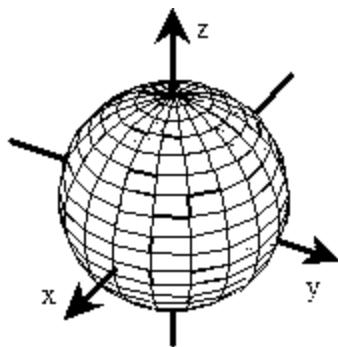
A)



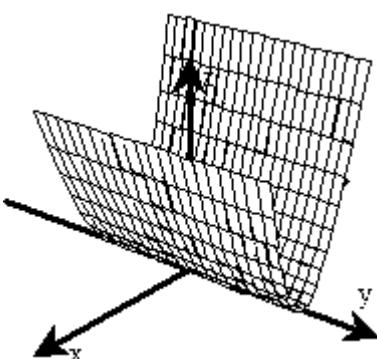
B)



C)



D)



**Find the extreme values of the function subject to the given constraint.**

133)  $f(x, y) = 4x + 6y, \quad x^2 + y^2 = 13$

133) \_\_\_\_\_

- A) Maximum: 36 at (3, 4); minimum: 0 at (0, 0)
- B) Maximum: 26 at (2, 3); minimum: 0 at (0, 0)
- C) Maximum: 26 at (2, 3); minimum: -26 at (-2, -3)
- D) Maximum: 36 at (3, 4); minimum: -36 at (-3, -4)

**Use implicit differentiation to find the specified derivative at the given point.**

134) Find  $\frac{\partial y}{\partial x}$  at the point (4, 6, 4) for  $-6x^2 + 4 \ln xz - 2yz^2 - 7e^z = 0$ .

134) \_\_\_\_\_

- A)  $-\frac{47}{32}$
- B)  $-\frac{47}{8}$
- C)  $\frac{47}{32}$
- D)  $-\frac{49}{32}$

**Determine whether the given function satisfies the wave equation.**

135)  $w(x, t) = e^x - ct$

135) \_\_\_\_\_

- A) No
- B) Yes

**Find the requested partial derivative.**

136)  $(\partial w / \partial x)_{y,z}$  if  $w = x^3 + y^3 + z^3 + 18xyz$

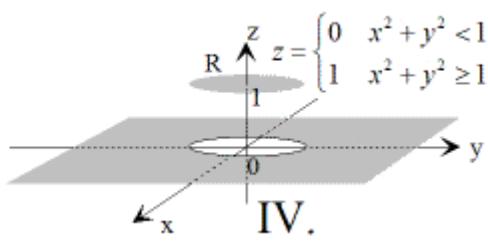
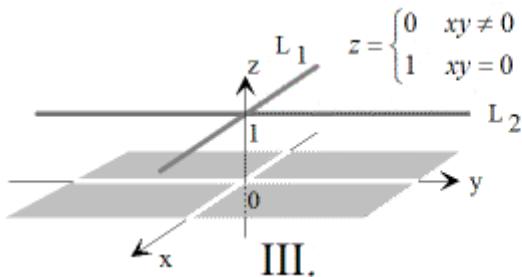
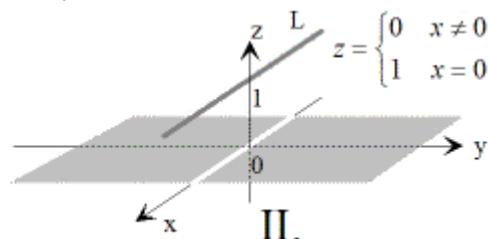
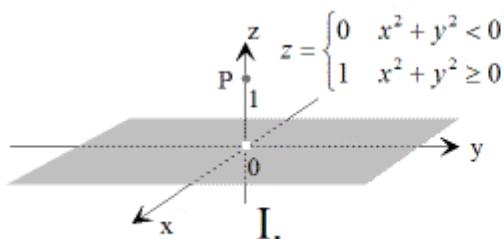
136) \_\_\_\_\_

- A)  $3(x^2 + 12xyz)$
- B)  $3(x^2 + 6yz)$
- C)  $3(x^2 + 6xyz)$
- D)  $3(x^2 + 12yz)$

**Answer the question.**

137) For which of the following functions do both  $f_x$  and  $f_y$  exist?

137) \_\_\_\_\_



A) II., III., and IV.

B) II. and IV.

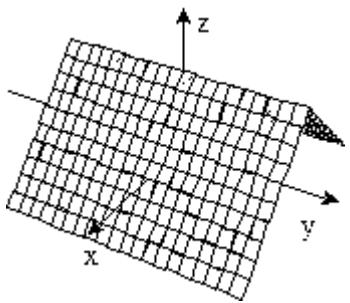
C) IV. only

D) I. and II.

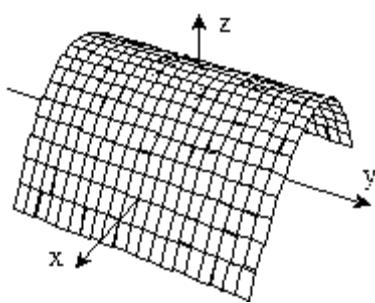
Sketch the surface  $z = f(x,y)$ .

138)  $f(x, y) = |x + y|$

A)

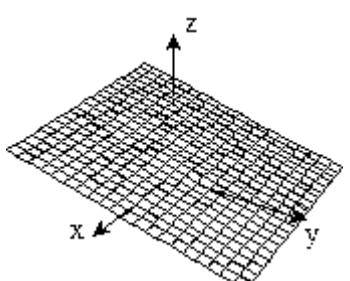


B)

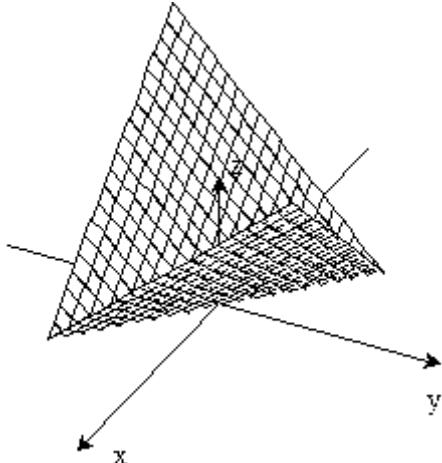


138) \_\_\_\_\_

C)



D)



Find the linearization of the function at the given point.

139)  $f(x, y, z) = \tan^{-1} xyz$  at  $(10, 10, 10)$

139) \_\_\_\_\_

A)  $L(x, y, z) = \frac{100}{1000001}x + \frac{100}{1000001}y + \frac{100}{1000001}z + \tan^{-1} a^3 - \frac{3000}{1000001}$

B)  $L(x, y, z) = \frac{100}{1000001}x + \frac{100}{1000001}y + \frac{100}{1000001}z + \tan^{-1} a^2 - \frac{300}{1000001}$

C)  $L(x, y, z) = \frac{100}{10001}x + \frac{100}{10001}y + \frac{100}{10001}z + \tan^{-1} a^2 - \frac{300}{10001}$

D)  $L(x, y, z) = \frac{100}{10001}x + \frac{100}{10001}y + \frac{100}{10001}z + \tan^{-1} a^3 - \frac{300}{10001}$

Solve the problem.

140) Find the points on the curve  $xy^2 = 128$  that are closest to the origin.

140) \_\_\_\_\_

A)  $(4, 4\sqrt{2})$

B)  $(64, \pm\sqrt{2})$

C)  $(4, \pm 4\sqrt{2})$

D)  $(64, \sqrt{2})$

141) Find the derivative of the function  $f(x, y, z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$  at the point  $(-5, 5, -5)$  in the direction in which the function decreases most rapidly. 141) \_\_\_\_\_

which the function decreases most rapidly.

A)  $-\frac{2}{5}\sqrt{2}$

B)  $-\frac{3}{5}\sqrt{3}$

C)  $-\frac{2}{5}\sqrt{3}$

D)  $-\frac{3}{5}\sqrt{2}$

**Find the derivative of the function at the given point in the direction of A.**

142)  $f(x, y, z) = \ln(x^2 - 5y^2 - 2z^2)$ ,  $(-5, -5, -5)$ ,  $\mathbf{A} = 3\mathbf{i} + 4\mathbf{j}$  142) \_\_\_\_\_

A)  $-\frac{17}{150}$

B)  $-\frac{17}{50}$

C)  $-\frac{17}{75}$

D)  $-\frac{17}{100}$

**Solve the problem.**

143) Find the equation for the tangent plane to the surface  $z = e^{9x^2 + 4y^2}$  at the point  $(0, 0, 1)$ . 143) \_\_\_\_\_

A)  $z = -1$

B)  $z = 1$

C)  $z = 0$

D)  $z = 2$

**Find the equation for the level surface of the function through the given point.**

144)  $f(x, y, z) = e^{(x^2 + y^2 - z)}$ ,  $(1, 3, 1)$  144) \_\_\_\_\_

A)  $x^2 + y^2 - z = e^9$

B)  $x^2 + y^2 - z = \ln(9)$

C)  $e^{(x^2 + y^2 - z)} = \ln(9)$

D)  $\ln(x^2 + y^2 - z) = 9$

**Solve the problem.**

145) The resistance  $R$  produced by wiring resistors of  $R_1$ ,  $R_2$ , and  $R_3$  ohms in parallel can be calculated from the formula 145) \_\_\_\_\_

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

If  $R_1$ ,  $R_2$ , and  $R_3$  are measured to be 10 ohms, 8 ohms, and 5 ohms respectively, and if these measurements are accurate to within 0.05 ohms, estimate the maximum possible error in computing  $R$ .

A) 0.011

B) 0.015

C) 0.022

D) 0.018

**Find the equation for the level surface of the function through the given point.**

146)  $f(x, y, z) = \sum_{n=0}^{\infty} \frac{(zy + x)^n}{2^n x^n}$ ,  $(5, 1, 2)$  146) \_\_\_\_\_

A)  $\frac{(zy + x)^n}{2^n x^n} = \frac{7}{10}$

B)  $\sum_{n=0}^{\infty} \frac{(zy + x)^n}{2^n x^n} = \frac{7}{10}$

C)  $\frac{zy + x}{2x} = \frac{7}{10}$

D) The level surfaces cannot be determined.

**Use implicit differentiation to find the specified derivative at the given point.**

147) Find  $\frac{dy}{dx}$  at the point  $(2, 1)$  for  $\ln x + xy^2 + \ln y = 0$ . 147) \_\_\_\_\_

A) -1

B) 1

C)  $\frac{3}{10}$

D)  $-\frac{3}{10}$

**Find the limit.**

$$148) \lim_{\substack{(x,y) \rightarrow (0,0) \\ x \neq y}} \frac{8\sqrt{y} - 8\sqrt{x} + \sqrt{xy} - x}{\sqrt{y} - \sqrt{x}}$$

148) \_\_\_\_\_

A) 0

B) 8

C) 16

D) No limit

**Use Taylor's formula to find the requested approximation of  $f(x, y)$  near the origin.**

$$149) \text{Quadratic approximation to } f(x, y) = \frac{1}{(1 + 3x + y)^2}$$

149) \_\_\_\_\_

A)  $1 - 2x - 6y + 3x^2 - 18xy + 27y^2$

B)  $1 - 2x - 6y + 3x^2 + 18xy + 27y^2$

C)  $1 + 2x + 6y + 3x^2 - 18xy + 27y^2$

D)  $1 + 2x + 6y + 3x^2 + 18xy + 27y^2$

**Determine whether the given function satisfies Laplace's equation.**

$$150) f(x, y, z) = \cos(8x) \sin(8y) e(\sqrt{128}z)$$

150) \_\_\_\_\_

A) No

B) Yes

**At what points is the given function continuous?**

$$151) f(x, y) = \frac{x - y}{2x^2 + x - 6}$$

151) \_\_\_\_\_

A) All  $(x, y)$  such that  $x \neq \frac{3}{2}$  and  $x \neq -2$

B) All  $(x, y)$  such that  $x \neq 0$

C) All  $(x, y)$  satisfying  $x - y \neq 0$

D) All  $(x, y)$

**Solve the problem.**

152) The Redlich-Kwong equation provides an approximate model for the behavior of real gases. The equation is  $P(V, T) = \frac{RT}{V - b} - \frac{a}{T^{1/2}V(V + b)}$ , where  $P$  is pressure,  $V$  is volume,  $T$  is Kelvin

152) \_\_\_\_\_

temperature, and  $a, b$ , and  $R$  are constants. Find the derivative of the function with respect to each variable.

A)  $P_V = \frac{a(2V + b)}{2V^2(v + b)^2T^{1/2}} - \frac{RT}{(V - b)^2}; P_T = \frac{a}{T^{3/2}V(V + b)} + \frac{R}{V - b}$

B)  $P_V = \frac{a(2V + b)}{V^2(v + b)^2T^{1/2}} + \frac{RT}{(V - b)^2}; P_T = \frac{a}{2T^{3/2}V(V + b)} - \frac{R}{V - b}$

C)  $P_V = \frac{a(2V + b)}{V^2(v + b)^2T^{1/2}} + \frac{RT}{(V - b)^2}; P_T = -\frac{a}{2T^{3/2}V(V + b)} + \frac{R}{V - b}$

D)  $P_V = \frac{a(2V + b)}{V^2(v + b)^2T^{1/2}} - \frac{RT}{(V - b)^2}; P_T = \frac{a}{2T^{3/2}V(V + b)} + \frac{R}{V - b}$

153) Evaluate  $\frac{dw}{dt}$  at  $t = 8$  for the function  $w(x, y) = e^y - \ln x; x = t^2, y = \ln t$ .

153) \_\_\_\_\_

A) 6

B)  $\frac{3}{4}$

C)  $-\frac{3}{4}$

D)  $\frac{7}{8}$

**Find the requested partial derivative.**

154)  $(\partial w / \partial y)_x$  if  $w = 5x + 6y + 2z$  and  $x + y = z$

154) \_\_\_\_\_

A) 13

B) 8

C) 6

D) 1

Use Taylor's formula to find the requested approximation of  $f(x, y)$  near the origin.

155) Quadratic approximation to  $f(x, y) = e^x + 3y$

155) \_\_\_\_\_

A)  $1 + x + 3y + \frac{1}{2}x^2 + \frac{9}{2}y^2$

B)  $x + 3y + \frac{1}{2}x^2 + \frac{9}{2}y^2$

C)  $1 + x + 3y + \frac{1}{2}x^2 + 3xy + \frac{9}{2}y^2$

D)  $x + 3y + \frac{1}{2}x^2 + \frac{3}{2}xy + \frac{9}{2}y^2$

Solve the problem.

156) About how much will  $f(x, y, z) = xy^3z^2$  change if the point  $(x, y, z)$  moves from  $(3, 9, -10)$  a distance of  $ds = \frac{1}{10}$  unit in the direction of  $2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ ?

156) \_\_\_\_\_

A) 11178

B) 12798

C) 14418

D) 9558

Find the linearization of the function at the given point.

157)  $f(x, y, z) = \ln(-4x + 8y + 7z)$  at  $\left(\frac{1}{-4}, \frac{1}{8}, \frac{1}{7}\right)$

157) \_\_\_\_\_

A)  $L(x, y, z) = -2x + 4y + \frac{7}{2}z + \ln 2 - 1$

B)  $L(x, y, z) = -\frac{4}{3}x + \frac{8}{3}y + \frac{7}{3}z + \ln 2 - 1$

C)  $L(x, y, z) = -2x + 4y + \frac{7}{2}z + \ln 3 - 1$

D)  $L(x, y, z) = -\frac{4}{3}x + \frac{8}{3}y + \frac{7}{3}z + \ln 3 - 1$

Solve the problem.

158) Find  $F'(x)$  if  $F(x) = \int_0^{x^2} \sqrt{t^2 + x^2} dt$ .

158) \_\_\_\_\_

A)  $2x\sqrt{x^2 + 1} + \int_0^{x^2} \frac{x}{\sqrt{t^2 + x^2}} dt$

B)  $2x^2\sqrt{x^2 + 1} + \int_0^{x^2} \frac{x^2}{\sqrt{t^2 + x^2}} dt$

C)  $2x^2\sqrt{x^2 + 1} + \int_0^{x^2} \frac{x}{\sqrt{t^2 + x^2}} dt$

D)  $x\sqrt{x^2 + 1} + \int_0^{x^2} \frac{x}{\sqrt{t^2 + x^2}} dt$

Use Taylor's formula to find the requested approximation of  $f(x, y)$  near the origin.

159) Quadratic approximation to  $f(x, y) = \sin(8x + y)$

159) \_\_\_\_\_

A)  $8x + y + 4x^2 + y^2$

B)  $8x + y + 4x^2 + \frac{1}{2}xy + \frac{1}{2}y^2$

C)  $8x + y + 4x^2 + \frac{1}{2}y^2$

D)  $8x + y$

Find the extreme values of the function subject to the given constraint.

160)  $f(x, y, z) = 4x - 3y + 2z, x^2 + y^2 = 6z$

160) \_\_\_\_\_

A) Maximum: none; minimum:  $-\frac{33}{4}$  at  $\left(6, \frac{9}{2}, -\frac{225}{24}\right)$

B) Maximum: none; minimum:  $-\frac{75}{4}$  at  $\left(-6, \frac{9}{2}, \frac{225}{24}\right)$

C) Maximum: none; minimum:  $\frac{117}{4}$  at  $\left(6, \frac{9}{2}, \frac{225}{24}\right)$

D) Maximum: none; minimum:  $\frac{225}{4}$  at  $\left(6, -\frac{9}{2}, \frac{225}{24}\right)$

**Solve the problem.**

161) Find the point on the line  $x - 3y = 6$  that is closest to the origin.

A)  $\left(\frac{3}{5}, -\frac{9}{5}\right)$

B)  $\left(\frac{9}{5}, -\frac{7}{5}\right)$

C)  $\left(-\frac{9}{5}, -\frac{13}{5}\right)$

D)  $\left(-\frac{3}{5}, -\frac{11}{5}\right)$

161) \_\_\_\_\_

**Write a chain rule formula for the following derivative.**

162)  $\frac{\partial u}{\partial x}$  for  $u = f(r, s, t); r = g(y), s = h(z), t = k(x, z)$

A)  $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t}$

B)  $\frac{\partial u}{\partial x} = 0$

C)  $\frac{\partial u}{\partial x} = \frac{\partial t}{\partial x}$

D)  $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} \frac{\partial t}{\partial x}$

162) \_\_\_\_\_

**Find the limit.**

163)  $\lim_{(x, y) \rightarrow (4, 3)} x \ln y$

A) 81

B)  $\ln 81$

C)  $\ln(3) - 4$

D) No limit

163) \_\_\_\_\_

**Compute the gradient of the function at the given point.**

164)  $f(x, y, z) = 9xy^3z^2, (9, 729, 81)$

A)  $2.287679245e+13\mathbf{i} + 5.64859073e+11\mathbf{j} + 7.625597485e+12\mathbf{k}$

B)  $2.541865828e+12\mathbf{i} + 8.472886094e+11\mathbf{j} + 6.276211922e+10\mathbf{k}$

C)  $2.287679245e+13\mathbf{i} + 8.472886094e+11\mathbf{j} + 5.083731657e+12\mathbf{k}$

D)  $2.541865828e+12\mathbf{i} + 5.64859073e+11\mathbf{j} + 9.414317883e+10\mathbf{k}$

164) \_\_\_\_\_

**Solve the problem.**

165) Find the maximum value of  $f(x, y, z, w) = x + y + z + w$  subject to  $x^2 + y^2 + z^2 + w^2 = 1$ .

A) 2

B) 1

C) 10

D) 4

165) \_\_\_\_\_

**Find all the first order partial derivatives for the following function.**

166)  $f(x, y, z) = \cos x \sin^2 yz$

A)  $\frac{\partial f}{\partial x} = -\sin x \sin^2 yz; \frac{\partial f}{\partial y} = z \cos x \sin yz \cos yz; \frac{\partial f}{\partial z} = y \cos x \sin yz \cos yz$

B)  $\frac{\partial f}{\partial x} = \sin x \sin^2 yz; \frac{\partial f}{\partial y} = -2z \cos x \sin yz \cos yz; \frac{\partial f}{\partial z} = -2y \cos x \sin yz \cos yz$

C)  $\frac{\partial f}{\partial x} = -\sin x \sin^2 yz; \frac{\partial f}{\partial y} = 2z \cos x \sin yz \cos yz; \frac{\partial f}{\partial z} = 2y \cos x \sin yz \cos yz$

D)  $\frac{\partial f}{\partial x} = \sin x \sin^2 yz; \frac{\partial f}{\partial y} = -\cos x \sin yz \cos yz; \frac{\partial f}{\partial z} = -\cos x \sin yz \cos yz$

166) \_\_\_\_\_

**Find all local extreme values of the given function and identify each as a local maximum, local minimum, or saddle point.**

167)  $f(x, y) = 2xy - 6x + 6y$

A)  $f(-3, -3) = 18$ , saddle point;  $f(3, 3) = 18$ , saddle point

B)  $f(-3, -3) = 18$ , local minimum;  $f(3, 3) = 18$ , local minimum

C)  $f(-3, 3) = 18$ , saddle point

D)  $f(3, -3) = -54$ , local maximum

167) \_\_\_\_\_

Use Taylor's formula to find the requested approximation of  $f(x, y)$  near the origin.

168) Cubic approximation to  $f(x, y) = \frac{1}{1 + 7x + y}$

168) \_\_\_\_\_

- A)  $1 - 7x - y + 49x^2 + 7xy + y^2 - 343x^3 - 147x^2y - 21xy^2 - y^3$
- B)  $1 - 7x - y + 49x^2 + 7xy + y^2 - 343x^3 + 147x^2y - 21xy^2 + y^3$
- C)  $1 - 7x - y + 49x^2 + 14xy + y^2 - 343x^3 + 147x^2y - 21xy^2 + y^3$
- D)  $1 - 7x - y + 49x^2 + 14xy + y^2 - 343x^3 - 147x^2y - 21xy^2 - y^3$

169) Cubic approximation to  $f(x, y) = e^x + 2y$

169) \_\_\_\_\_

- A)  $1 + x + 2y + \frac{1}{2}x^2 + 2xy + 2y^2 + \frac{1}{6}x^3 + 1x^2y + 2xy^2 + \frac{4}{3}y^3$
- B)  $1 + x + 2y + \frac{1}{2}x^2 + 1xy + 2y^2 + \frac{1}{6}x^3 + \frac{2}{3}x^2y + \frac{4}{3}xy^2 + \frac{4}{3}y^3$
- C)  $1 + x + 2y + \frac{1}{2}x^2 + 2xy + 2y^2 + \frac{1}{6}x^3 + \frac{2}{3}x^2y + \frac{4}{3}xy^2 + \frac{4}{3}y^3$
- D)  $1 + x + 2y + \frac{1}{2}x^2 + 1xy + 2y^2 + \frac{1}{6}x^3 + 1x^2y + 2xy^2 + \frac{4}{3}y^3$

At what points is the given function continuous?

170)  $f(x, y) = \sqrt{6x + 8y}$

170) \_\_\_\_\_

- A) All  $(x, y)$
- B) All  $(x, y)$  such that  $6x + 8y \geq 0$
- C) All  $(x, y)$  such that  $x + y \geq 0$
- D) All  $(x, y)$  such that  $6x + 8y \neq 0$

Find the requested partial derivative.

171)  $\frac{\partial z}{\partial y}$  at  $(x, y, z) = (1, 1, 1)$  if  $z^3 = z + xy - 1$  and  $y^3 = x + y - 1$

171) \_\_\_\_\_

- A)  $\frac{2}{3}$
- B)  $\frac{1}{4}$
- C)  $\frac{1}{2}$
- D)  $\frac{3}{2}$

Use implicit differentiation to find the specified derivative at the given point.

172) Find  $\frac{dy}{dx}$  at the point  $(1, -1)$  for  $-5xy^2 + 4x^2y - 2x = 0$ .

172) \_\_\_\_\_

- A)  $-\frac{15}{14}$
- B)  $\frac{15}{14}$
- C)  $-\frac{5}{3}$
- D)  $\frac{11}{9}$

Find all the first order partial derivatives for the following function.

173)  $f(x, y) = \frac{e^{-x}}{x^2 + y^2}$

173) \_\_\_\_\_

- A)  $\frac{\partial f}{\partial x} = -\frac{2xe^{-x}}{(x^2 + y^2)^2}; \frac{\partial f}{\partial y} = -\frac{2ye^{-x}}{(x^2 + y^2)^2}$
- B)  $\frac{\partial f}{\partial x} = -\frac{e^{-x}(x^2 + y^2 + x)}{(x^2 + y^2)^2}; \frac{\partial f}{\partial y} = -\frac{ye^{-x}}{(x^2 + y^2)^2}$
- C)  $\frac{\partial f}{\partial x} = -\frac{e^{-x}(x^2 + y^2 + 2x)}{(x^2 + y^2)^2}; \frac{\partial f}{\partial y} = -\frac{2ye^{-x}}{(x^2 + y^2)^2}$
- D)  $\frac{\partial f}{\partial x} = \frac{e^{-x}(x^2 + y^2 + 2x)}{(x^2 + y^2)^2}; \frac{\partial f}{\partial y} = \frac{2ye^{-x}}{(x^2 + y^2)^2}$

**Solve the problem.**

- 174) Find the point on the sphere  $x^2 + y^2 + z^2 = 4$  that is farthest from the point  $(3, 1, -1)$ .

174) \_\_\_\_\_

A)  $\left( -\frac{6}{\sqrt{11}}, -\frac{2}{\sqrt{11}}, -\frac{2}{\sqrt{11}} \right)$   
 C)  $\left( -\frac{6}{\sqrt{11}}, \frac{2}{\sqrt{11}}, -\frac{2}{\sqrt{11}} \right)$

B)  $\left( -\frac{6}{\sqrt{11}}, \frac{2}{\sqrt{11}}, \frac{2}{\sqrt{11}} \right)$   
 D)  $\left( -\frac{6}{\sqrt{11}}, -\frac{2}{\sqrt{11}}, \frac{2}{\sqrt{11}} \right)$

**Find all the second order partial derivatives of the given function.**

- 175)  $f(x, y) = \cos xy^2$

175) \_\_\_\_\_

- A)  $\frac{\partial^2 f}{\partial x^2} = -y^4 \cos xy^2; \frac{\partial^2 f}{\partial y^2} = -2x[2xy^2 \cos(xy^2) + \sin(xy^2)]; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -2y[xy^2 \cos(xy^2) + \sin(xy^2)];$   
 B)  $\frac{\partial^2 f}{\partial x^2} = -y^2 \sin xy^2; \frac{\partial^2 f}{\partial y^2} = 2[\sin(xy^2) - 2y^2 \cos(xy^2)]; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 2y[\sin(xy^2) - y^2 \cos(xy^2)];$   
 C)  $\frac{\partial^2 f}{\partial x^2} = y^2 \sin xy^2; \frac{\partial^2 f}{\partial y^2} = 2[2y^2 \cos(xy^2) - \sin(xy^2)]; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 2y[y^2 \cos(xy^2) - \sin(xy^2)];$   
 D)  $\frac{\partial^2 f}{\partial x^2} = -y^2 \sin xy^2; \frac{\partial^2 f}{\partial y^2} = 2y[2y^2 \cos(xy^2) - \sin(xy^2)]; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 2[y^2 \cos(xy^2) - \sin(xy^2)]$

**Solve the problem.**

- 176) About how much will  $f(x, y) = \tan^{-1} xy$  change if the point  $(x, y, z)$  moves from  $\left(-10\sqrt{2}, \frac{-9}{\sqrt{2}}\right)$  a

176) \_\_\_\_\_

distance of  $ds = \frac{1}{10}$  unit in the direction of  $\mathbf{i} + \mathbf{j}$ ?

A)  $-\frac{29}{162020}$

B)  $-\frac{39}{162020}$

C)  $-\frac{19}{162020}$

D)  $-\frac{49}{162020}$

**Determine whether the given function satisfies Laplace's equation.**

- 177)  $f(x, y, z) = 7x + 3y^3z^2$

177) \_\_\_\_\_

A) Yes

B) No

**Use implicit differentiation to find the specified derivative at the given point.**

- 178) Find  $\frac{\partial z}{\partial y}$  at the point  $(6, 1, -1)$  for  $\ln\left(\frac{yz}{x}\right) - e^{xy+z^2} = 0$ .

178) \_\_\_\_\_

A)  $\frac{1 - 2e^7}{1 - 6e^7}$

B)  $\frac{2e^7 - 1}{1 - 6e^7}$

C)  $\frac{1 - 6e^7}{1 - 2e^7}$

D)  $\frac{6e^7 - 1}{1 - 2e^7}$

**Solve the problem.**

- 179) A rectangular box is to be inscribed inside the ellipsoid  $2x^2 + y^2 + 4z^2 = 12$ . Find the largest possible volume for the box.

179) \_\_\_\_\_

A)  $16\sqrt{2}$

B)  $15\sqrt{2}$

C)  $12\sqrt{2}$

D)  $18\sqrt{2}$

180) Find the equation for the tangent plane to the surface  $z = \ln(3x^2 + 6y^2 + 1)$  at the point  $(0, 0, 0)$ .

A)  $x - y = 0$

B)  $z = 0$

C)  $x + y = 0$

D)  $x + y + z = 0$

180) \_\_\_\_\_

Find the linearization of the function at the given point.

181)  $f(x, y) = 3 \sin x + 9 \cos y$  at  $(0, \frac{\pi}{2})$

181) \_\_\_\_\_

A)  $L(x, y) = 3x - 9y + \frac{9}{2}\pi$

B)  $L(x, y) = -3x - 9y + \frac{9}{2}\pi$

C)  $L(x, y) = 3x + 9y + \frac{9}{2}\pi$

D)  $L(x, y) = -3x + 9y + \frac{9}{2}\pi$

Solve the problem.

182) Find the distance between the skew lines

$x = t - 6, y = t, z = 2t$

and

$x = t, y = t, z = -t$ .

A)  $3\sqrt{3}$

B)  $3\sqrt{2}$

C)  $2\sqrt{2}$

D)  $2\sqrt{3}$

182) \_\_\_\_\_

Find the limit.

183)  $\lim_{(x, y) \rightarrow (5, 2)} \left( \frac{2}{x} - \frac{3}{y} \right)$

183) \_\_\_\_\_

A) -11

B)  $-\frac{11}{10}$

C)  $\frac{11}{10}$

D) No limit

Solve the problem.

184) Find the square of the distance between the paraboloid  $z = 10x^2 + 8y^2$  and the point  $(42, 85, 239)$ .

Use the fact that the line connecting the given point and the point closest to it on the paraboloid is normal to the paraboloid.

184) \_\_\_\_\_

A) 6785

B) 9345

C) 5697

D) 8001

185) A simple electrical circuit consists of a resistor connected between the terminals of a battery. The voltage  $V$  (in volts) is dropping as the battery wears out. At the same time, the resistance  $R$  (in ohms) is increasing as the resistor heats up. The power  $P$  (in watts) dissipated by the circuit is

185) \_\_\_\_\_

given by  $P = \frac{V^2}{R}$ . Use the equation

$$\frac{dP}{dt} = \frac{\partial P}{\partial V} \frac{dV}{dt} + \frac{\partial P}{\partial R} \frac{dR}{dt}$$

to find how much the power is changing at the instant when  $R = 3$  ohms,  $V = 2$  volts,  
 $dR/dt = 2$  ohms/sec and  $dV/dt = -0.01$  volts/sec.

A) -0.88 watts

B) 0.88 watts

C) 0.9 watts

D) -0.9 watts

Find the extreme values of the function subject to the given constraint.

186)  $f(x, y) = x^2y, x^2 + 2y^2 = 6$

186) \_\_\_\_\_

A) Maximum: 18 at  $(3, 2)$ ; minimum: -18 at  $(-3, -2)$

B) Maximum: 4 at  $(\pm 2, 1)$ ; minimum: -4 at  $(\pm 2, -1)$

C) Maximum: 18 at  $(\pm 3, 2)$ ; minimum: -18 at  $(\pm 3, -2)$

D) Maximum: 4 at  $(2, 1)$ ; minimum: -4 at  $(-2, -1)$

**Write a chain rule formula for the following derivative.**

187)  $\frac{\partial u}{\partial r}$  for  $u = f(x)$ ;  $x = g(p, q, r)$

187) \_\_\_\_\_

A)  $\frac{\partial u}{\partial r} = \frac{du}{dx} \frac{\partial x}{\partial r}$

B)  $\frac{\partial u}{\partial r} = \frac{du}{dx} \frac{\partial x}{\partial p} + \frac{du}{dx} \frac{\partial x}{\partial q} + \frac{du}{dx} \frac{\partial x}{\partial r}$

C)  $\frac{\partial u}{\partial r} = \frac{\partial x}{\partial r}$

D)  $\frac{\partial u}{\partial r} = \frac{du}{dx}$

**Solve the problem.**

188) Find the least squares line for the points  $(1, 1)$ ,  $(2, 4)$ ,  $(3, 9)$ ,  $(4, 16)$ .

188) \_\_\_\_\_

A)  $y = -5 + 5x$

B)  $y = -3 + 4x$

C)  $y = -\frac{10}{3} + 4x$

D)  $y = -2 + 3x$

**At what points is the given function continuous?**

189)  $f(x, y, z) = \frac{e^x}{e^{y+z}}$

189) \_\_\_\_\_

A) All  $(x, y, z)$  such that  $x - y - z \geq 0$

B) All  $(x, y, z)$  such that  $x \geq 0$  and  $y + z \geq 0$

C) All  $(x, y, z)$  in the first octant

D) All  $(x, y, z)$

**Use implicit differentiation to find the specified derivative at the given point.**

190) Find  $\frac{dy}{dx}$  at the point  $(-1, 1)$  for  $6x - \frac{3}{y} + 7x^2y^2 = 0$ .

190) \_\_\_\_\_

A)  $\frac{4}{9}$

B)  $-\frac{20}{17}$

C)  $\frac{8}{17}$

D)  $-\frac{8}{17}$

**Solve the problem.**

191) Find an equation for the level surface of the function  $f(x, y, z) = \ln\left(\frac{xy}{z}\right)$  that passes through the point  $(e^4, e^{12}, e^3)$ .

191) \_\_\_\_\_

A)  $\ln\left(\frac{xy}{z}\right) = \frac{1}{16}$

B)  $\frac{xy}{z} = e^{16}$

C)  $\ln\left(\frac{xy}{z}\right) = 16$

D)  $\frac{xy}{z} = e^{13}$

192) Find the distance between the line  $y = x - 4$  and the parabola  $y = 3x^2$ . Use the fact that the line connecting the closest points on each curve is normal to both curves.

192) \_\_\_\_\_

A)  $\frac{15}{4}\sqrt{2}$

B)  $\frac{15}{8}\sqrt{2}$

C)  $\frac{47}{12}\sqrt{2}$

D)  $\frac{47}{24}\sqrt{2}$

193) Find the derivative of the function  $f(x, y) = e^{xy}$  at the point  $(0, 7)$  in the direction in which the function decreases most rapidly.

193) \_\_\_\_\_

A) -7

B) -6

C) -14

D) -21

**Determine whether the given function satisfies Laplace's equation.**

194)  $f(x, y, z) = 12x^2 - 6y^2 - 6z^2$

194) \_\_\_\_\_

A) No

B) Yes

**Find the absolute maxima and minima of the function on the given domain.**

195)  $f(x, y) = 8x + 4y$  on the trapezoidal region with vertices  $(0, 0)$ ,  $(1, 0)$ ,  $(0, 2)$ , and  $(1, 1)$

195) \_\_\_\_\_

- A) Absolute maximum: 12 at  $(1, 1)$ ; absolute minimum: 0 at  $(0, 0)$
- B) Absolute maximum: 8 at  $(0, 2)$ ; absolute minimum: 0 at  $(0, 0)$
- C) Absolute maximum: 16 at  $(2, 0)$ ; absolute minimum: 8 at  $(1, 0)$
- D) Absolute maximum: 12 at  $(1, 1)$ ; absolute minimum: 8 at  $(1, 0)$

**At what points is the given function continuous?**

196)  $f(x, y, z) = yz \cos\left(\frac{1}{x}\right)$

196) \_\_\_\_\_

- A) All  $(x, y, z)$
- B) All  $(x, y, z)$  such that  $x \neq 0$
- C) All  $(x, y, z)$  such that  $x \neq \frac{2}{\pi}$
- D) All  $(x, y, z)$  such that  $0 \leq \frac{1}{x} \leq 2\pi$

**Find the extreme values of the function subject to the given constraint.**

197)  $f(x, y, z) = (x - 1)^2 + (y - 2)^2 + (z - 2)^2$ ,  $x^2 + y^2 + z^2 = 36$

197) \_\_\_\_\_

- A) Maximum: 77 at  $(-4, -4, -2)$ ; minimum: 13 at  $(4, 4, 2)$
- B) Maximum: 81 at  $(-2, -4, -4)$ ; minimum: 9 at  $(2, 4, 4)$
- C) Maximum: 77 at  $(-4, -2, -4)$ ; minimum: 13 at  $(4, 2, 4)$
- D) Maximum: 49 at  $(-2, 4, -4)$ ; minimum: 41 at  $(2, -4, 4)$

**Answer the question.**

198) Which order of differentiation will calculate  $f_{xy}$  faster, x first or y first?

198) \_\_\_\_\_

$$f(x, y) = \frac{1}{g(y)}, g(y) \neq 0$$

- A) y first
- B) x first

**Find all local extreme values of the given function and identify each as a local maximum, local minimum, or saddle point.**

199)  $f(x, y) = x^2 + 2x + y^2 + 8y + 9$

199) \_\_\_\_\_

- A)  $f(1, 4) = 60$ , local maximum
- B)  $f(-1, -4) = -8$ , local minimum
- C)  $f(-1, 4) = 56$ , saddle point
- D)  $f(1, -4) = -4$ , saddle point

**Compute the gradient of the function at the given point.**

200)  $f(x, y) = -6x + 3y$ ,  $(-6, -3)$

200) \_\_\_\_\_

- A)  $-6\mathbf{i} + 3\mathbf{j}$
- B)  $-24\mathbf{i} + 9\mathbf{j}$
- C)  $4\mathbf{i} - 3\mathbf{j}$
- D)  $-24\mathbf{i} - 9\mathbf{j}$

**Find the extreme values of the function subject to the given constraint.**

201)  $f(x, y) = xy, \quad 9x^2 + 4y^2 = 36$

201)

- A) Maximum: 3 at  $\left(\sqrt{2}, \frac{3}{2}\sqrt{2}\right)$ ; minimum: -3 at  $\left(-\sqrt{2}, -\frac{3}{2}\sqrt{2}\right)$
- B) Maximum: 3 at  $\left(\sqrt{2}, \frac{3}{2}\sqrt{2}\right)$ ; minimum: -3 at  $\left(\sqrt{2}, -\frac{3}{2}\sqrt{2}\right)$
- C) Maximum: 3 at  $\left(\sqrt{2}, \frac{3}{2}\sqrt{2}\right)$  and  $\left(-\sqrt{2}, -\frac{3}{2}\sqrt{2}\right)$ ; minimum: -3 at  $\left(\sqrt{2}, -\frac{3}{2}\sqrt{2}\right)$  and  $\left(-\sqrt{2}, \frac{3}{2}\sqrt{2}\right)$
- D) Maximum: 3 at  $\left(\sqrt{2}, -\frac{3}{2}\sqrt{2}\right)$ ; minimum: -3 at  $\left(-\sqrt{2}, \frac{3}{2}\sqrt{2}\right)$

**Solve the problem.**

202) The surface area of a hollow cylinder (tube) is given by

202)

$$S = 2\pi(R_1 + R_2)(h + R_1 - R_2),$$

where  $h$  is the length of the cylinder and  $R_1$  and  $R_2$  are the outer and inner radii. If  $h$ ,  $R_1$ , and  $R_2$  are measured to be 10 inches, 3 inches, and 5 inches respectively, and if these measurements are accurate to within 0.1 inches, estimate the maximum percentage error in computing  $S$ .

A) 0.037%

B) 0.052%

C) 0.042%

D) 0.033%

203) Find parametric equations for the normal line to the surface  $x^2 + 7xyz + y^2 = 9z^2$  at the point (1, 1, 1).

203)

A)  $x = t + 9, y = t + 9, z = t - 11$

B)  $x = 9t + 1, y = -9t + 1, z = -11t + 1$

C)  $x = t - 9, y = t - 9, z = t + 11$

D)  $x = 9t + 1, y = 9t + 1, z = -11t + 1$

**Provide an appropriate answer.**

204) Find the value(s) of  $t$  corresponding to the extrema of  $f(x, y, z) = \sin(x^2 + y^2)\cos(z)$  subject to the constraints  $x^2 + y^2 = 6t$ ,  $0 \leq t \leq \pi$ , and  $z = \frac{\pi}{6}$ . Classify each extremum as a minimum or maximum.

204)

A)  $t = \frac{\pi}{12}$ , minimum;  $t = -\frac{\pi}{12}$ , maximum

B)  $t = \frac{\pi}{12}$ , minimum

C)  $t = -\frac{\pi}{12}$ , maximum

D)  $t = \frac{\pi}{12}$ , minimum;  $t = 0$ , maximum

**Find the absolute maximum and minimum values of the function on the given curve.**

205) Function:  $f(x, y) = xy$ ; curve:  $x^2 + y^2 = 64$ ,  $x \geq 0$ ,  $y \geq 0$ . (Use the parametric equations  $x = 8 \cos t$ ,  $y = 8 \sin t$ .)

205)

A) Absolute maximum: 16 at  $t = \frac{\pi}{4}$ ; absolute minimum: 0 at  $t = 0$  and  $t = \frac{\pi}{2}$

B) Absolute maximum: 16 at  $t = \frac{\pi}{4}$ ; absolute minimum: 0 at  $t = 0$  and  $t = \frac{\pi}{4}$

C) Absolute maximum: 32 at  $t = \frac{\pi}{4}$ ; absolute minimum: 0 at  $t = 0$  and  $t = \frac{\pi}{2}$

D) Absolute maximum: 32 at  $t = \frac{\pi}{4}$ ; absolute minimum: 0 at  $t = 0$  and  $t = \frac{\pi}{4}$

**Solve the problem.**

- 206) Find an equation for the level surface of the function  $f(x, y, z) = \int_z^y e^\theta d\theta - \int_x^{\sqrt{2}} t dt$  that passes through the point  $(0, \ln 6, \ln 3)$ . 206) \_\_\_\_\_

through the point  $(0, \ln 6, \ln 3)$ .

A)  $e^y + e^z - \frac{x^2}{2} = 3$

B)  $e^y + e^z - \frac{x^2}{2} = -3$

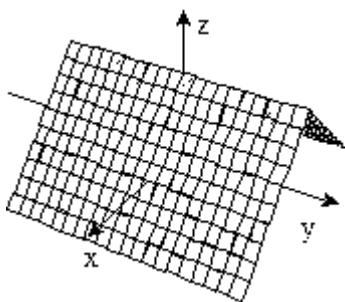
C)  $e^y - e^z + \frac{x^2}{2} = 3$

D)  $e^y - e^z + \frac{x^2}{2} = -3$

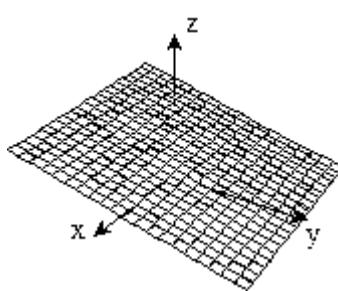
**Sketch the surface  $z = f(x,y)$ .**

- 207)  $f(x, y) = 1 - |x|$  207) \_\_\_\_\_

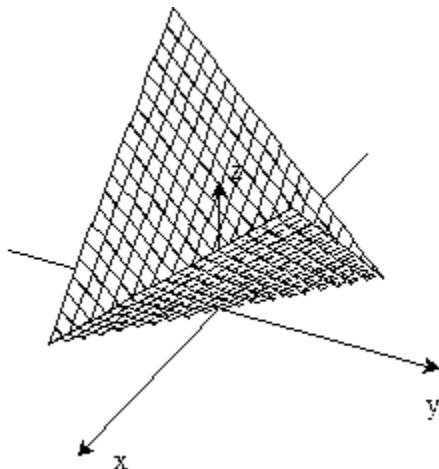
A)



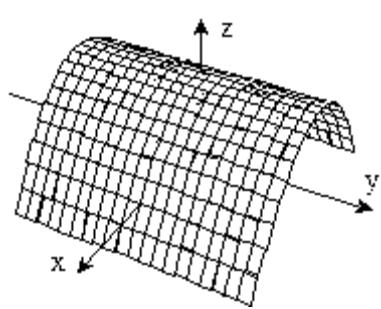
B)



C)



D)



**Solve the problem.**

- 208) The radius  $r$  and height  $h$  of a cylinder are changing with time. At the instant in question,  $r = 4$  cm,  $h = 7$  cm,  $dr/dt = 0.02$  cm/sec and  $dh/dt = -0.02$  cm/sec. At what rate is the cylinder's volume changing at that instant? 208) \_\_\_\_\_

A)  $2.51 \text{ cm}^3/\text{sec}$

B)  $1.51 \text{ cm}^3/\text{sec}$

C)  $4.52 \text{ cm}^3/\text{sec}$

D)  $0.75 \text{ cm}^3/\text{sec}$

**Find the derivative of the function at the given point in the direction of A.**

209)  $f(x, y) = \tan^{-1} \frac{-9x}{y}$ ,  $(4, -7)$ ,  $A = 12\mathbf{i} - 5\mathbf{j}$

209) \_\_\_\_\_

A)  $\frac{576}{17485}$

B)  $\frac{432}{17485}$

C)  $\frac{648}{17485}$

D)  $\frac{36}{1345}$

210)  $f(x, y) = 6x^2 - 3y$ ,  $(-6, 7)$ ,  $A = 3\mathbf{i} - 4\mathbf{j}$

210) \_\_\_\_\_

A)  $-84$

B)  $-\frac{348}{5}$

C)  $-\frac{204}{5}$

D)  $-\frac{276}{5}$

**Solve the problem.**

211) Determine the point on the paraboloid  $z = 9x^2 + 10y^2$  that is closest to the point  $(57, -42, 120)$ .

211) \_\_\_\_\_

A)  $(2, 3, 126)$

B)  $(3, -2, 121)$

C)  $(3, 2, 121)$

D)  $(-2, 3, 126)$

**Use polar coordinates to find the limit of the function as  $(x, y)$  approaches  $(0, 0)$ .**

212)  $f(x, y) = \cos\left(\frac{x^2}{x^2 + y^2}\right)$

212) \_\_\_\_\_

A)  $\frac{\pi}{2}$

B)  $0$

C)  $1$

D) No limit

**Find all the first order partial derivatives for the following function.**

213)  $f(x, y) = \ln y^x$

213) \_\_\_\_\_

A)  $\frac{\partial f}{\partial x} = \ln y; \frac{\partial f}{\partial y} = \frac{x}{y}$

B)  $\frac{\partial f}{\partial x} = x \ln y; \frac{\partial f}{\partial y} = -\frac{x}{y}$

C)  $\frac{\partial f}{\partial x} = \ln y; \frac{\partial f}{\partial y} = -x \ln y$

D)  $\frac{\partial f}{\partial x} = 0; \frac{\partial f}{\partial y} = -\frac{x}{y}$

**Find the domain and range and describe the level curves for the function  $f(x, y)$ .**

214)  $f(x, y) = \cos^{-1}(x^2 + y^2)$

214) \_\_\_\_\_

A) Domain: all points in the  $x$ - $y$  plane satisfying  $x^2 + y^2 \leq 1$ ; range: real numbers  $-\frac{\pi}{2} \leq z \leq \frac{\pi}{2}$ ;

level curves: circles with centers at  $(0, 0)$  and radii  $r$ ,  $0 < r \leq 1$

B) Domain: all points in the  $x$ - $y$  plane satisfying  $x^2 + y^2 \leq 1$ ; range: real numbers  $-\frac{\pi}{2} \leq z \leq \frac{\pi}{2}$ ;

level curves: circles with centers at  $(0, 0)$

C) Domain: all points in the  $x$ - $y$  plane; range: all real numbers; level curves: circles with centers at  $(0, 0)$

D) Domain: all points in the  $x$ - $y$  plane; range: real numbers  $-\frac{\pi}{2} \leq z \leq \frac{\pi}{2}$ ; level curves: circles with centers at  $(0, 0)$

**Find the absolute maxima and minima of the function on the given domain.**

215)  $f(x, y) = 3x + 4y$  on the closed triangular region with vertices  $(0, 0)$ ,  $(1, 0)$ , and  $(0, 1)$

215) \_\_\_\_\_

A) Absolute maximum: 7 at  $(1, 1)$ ; absolute minimum: 3 at  $(1, 0)$

B) Absolute maximum: 4 at  $(0, 1)$ ; absolute minimum: 3 at  $(1, 0)$

C) Absolute maximum: 4 at  $(0, 1)$ ; absolute minimum: 0 at  $(0, 0)$

D) Absolute maximum: 3 at  $(1, 0)$ ; absolute minimum: 0 at  $(0, 0)$

**Solve the problem.**

216) Evaluate  $\frac{\partial w}{\partial v}$  at  $(u, v) = (1, 5)$  for the function  $w(x, y) = xy^2 - \ln x$ ;  $x = e^{u+v}$ ,  $y = uv$ .

216) \_\_\_\_\_

- A) -1      B)  $35e^6 - 1$       C)  $30e^6 - 1$       D)  $75e^6 - 1$

217) Find the equation for the tangent plane to the surface  $x^2 - 8xyz + y^2 = 6z^2$  at the point  $(1, 1, 1)$ .

217) \_\_\_\_\_

- A)  $6x + 6y - 4z = 1$       B)  $6x + 6y - 4z = 8$       C)  $x + y + z = 1$       D)  $x + y + z = 8$

**Find the limit.**

$$218) \lim_{\substack{(x, y) \rightarrow (1, -1) \\ x \neq -y}} \frac{8x^2 + 16xy + 8y^2}{x + y}$$

218) \_\_\_\_\_

- A) 1      B) 0      C)  $\frac{1}{2}$       D) No limit

**Write a chain rule formula for the following derivative.**

$$219) \frac{\partial u}{\partial t} \text{ for } u = f(v); v = h(s, t)$$

219) \_\_\_\_\_

- A)  $\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t}$       B)  $\frac{\partial u}{\partial t} = \frac{\partial v}{\partial u} \frac{\partial u}{\partial t}$       C)  $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial v}$       D)  $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial v} \frac{\partial v}{\partial t}$

**At what points is the given function continuous?**

$$220) f(x, y) = \tan(x + y)$$

220) \_\_\_\_\_

- A) All  $(x, y)$   
 B) All  $(x, y) \neq \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$   
 C) All  $(x, y) \neq (0, 0)$   
 D) All  $(x, y)$  such that  $x + y \neq \frac{(2n+1)\pi}{2}$ , where  $n$  is an integer

**Provide an appropriate answer.**

221) Suppose that  $w = x^2 + y^2 + 12z + t$  and  $x + 3z + t = 5$ . Assuming that the independent variables are  $x, y$ , and  $z$ , find  $\frac{\partial w}{\partial x}$ .

221) \_\_\_\_\_

- A)  $\frac{\partial w}{\partial x} = 2x - 1$       B)  $\frac{\partial w}{\partial x} = 2x + 4$       C)  $\frac{\partial w}{\partial x} = 2x + 1$       D)  $\frac{\partial w}{\partial x} = 2x - 4$

**Solve the problem.**

222) Find the point on the line  $2x + 3y = 5$  that is closest to the point  $(1, 2)$ .

222) \_\_\_\_\_

- A)  $\left(\frac{7}{17}, \frac{13}{17}\right)$       B)  $\left(\frac{7}{13}, \frac{17}{13}\right)$       C)  $\left(\frac{5}{13}, \frac{15}{13}\right)$       D)  $\left(\frac{5}{17}, \frac{15}{17}\right)$

**Answer the question.**

- 223) Find the direction in which the function is increasing or decreasing most rapidly at the point  $P_0$ . 223) \_\_\_\_\_

$$f(x, y) = xe^y - \ln(x), P_0(4, 0)$$

A)  $\left\{ \begin{array}{l} \frac{3}{\sqrt{265}} \\ \frac{16}{\sqrt{265}} \end{array} \right\} \mathbf{i} + \left\{ \begin{array}{l} \frac{16}{\sqrt{265}} \\ \frac{3}{\sqrt{265}} \end{array} \right\} \mathbf{j}$

B)  $\left\{ \begin{array}{l} \frac{-3}{\sqrt{265}} \\ \frac{16}{\sqrt{265}} \end{array} \right\} \mathbf{i} + \left\{ \begin{array}{l} \frac{16}{\sqrt{265}} \\ \frac{-3}{\sqrt{265}} \end{array} \right\} \mathbf{j}$

C)  $\left\{ \begin{array}{l} \frac{16}{\sqrt{265}} \\ \frac{-3}{\sqrt{265}} \end{array} \right\} \mathbf{i} + \left\{ \begin{array}{l} \frac{3}{\sqrt{265}} \\ \frac{16}{\sqrt{265}} \end{array} \right\} \mathbf{j}$

D)  $\left\{ \begin{array}{l} \frac{-3}{\sqrt{265}} \\ \frac{-3}{\sqrt{265}} \end{array} \right\} \mathbf{i}$

**Solve the problem.**

- 224) Find an equation for the level surface of the function  $f(x, y, z) = x + e^{y+z}$  that passes through the point  $(1, \ln(5), \ln(8))$ . 224) \_\_\_\_\_

A)  $x + e^{y+z} = 41$

B)  $\ln(x) + y + z = 41$

C)  $x + e^{y+z} = 40$

D)  $x + e^{y+z} = 14$

- 225) What is the distance from the surface  $xy - z^2 - 6y + 36 = 0$  to the origin? 225) \_\_\_\_\_

A)  $2\sqrt{3}$

B)  $2\sqrt{6}$

C)  $6\sqrt{3}$

D)  $6\sqrt{2}$

**Find all local extreme values of the given function and identify each as a local maximum, local minimum, or saddle point.**

- 226)  $f(x, y) = 4 - x^4y^4$  226) \_\_\_\_\_

A)  $f(0, 0) = 4$ , local maximum

B)  $f(0, 0) = 4$ , local maximum;  $f(4, 4) = -65,532$ , local minimum

C)  $f(4, 4) = -65,532$ , local minimum

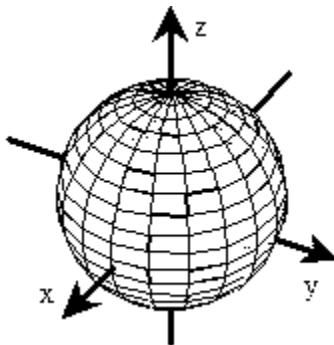
D)  $f(4, 0) = 4$ , saddle point;  $f(0, 4) = 4$ , saddle point

**Sketch a typical level surface for the function.**

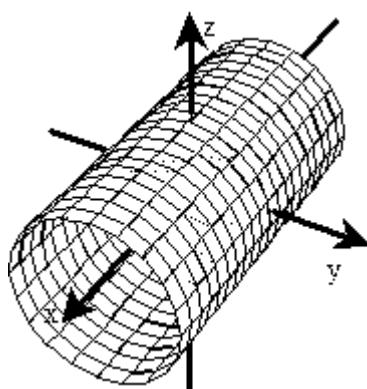
227)  $f(x, y, z) = \sqrt{x^2 - z}$

227) \_\_\_\_\_

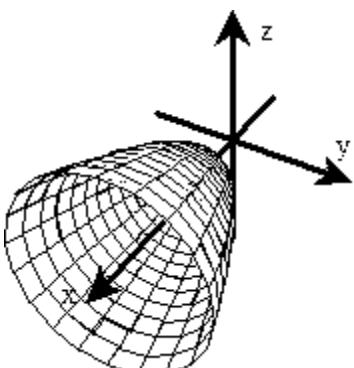
A)



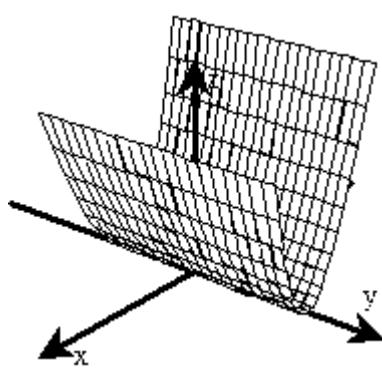
B)



C)



D)



**Find an upper bound for the magnitude  $|E|$  of the error in the linear approximation  $L$  to  $f$  at the given point over the given region.**

228)  $f(x, y) = \ln(6x + 3y)$  at  $(1, 1)$

228) \_\_\_\_\_

R:  $|x - 1| \leq 0.1, |y - 1| \leq 0.1$

A)  $|E| \leq 0.0139$

B)  $|E| \leq 0.011$

C)  $|E| \leq 0.0089$

D)  $|E| \leq 0.0073$

**Compute the gradient of the function at the given point.**

229)  $f(x, y, z) = \tan^{-1} \frac{10x}{9y + 8z}, (-6, 0, 0)$

229) \_\_\_\_\_

A)  $\frac{2}{15}\mathbf{j} - \frac{3}{20}\mathbf{k}$

B)  $\frac{2}{15}\mathbf{j} + \frac{3}{20}\mathbf{k}$

C)  $-\frac{3}{20}\mathbf{j} + \frac{2}{15}\mathbf{k}$

D)  $\frac{3}{20}\mathbf{j} + \frac{2}{15}\mathbf{k}$

**Find the limit.**

230)  $\lim_{(x, y) \rightarrow (1, 1)} \ln \left| \frac{x+y}{xy} \right|$

230) \_\_\_\_\_

A)  $-\ln 2$

B)  $\ln 2$

C) 0

D) No limit

**At what points is the given function continuous?**

231)  $f(x, y, z) = \frac{1}{|x+4| + |y-3| + |z+2|}$

231) \_\_\_\_\_

A) All  $(x, y, z) \neq (0, 0, 0)$

B) All  $(x, y, z) \neq \pm (-4, 3, -2)$

C) All  $(x, y, z) \neq (-4, 3, -2)$

D) All  $(x, y, z)$

**Find the limit.**

$$232) \lim_{\substack{(x, y) \rightarrow (9, 5) \\ y \neq 5}} \frac{xy + 6y - 5x - 30}{y - 5}$$

232) \_\_\_\_\_

- A) 3      B) 0      C) 15      D) 1

**Solve the problem.**

233) Find any local extrema (maxima, minima, or saddle points) of  $f(x, y)$  given that

$$f_x = 3x + 9y \text{ and } f_y = 8x + 2y.$$

233) \_\_\_\_\_

- A) Local maximum at  $\left(-3, -\frac{1}{4}\right)$       B) Saddle point at  $(0, 0)$   
C) Saddle point at  $(6, 72)$       D) Local minimum at  $\left(-3, -\frac{1}{4}\right)$

**Determine whether the given function satisfies Laplace's equation.**

234)  $f(x, y) = e^{-5y} \sin -5x$

234) \_\_\_\_\_

- A) Yes      B) No

**Solve the problem.**

235) Find the derivative of the function  $f(x, y) = \tan^{-1} \frac{y}{x}$  at the point  $(-9, 9)$  in the direction in which

235) \_\_\_\_\_

the function increases most rapidly.

- A)  $\frac{\sqrt{3}}{18}$       B)  $\frac{\sqrt{2}}{18}$       C)  $\frac{\sqrt{2}}{27}$       D)  $\frac{\sqrt{3}}{27}$

236) Find any local extrema (maxima, minima, or saddle points) of  $f(x, y)$  given that

236) \_\_\_\_\_

$$f_x = -10x + 3y \text{ and } f_y = 4x - 4y.$$

- A) Local minimum at  $\left(\frac{3}{10}, 1\right)$       B) Local minimum at  $(0, 0)$   
C) Saddle point at  $(0, 0)$       D) Local maximum at  $\left(\frac{3}{10}, 1\right)$

237) A normal line to the paraboloid  $z = 9x^2 + 3y^2$  also passes through the point  $(38, 35, 110)$ . Find the point on the paraboloid that the normal line passes through.

237) \_\_\_\_\_

- A)  $(2, 5, 111)$       B)  $(2, 111, 5)$       C)  $(5, 2, 38)$       D)  $(5, 38, 2)$

238) Evaluate  $\frac{\partial u}{\partial x}$  at  $(x, y, z) = (5, 4, 5)$  for the function  $u(p, q, r) = p^2 - q^2 - r$ ;  $p = xy$ ,  $q = y^2$ ,  $r = xz$ .

238) \_\_\_\_\_

- A) 45      B) 165      C) 155      D) 35

**Find the domain and range and describe the level curves for the function  $f(x, y)$ .**

239)  $f(x, y) = e^{x+y}$

239) \_\_\_\_\_

- A) Domain: all points in the  $x$ - $y$  plane; range: all real numbers; level curves: lines  $x + y = c$   
B) Domain: all points in the first quadrant of the  $x$ - $y$  plane; range: real numbers  $z > 0$ ; level curves: lines  $x + y = c$   
C) Domain: all points in the first quadrant of the  $x$ - $y$  plane; range: all real numbers; level curves: lines  $x + y = c$   
D) Domain: all points in the  $x$ - $y$  plane; range: real numbers  $z > 0$ ; level curves: lines  $x + y = c$

**Find the linearization of the function at the given point.**

240)  $f(x, y, z) = -7x^2 + 5y^2 - 5z^2$  at  $(1, -2, 3)$

240) \_\_\_\_\_

- A)  $L(x, y, z) = -14x - 20y - 30z - 32$   
C)  $L(x, y, z) = -14x + 20y - 30z - 32$

- B)  $L(x, y, z) = -14x - 20y - 30z + 32$   
D)  $L(x, y, z) = -14x + 20y - 30z + 32$

241)  $f(x, y) = -7x^2y^3$  at  $(4, 8)$

241) \_\_\_\_\_

- A)  $L(x, y) = -7168x - 21,504y + 229,376$   
C)  $L(x, y) = -28,672x - 5376y + 229,376$

- B)  $L(x, y) = -28,672x - 21,504y + 229,376$   
D)  $L(x, y) = -7168x - 5376y + 229,376$

**Find the absolute maximum and minimum values of the function on the given curve.**

242) Function:  $f(x, y) = x^2 + y^2$ ; curve:  $x = 10t + 1$ ,  $y = 10t - 1$ ,  $0 \leq t \leq 1$ .

242) \_\_\_\_\_

- A) Absolute maximum: 201 at  $t = 1$ ; absolute minimum: 11 at  $t = 0$ .  
B) Absolute maximum: 201 at  $t = 1$ ; absolute minimum: 2 at  $t = 0$ .  
C) Absolute maximum: 202 at  $t = 1$ ; absolute minimum: 11 at  $t = 0$ .  
D) Absolute maximum: 202 at  $t = 1$ ; absolute minimum: 2 at  $t = 0$ .

**Determine whether the given function satisfies Laplace's equation.**

243)  $f(x, y) = \frac{x^2}{y}$

243) \_\_\_\_\_

- A) No  
B) Yes

**Find the limit.**

244)  $\lim_{P \rightarrow (9, 0, -9)} ey^2 \sin^{-1} \left( -\frac{z}{x} \right)$

244) \_\_\_\_\_

- A)  $\frac{\pi}{2}$   
B)  $\pi$   
C) 1  
D) 0

**Solve the problem.**

245) Find parametric equations for the normal line to the surface  $z = 4x^2 + 2y^2$  at the point  $(2, 1, 18)$ .

245) \_\_\_\_\_

- A)  $x = 8t + 2$ ,  $y = 2t + 1$ ,  $z = -t + 10$   
B)  $x = 2t + 8$ ,  $y = t + 2$ ,  $z = -t + 18$   
C)  $x = 2t + 16$ ,  $y = t + 8$ ,  $z = 18t - 1$   
D)  $x = 16t + 2$ ,  $y = 4t + 1$ ,  $z = -t + 18$

**Answer the question.**

246) Which order of differentiation will calculate  $f_{xy}$  faster, x first or y first?

246) \_\_\_\_\_

- $f(x, y) = y \ln(x) - 3 \sin(x)$   
A) y first  
B) x first

**Find the extreme values of the function subject to the given constraint.**

247)  $f(x, y) = y^2 - x^2$ ,  $x^2 + y^2 = 25$

247) \_\_\_\_\_

- A) Maximum: 50 at  $(0, \pm 5\sqrt{2})$ ; minimum: -25 at  $(\pm 5, 0)$   
B) Maximum: 50 at  $(0, \pm 5\sqrt{2})$ ; minimum: -50 at  $(\pm 5\sqrt{2}, 0)$   
C) Maximum: 25 at  $(0, \pm 5)$ ; minimum: -50 at  $(\pm 5\sqrt{2}, 0)$   
D) Maximum: 25 at  $(0, \pm 5)$ ; minimum: -25 at  $(\pm 5, 0)$

**Find the limit.**

248)  $\lim_{(x, y) \rightarrow (2, 2)} e^{-x^2 - y^2}$

A) 1

B)  $e^8$

C)  $e^{-8}$

D) 0

248) \_\_\_\_\_

**Solve the problem.**

249) Find the extreme values of  $f(x, y, z) = xyz$  subject to  $x^2 + y^2 + z^2 = 4$  and  $x + y = 2$ .

249) \_\_\_\_\_

- A) Maximum:  $\sqrt{2}$  at  $(1, -1, -\sqrt{2})$ ; minimum:  $-\sqrt{2}$  at  $(1, -1, \sqrt{2})$
- B) Maximum:  $\sqrt{2}$  at  $(1, 1, \sqrt{2})$ ; minimum:  $-\sqrt{2}$  at  $(1, 1, -\sqrt{2})$
- C) Maximum:  $\sqrt{3}$  at  $(1, -1, -\sqrt{3})$ ; minimum:  $-\sqrt{3}$  at  $(1, -1, \sqrt{3})$
- D) Maximum:  $\sqrt{3}$  at  $(1, 1, \sqrt{3})$ ; minimum:  $-\sqrt{3}$  at  $(1, 1, -\sqrt{3})$

**Find the limit.**

250)  $\lim_{(x, y) \rightarrow \left(\frac{\pi}{4}, 1\right)} \frac{y \tan x}{y + 1}$

A)  $\frac{1}{2}$

B) 2

C) 0

D)  $\frac{4 \tan 1}{5\pi}$

250) \_\_\_\_\_

**Solve the problem.**

251) Evaluate  $\frac{\partial w}{\partial u}$  at  $(u, v) = (1, 5)$  for the function  $w(x, y, z) = xz + yz - z^2$ ;  $x = uv$ ,  $y = uv$ ,  $z = u$ .

251) \_\_\_\_\_

A) 0

B) -2

C) 20

D) 18

252) Find  $F'(x)$  if  $F(x) = \int_x^1 \sqrt{t^4 + x} dt$ .

252) \_\_\_\_\_

A)  $\int_x^1 \frac{1}{\sqrt{t^4 + x}} dt - \sqrt{x^4 + x}$

B)  $\sqrt{x^4 + x} - \int_x^1 \frac{1}{2\sqrt{t^4 + x}} dt$

C)  $\int_x^1 \frac{1}{2\sqrt{t^4 + x}} dt - \sqrt{x^4 + x}$

D)  $\int_x^1 \frac{1}{\sqrt{t^4 + x}} dt - x\sqrt{x^4 + x}$

**Provide an appropriate response.**

253) Define  $f(0,0)$  in such a way that extends  $f(x, y) = \frac{x^2 y^2}{x^2 + y^2}$  to be continuous at the origin.

253) \_\_\_\_\_

A) No definition makes  $f(x, y)$  continuous at the origin.

B)  $f(0, 0) = 2$

C)  $f(0, 0) = 1$

D)  $f(0, 0) = 0$

**Find the limit.**

254)  $\lim_{(x, y) \rightarrow (0, 0)} \frac{4x^2 + 4y^2 + 2}{4x^2 - 4y^2 + 1}$

254) \_\_\_\_\_

A) 1

B) -1

C) 2

D) No limit

Determine whether the given function satisfies Laplace's equation.

255)  $f(x, y) = \cos(x) \sin(-y)$

255) \_\_\_\_\_

A) No

B) Yes

Find the absolute maximum and minimum values of the function on the given curve.

256) Function:  $f(x, y) = x^2 + 2y^2$ ; curve:  $\frac{x^2}{4} + \frac{y^2}{64} = 1, x \geq 0, y \geq 0$ . (Use the parametric equations

256) \_\_\_\_\_

$x = 2 \cos t, y = 8 \sin t$ .)

A) Absolute maximum: 128 at  $t = \frac{\pi}{2}$ ; absolute minimum: 8 at  $t = 0$

B) Absolute maximum: 64 at  $t = \frac{\pi}{2}$ ; absolute minimum: 4 at  $t = 0$

C) Absolute maximum: 64 at  $t = \frac{\pi}{2}$ ; absolute minimum: 8 at  $t = 0$

D) Absolute maximum: 128 at  $t = \frac{\pi}{2}$ ; absolute minimum: 4 at  $t = 0$

Use polar coordinates to find the limit of the function as  $(x, y)$  approaches  $(0, 0)$ .

257)  $f(x, y) = \frac{x+y}{x^2+y+y^2}$

257) \_\_\_\_\_

A) 1

B) 2

C) 0

D) No limit

Solve the problem.

258) Find the maximum value of  $f(x, y, z) = x + 2y + 3z$  subject to  $x - y + z = 1$  and  $x^2 + y^2 = 1$ .

258) \_\_\_\_\_

A)  $3 - \sqrt{29}$

B)  $3 + \sqrt{37}$

C)  $3 + \sqrt{29}$

D)  $3 - \sqrt{37}$

Find the derivative of the function at the given point in the direction of A.

259)  $f(x, y, z) = -3x + 6y - 9z, (-9, 10, -2), A = 3\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$

259) \_\_\_\_\_

A)  $-\frac{45}{7}$

B)  $-\frac{27}{7}$

C)  $-\frac{9}{7}$

D)  $-\frac{3}{7}$

Provide an appropriate answer.

260) Find the extrema of  $f(x, y, z) = x + yz$  on the line defined by  $x = 6(6 + t)$ ,  $y = t - 6$ , and  $z = t + 6$ .

260) \_\_\_\_\_

Classify each extremum as a minimum or maximum.

A)  $(54, -3, 9)$ , minimum;  $(0, -6, 0)$ , minimum

B)  $(18, -9, 3)$ , minimum;  $(54, -3, 9)$ , maximum

C)  $(18, -9, 3)$ , minimum

D)  $(54, -3, 9)$ , minimum

Find the limit.

261)  $\lim_{P \rightarrow (5, 5, 1)} \ln z \sqrt{x^2 + y^2}$

261) \_\_\_\_\_

A)  $\ln 5\sqrt{2}$

B)  $\ln 25$

C) 0

D)  $\ln \sqrt{2}$

Solve the problem.

262) A rectangle with sides parallel to the axes is inscribed in the region bounded by the axes and the line  $x + 2y = 2$ . Find the maximum area of this rectangle.

262) \_\_\_\_\_

A)  $\frac{1}{3}$

B)  $\frac{1}{4}$

C)  $\frac{1}{2}$

D)  $\frac{2}{3}$

**Find the requested partial derivative.**

263)  $(\partial w / \partial z)_{x,y}$  at  $(x, y, z, w) = (1, 2, 9, 266)$  if  $w = x^2 + y^2 + z^2 + 10xyz$

263) \_\_\_\_\_

A) 38

B) 33

C) 28

D) 48

**Solve the problem.**

264) Evaluate  $\frac{dw}{dt}$  at  $t = \frac{1}{2}\pi$  for the function  $w(x, y) = x^2 - y^2 + 10x$ ;  $x = \cos t$ ,  $y = \sin t$ .

264) \_\_\_\_\_

A) -10

B) 8

C) 6

D) 3

**Find the requested partial derivative.**

265)  $\frac{\partial w}{\partial x}$  if  $w = x^3 + y^3 + z^3 + 18xyz$  and  $\left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial x}\right)^2 = 0$ .

265) \_\_\_\_\_

A)  $3x^2$

B)  $18(x^2 + y^2 + z^2)$

C)  $3(x^2 + y^2 + z^2)$

D)  $18x^2$

**Solve the problem.**

266) Find the point on the curve of intersection of the paraboloid  $x^2 + y^2 + 2z = 4$  and the plane  $x - y + 2z = 0$  that is farthest from the origin.

266) \_\_\_\_\_

A)  $(2, -2, -2)$

B)  $(-2, -2, 2)$

C)  $(-2, 2, 2)$

D)  $(2, -2, 2)$

267) The resistance  $R$  produced by wiring resistors of  $R_1$  and  $R_2$  ohms in parallel can be calculated from the formula

267) \_\_\_\_\_

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

If  $R_1$  and  $R_2$  are measured to be 3 ohms and 6 ohms respectively and if these measurements are accurate to within 0.05 ohms, estimate the maximum possible error in computing  $R$ .

A) 0.033

B) 0.017

C) 0.022

D) 0.028

**Determine whether the given function satisfies the wave equation.**

268)  $w(x, t) = e^{9ct} \cos 9x$

268) \_\_\_\_\_

A) No

B) Yes

**Find all the second order partial derivatives of the given function.**

269)  $f(x, y) = e^x/y$

269) \_\_\_\_\_

A)  $\frac{\partial^2 f}{\partial x^2} = \frac{e^x/y}{y^2}; \frac{\partial^2 f}{\partial y^2} = \left( \frac{x^2 + 2xy}{y^4} \right); \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{y+x}{y^3}$

B)  $\frac{\partial^2 f}{\partial x^2} = \frac{e^x/y}{y^2}; \frac{\partial^2 f}{\partial y^2} = e^x/y \left( \frac{x^2 + 2xy}{y^4} \right); \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -e^x/y \left( \frac{y+x}{y^3} \right)$

C)  $\frac{\partial^2 f}{\partial x^2} = \frac{e^x/y}{y^2}; \frac{\partial^2 f}{\partial y^2} = -e^x/y \left( \frac{x^2 + 2xy}{y^3} \right); \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = e^x/y \left( \frac{y+x}{y^3} \right)$

D)  $\frac{\partial^2 f}{\partial x^2} = \frac{e^x/y}{y^2}; \frac{\partial^2 f}{\partial y^2} = e^x/y \left( \frac{x^2 + 2xy}{y^3} \right); \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -e^x/y \left( \frac{y+x}{y^3} \right)$

**Find all the first order partial derivatives for the following function.**

270)  $f(x, y) = \sin^2(8xy^2 - y)$

270) \_\_\_\_\_

A)  $\frac{\partial f}{\partial x} = 16y^2 \sin(8xy^2 - y) \cos(8xy^2 - y); \frac{\partial f}{\partial y} = (32xy - 2) \sin(8xy^2 - y) \cos(8xy^2 - y)$

B)  $\frac{\partial f}{\partial x} = 16y^2 \sin(8xy^2 - y) \cos(8xy^2 - y); \frac{\partial f}{\partial y} = 2 \sin(8xy^2 - y) \cos(8xy^2 - y)$

C)  $\frac{\partial f}{\partial x} = 2 \sin(8xy^2 - y) \cos(8xy^2 - y); \frac{\partial f}{\partial y} = 2 \sin(8xy^2 - y) \cos(8xy^2 - y)$

D)  $\frac{\partial f}{\partial x} = 2 \sin(8xy^2 - y) \cos(8xy^2 - y); \frac{\partial f}{\partial y} = (32x - 2) \sin(8xy^2 - y) \cos(8xy^2 - y)$

**Compute the gradient of the function at the given point.**

271)  $f(x, y) = 2x^2 - 8y, (4, 8)$

271) \_\_\_\_\_

A)  $64\mathbf{i} - 64\mathbf{j}$

B)  $16\mathbf{i} - 8\mathbf{j}$

C)  $16\mathbf{i} - 64\mathbf{j}$

D)  $32\mathbf{i} - 64\mathbf{j}$

**Find the limit.**

272)  $\lim_{(x, y) \rightarrow (3, 5)} \sqrt{\frac{1}{xy}}$

272) \_\_\_\_\_

A) 15

B)  $\frac{\sqrt{15}}{15}$

C)  $\sqrt{15}$

D) No limit

**Determine whether the given function satisfies the wave equation.**

273)  $w(x, t) = \cos(4ct) \sin(4x)$

273) \_\_\_\_\_

A) No

B) Yes

**Solve the problem.**

274) Find parametric equations for the normal line to the surface  $z = \ln(8x^2 + 3y^2 + 1)$  at the point  $(0, 0, 0)$ .

274) \_\_\_\_\_

A)  $x=t, y=t, z=-1$

B)  $x=1, y=1, z=-t$

C)  $x=t, y=t, z=0$

D)  $x=0, y=0, z=t$

275) What points of the surface  $xy - z^2 - 6y + 36 = 0$  are closest to the origin?

275) \_\_\_\_\_

A)  $(-2, 4, \pm 2)$

B)  $(-2, -4, \pm 2)$

C)  $(2, 4, \pm 2)$

D)  $(2, -4, \pm 2)$

276) Let  $T(x, y) = 16x^2 - 2xy + 4y^2$  be the temperature at the point  $(x, y)$  on the ellipse  $x = 2 \cos t, y = 4 \sin t, 0 \leq t \leq 2\pi$ . Find the minimum and maximum temperatures,  $T_{\min}$  and  $T_{\max}$ , respectively, on the ellipse.

276) \_\_\_\_\_

A)  $T_{\min} = \frac{\pi}{4}; T_{\max} = \frac{3\pi}{4}$

B)  $T_{\min} = 56; T_{\max} = 72$

C)  $T_{\min} = 0; T_{\max} = 16$

D)  $T_{\min} = 120; T_{\max} = 136$

**Find all the first order partial derivatives for the following function.**

277)  $f(x, y) = 2x - 2y^2 - 7$

A)  $\frac{\partial f}{\partial x} = -4y; \frac{\partial f}{\partial y} = 2$

C)  $\frac{\partial f}{\partial x} = 2; \frac{\partial f}{\partial y} = -4y$

B)  $\frac{\partial f}{\partial x} = -5; \frac{\partial f}{\partial y} = -4y - 7$

D)  $\frac{\partial f}{\partial x} = 2x; \frac{\partial f}{\partial y} = -4y$

277)

278)  $f(x, y) = (8x^5y^2 + 4)^2$

A)  $\frac{\partial f}{\partial x} = 80x^4y^2(8x^5y^2 + 4); \frac{\partial f}{\partial y} = 32x^5y^1(8x^5y^2 + 4)$

B)  $\frac{\partial f}{\partial x} = 40x^4y^2; \frac{\partial f}{\partial y} = 16x^5y^1$

C)  $\frac{\partial f}{\partial x} = 2(8x^5y^2 + 4); \frac{\partial f}{\partial y} = 2(8x^5y^2 + 4)$

D)  $\frac{\partial f}{\partial x} = 32x^5y^1(8x^5y^2 + 4); \frac{\partial f}{\partial y} = 80x^4y^2(8x^5y^2 + 4)$

278)

279)  $f(x, y, z) = z(e^x)^y$

A)  $\frac{\partial f}{\partial x} = zye^{xy}; \frac{\partial f}{\partial y} = zx e^{xy}; \frac{\partial f}{\partial z} = ze^{xy}$

C)  $\frac{\partial f}{\partial x} = zye^{xy}; \frac{\partial f}{\partial y} = zx e^{xy}; \frac{\partial f}{\partial z} = e^{xy}$

B)  $\frac{\partial f}{\partial x} = zx e^{xy}; \frac{\partial f}{\partial y} = zye^{xy}; \frac{\partial f}{\partial z} = e^{xy}$

D)  $\frac{\partial f}{\partial x} = ze^{xy}; \frac{\partial f}{\partial y} = ze^{xy}; \frac{\partial f}{\partial z} = e^{xy}$

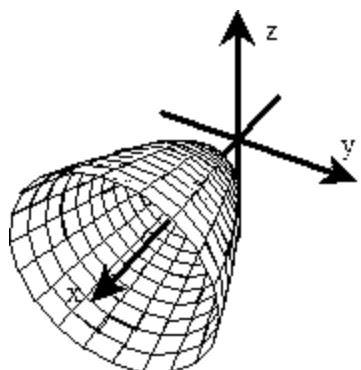
279)

**Sketch a typical level surface for the function.**

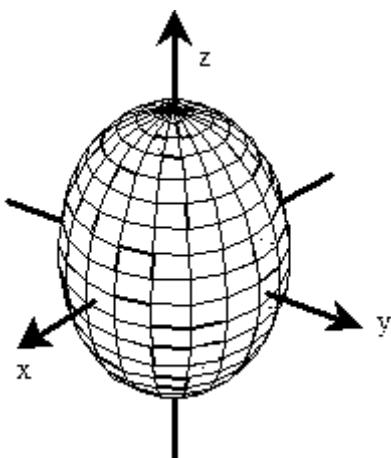
280)  $f(x, y, z) = x - y^2 - z^2$

280) \_\_\_\_\_

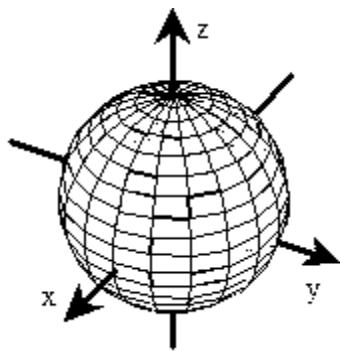
A)



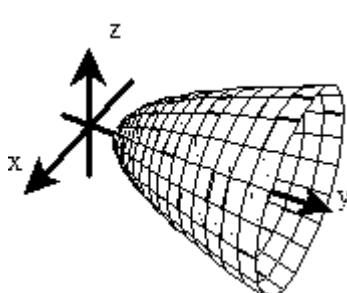
B)



C)



D)



**Solve the problem.**

- 281) Write parametric equations for the tangent line to the curve of intersection of the surfaces  
 $x + y^2 + 2z = 4$  and  $x = 1$  at the point  $(1, 1, 1)$ .

281) \_\_\_\_\_

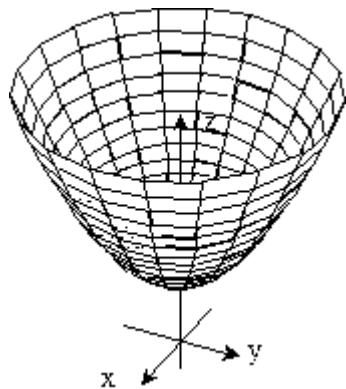
- A)  $x = 1, y = 4t + 1, z = -t + 1$   
C)  $x = 1, y = 4t + 1, z = -2t + 1$

- B)  $x = 1, y = 2t + 1, z = -t + 1$   
D)  $x = 1, y = 2t + 1, z = -2t + 1$

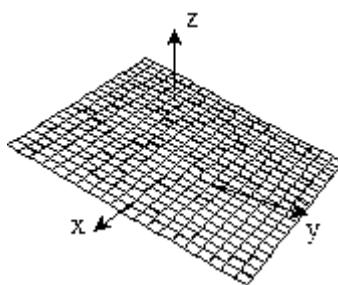
**Sketch the surface  $z = f(x,y)$ .**

282)  $f(x, y) = -x^2 - y^2$

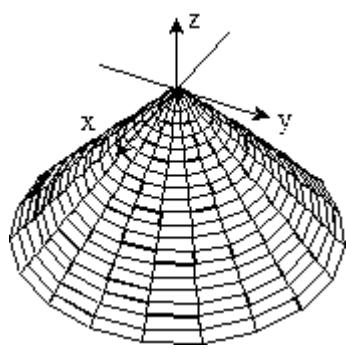
A)



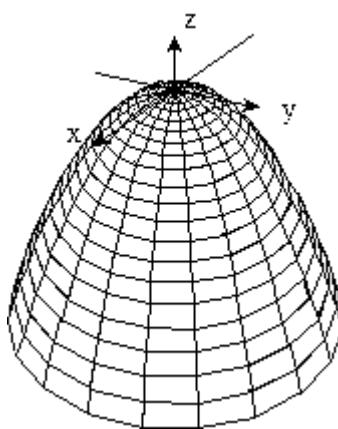
B)



C)



D)



**Find all the second order partial derivatives of the given function.**

283)  $f(x, y) = x^2 + y - e^{x+y}$

283)

A)  $\frac{\partial^2 f}{\partial x^2} = 2 - e^{x+y}; \frac{\partial^2 f}{\partial y^2} = -e^{x+y}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -e^{x+y}$

B)  $\frac{\partial^2 f}{\partial x^2} = 2 - y^2 e^{x+y}; \frac{\partial^2 f}{\partial y^2} = -x^2 e^{x+y}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -y^2 e^{x+y}$

C)  $\frac{\partial^2 f}{\partial x^2} = 2 + e^{x+y}; \frac{\partial^2 f}{\partial y^2} = e^{x+y}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = e^{x+y}$

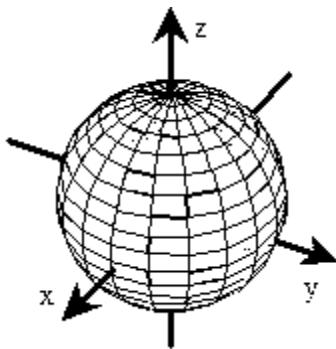
D)  $\frac{\partial^2 f}{\partial x^2} = 1 - e^{x+y}; \frac{\partial^2 f}{\partial y^2} = -e^{x+y}; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -e^{x+y}$

**Sketch a typical level surface for the function.**

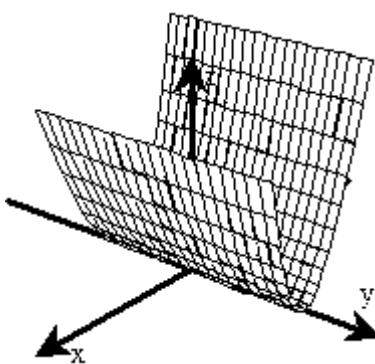
284)  $f(x, y, z) = 4e^{5(y^2 + z^2)}$

284) \_\_\_\_\_

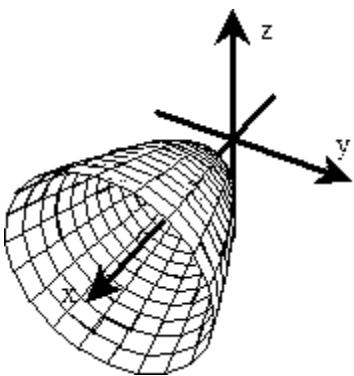
A)



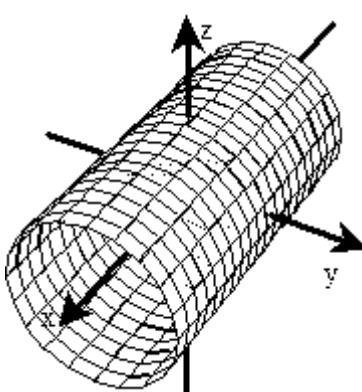
B)



C)



D)



**Determine whether the given function satisfies the wave equation.**

285)  $w(x, t) = \ln(-10cx^t)$

285) \_\_\_\_\_

A) Yes

B) No

**Find an upper bound for the magnitude  $|E|$  of the error in the linear approximation  $L$  to  $f$  at the given point over the given region.**

286)  $f(x, y, z) = 4xy + 5yz + 9zx$  at  $(1, 1, 1)$

286) \_\_\_\_\_

R:  $|x - 1| \leq 0.1, |y - 1| \leq 0.1, |z - 1| \leq 0.1$

A)  $|E| \leq 0.315$

B)  $|E| \leq 0.378$

C)  $|E| \leq 0.405$

D)  $|E| \leq 0.338$

**Use implicit differentiation to find the specified derivative at the given point.**

287) Find  $\frac{dy}{dx}$  at the point  $(1, 1)$  for  $3x^2 + 2y^3 + 2xy = 0$ .

287) \_\_\_\_\_

A) -1

B) 1

C)  $-\frac{5}{4}$

D) -2

**Use Taylor's formula to find the requested approximation of  $f(x, y)$  near the origin.**

288) Quadratic approximation to  $f(x, y) = \ln(1 + 7x + y)$

288) \_\_\_\_\_

A)  $7x + y - \frac{49}{2}x^2 - 7xy - \frac{1}{2}y^2$

B)  $7x + y - \frac{49}{2}x^2 - \frac{7}{2}xy - \frac{1}{2}y^2$

C)  $1 + 7x + y - \frac{49}{2}x^2 - \frac{7}{2}xy - \frac{1}{2}y^2$

D)  $1 + 7x + y - \frac{49}{2}x^2 - 7xy - \frac{1}{2}y^2$

**Estimate the error in the quadratic approximation of the given function at the origin over the given region.**

289)  $f(x, y) = \sin 5x \sin^2 6y, -0.1 \leq x, y \leq 0.1$

289) \_\_\_\_\_

- A)  $|E(x, y)| \leq 0.288$       B)  $|E(x, y)| \leq 0.05$       C)  $|E(x, y)| \leq 0.216$       D)  $|E(x, y)| \leq 0.864$

**Find the domain and range and describe the level curves for the function  $f(x, y)$ .**

290)  $f(x, y) = \frac{y+6}{x^2}$

290) \_\_\_\_\_

- A) Domain: all points in the  $x$ - $y$  plane excluding  $x = 0$ ; range: real numbers  $z \geq 0$ ; level curves: parabolas  $y = cx^2 - 6$   
B) Domain: all points in the  $x$ - $y$  plane excluding  $x = 0$ ; range: all real numbers; level curves: parabolas  $y = cx^2 - 6$   
C) Domain: all points in the  $x$ - $y$  plane; range: real numbers  $z \geq 0$ ; level curves: parabolas  $y = cx^2 - 6$   
D) Domain: all points in the  $x$ - $y$  plane; range: all real numbers; level curves: parabolas  $y = cx^2 - 6$

**Find the extreme values of the function subject to the given constraint.**

291)  $f(x, y, z) = x^2 + 2y^2 + 3z^2, x - y - z = 1$

291) \_\_\_\_\_

- A) Maximum: none; minimum:  $\frac{6}{11}$  at  $\left(\frac{6}{11}, -\frac{3}{11}, -\frac{2}{11}\right)$   
B) Maximum: none; minimum:  $\frac{6}{11}$  at  $\left(-\frac{6}{11}, -\frac{3}{11}, -\frac{2}{11}\right)$   
C) Maximum: none; minimum:  $\frac{6}{11}$  at  $\left(-\frac{6}{11}, \frac{3}{11}, -\frac{2}{11}\right)$   
D) Maximum: none; minimum:  $\frac{6}{11}$  at  $\left(-\frac{6}{11}, -\frac{3}{11}, \frac{2}{11}\right)$

**Find an upper bound for the magnitude  $|E|$  of the error in the linear approximation  $L$  to  $f$  at the given point over the given region.**

292)  $f(x, y) = 7x^2 - 3y^2 - 4$  at  $(-7, 8)$

292) \_\_\_\_\_

R:  $|x + 7| \leq 0.1, |y - 8| \leq 0.1$

- A)  $|E| \leq 0.12$       B)  $|E| \leq 0.03$       C)  $|E| \leq 0.06$       D)  $|E| \leq 0.09$

**Solve the problem.**

- 293) Amarillo Motors manufactures an economy car called the Citrus, which is notorious for its inability to hold a respectable resale value. The average resale value of a set of 1998 Amarillo Citrus's is summarized in the table below along with the age of the car at the time of resale and the number of cars included in the average. Fit a line of the form  $\ln(V) = m \cdot a + b$  to the data, where  $V$  is the resale value in thousands of dollars and  $a$  is the age of the car in years..

293)

Age (years)	Average Resale Value (1000's of dollars)	Frequency
0	18	3
1	14	8
2	11	2
3	8	4
4	6	10
5	4	6
6	3	5

- A)  $\ln(V) = 0.299a - 2.951$   
 B)  $\ln(V) = 0.314a - 2.986$   
 C)  $\ln(V) = 0.303a - 2.953$   
 D)  $\ln(V) = 0.315a$

**Find the limit.**

294)  $\lim_{(x, y) \rightarrow (0, 0)} \sin\left(\frac{x^3 + y^9}{x - y + 8}\right)$

294)

- A)  $\frac{1}{8}$   
 B) 0  
 C) 1  
 D) No limit

**Find the equation for the level surface of the function through the given point.**

295)  $f(x, y, z) = \int_y^z (\ln \theta + 1) d\theta + \int_0^x t e^t dt, (5, e^6, e^3)$

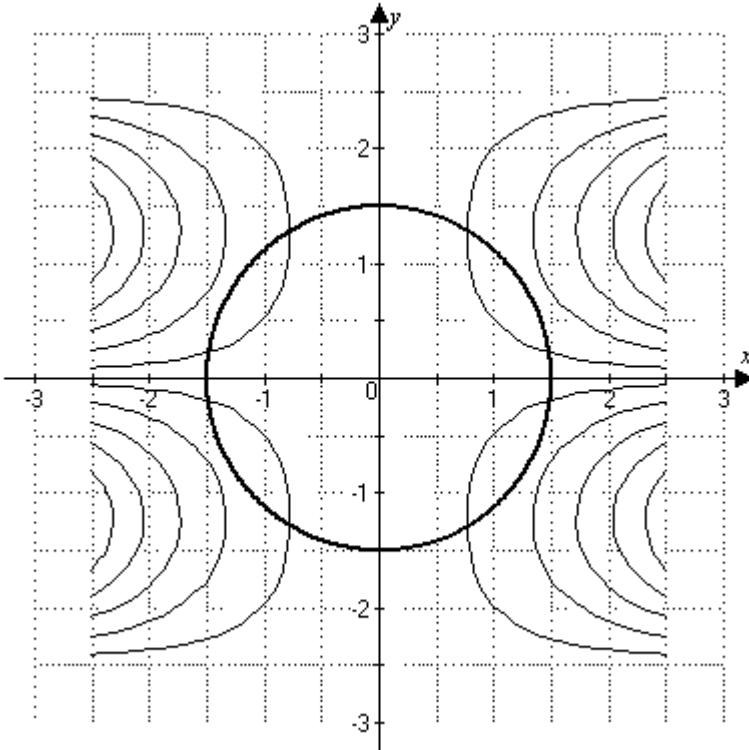
295)

- A)  $3e^3 - 6e^6 + 4e^5 - 2 = \int_y^z (\ln \theta + 1) d\theta + \int_0^x t e^t dt$   
 B)  $3e^3 - 6e^6 + 4e^5 + 1 = \int_y^z (\ln \theta + 1) d\theta + \int_0^x t e^t dt$   
 C)  $2e^3 - 5e^6 + 4e^5 + 1 = \int_y^z (\ln \theta + 1) d\theta + \int_0^x t e^t dt$   
 D)  $3e^3 - 6e^6 + 4e^5 + 1 = \ln \theta + 1 + t e^t$

**Answer the question.**

- 296) The graph below shows the level curves of a differentiable function  $f(x, y)$  (thin curves) as well as the constraint  $g(x, y) = \sqrt{x^2 + y^2} - \frac{3}{2} = 0$  (thick circle). Using the concepts of the orthogonal gradient theorem and the method of Lagrange multipliers, estimate the coordinates corresponding to the constrained extrema of  $f(x, y)$ .

296) \_\_\_\_\_



- A) (1.3, 0.7), (-1.3, 0.7), (-1.3, -0.7), (1.3, -0.7)
- B) (1.5, 0), (0, 1.5), (-1.5, 0), (0, -1.5)
- C) (1.1, 1.1), (-1.1, 1.1), (-1.1, -1.1), (1.1, -1.1)
- D) (1.5, 0.2), (0.7, 1.3), (-1.5, 0.2), (-0.7, 1.3), (-1.5, -0.2), (-0.7, -1.3), (1.5, -0.2), (0.7, -1.3)

**Solve the problem.**

- 297) Write an equation for the tangent line to the curve  $y^2 - x = 5$  at the point  $(1, \sqrt{6})$ .

297) \_\_\_\_\_

- A)  $x - 2\sqrt{6}y + 11 = 0$
- B)  $x - 2\sqrt{6}y + 1 = 0$
- C)  $x - 2y + 1 = 0$
- D)  $x - 2y + 11 = 0$

**Find all the second order partial derivatives of the given function.**

298)  $f(x, y) = xye^{-y^2}$

298) \_\_\_\_\_

A)  $\frac{\partial^2 f}{\partial x^2} = 0; \frac{\partial^2 f}{\partial y^2} = 2xye^{-y^2}(2y^2 - 3); \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = e^{-y^2}(1 - 2y^2)$

B)  $\frac{\partial^2 f}{\partial x^2} = ye^{-y^2}; \frac{\partial^2 f}{\partial y^2} = 2xye^{-y^2}(2y^2 - 6); \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = e^{-y^2}(1 - 2y^2)$

C)  $\frac{\partial^2 f}{\partial x^2} = 0; \frac{\partial^2 f}{\partial y^2} = 2xye^{-y^2}(2y^2 - 3); \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = e^{-y^2}(1 - y^2)$

D)  $\frac{\partial^2 f}{\partial x^2} = ye^{-y^2}; \frac{\partial^2 f}{\partial y^2} = 2xye^{-y^2}(y^2 - 1); \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = e^{-y^2}(1 - y^2)$

**Solve the problem.**

299) Find the least squares line for the points  $(10, 50), (11, -55), (12, 60), (13, -65)$ .

299) \_\_\_\_\_

A)  $y = 242 + 17x$

B)  $y = 262 + 17x$

C)  $y = 262 - 23x$

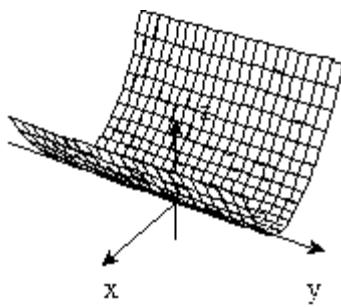
D)  $y = 242 - 23x$

**Sketch the surface  $z = f(x,y)$ .**

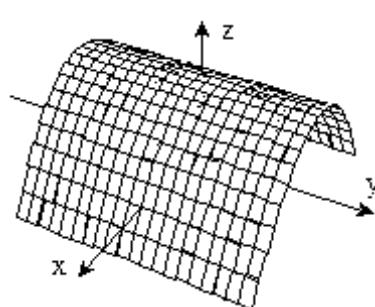
300)  $f(x, y) = 3 - x^2$

300) \_\_\_\_\_

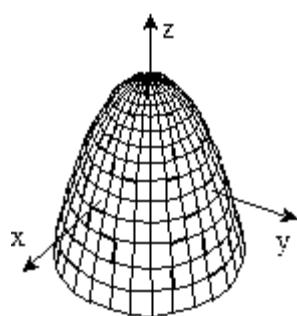
A)



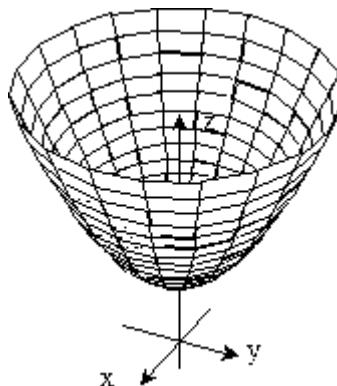
B)



C)



D)



**Find all local extreme values of the given function and identify each as a local maximum, local minimum, or saddle point.**

301)  $f(x, y) = x^2 + 10xy + y^2$

301) \_\_\_\_\_

- A)  $f(10, 0) = 100$ , local minimum;  $f(0, 10) = 100$ , local minimum
- B)  $f(10, 10) = 1200$ , local maximum
- C)  $f(0, 0) = 0$ , saddle point;  $f(10, 10) = 1200$ , local maximum
- D)  $f(0, 0) = 0$ , saddle point

**Solve the problem.**

302) Determine whether the function

302) \_\_\_\_\_

$$f(x, y) = 2x^2y^2 + 5x^4y^4$$

has a maximum, a minimum, or neither at the origin.

- A) Maximum
- B) Minimum
- C) Neither

**Use polar coordinates to find the limit of the function as  $(x, y)$  approaches  $(0, 0)$ .**

303)  $f(x, y) = \frac{x+y}{\sqrt{x+y}}$

303) \_\_\_\_\_

- A)  $\pi$
- B) 0
- C) 1
- D) No limit

**Find all local extreme values of the given function and identify each as a local maximum, local minimum, or saddle point.**

304)  $f(x, y) = -2xy(x + y) - 9$

304) \_\_\_\_\_

- A)  $f(0, 0) = -9$ , saddle point
- B)  $f(-2, -2) = 23$ , local maximum
- C)  $f(-2, 0) = -9$ , local minimum;  $f(0, -2) = -9$ , local minimum
- D)  $f(0, 0) = -9$ , saddle point;  $f(-2, -2) = 23$ , local maximum

**Find the extreme values of the function subject to the given constraint.**

305)  $f(x, y) = 3x - y + 1, \quad 3x^2 + y^2 = 9$

305) \_\_\_\_\_

- A) Maximum: 4 at  $\left(\frac{3}{2}, \frac{3}{2}\right)$ ; minimum: -5 at  $\left(-\frac{3}{2}, \frac{3}{2}\right)$
- B) Maximum: 7 at  $\left(\frac{3}{2}, -\frac{3}{2}\right)$ ; minimum: -2 at  $\left(-\frac{3}{2}, -\frac{3}{2}\right)$
- C) Maximum: 7 at  $\left(\frac{3}{2}, -\frac{3}{2}\right)$ ; minimum: -5 at  $\left(-\frac{3}{2}, \frac{3}{2}\right)$
- D) Maximum: 4 at  $\left(\frac{3}{2}, \frac{3}{2}\right)$ ; minimum: -2 at  $\left(-\frac{3}{2}, -\frac{3}{2}\right)$

**Estimate the error in the quadratic approximation of the given function at the origin over the given region.**

306)  $f(x, y) = e^{6x} \cos 3y, \quad -0.1 \leq x, y \leq 0.1$

306) \_\_\_\_\_

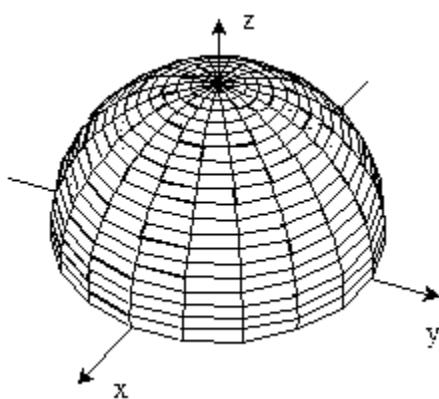
- A)  $|E(x, y)| \leq 0.5248$
- B)  $|E(x, y)| \leq 1.5743$
- C)  $|E(x, y)| \leq 0.3936$
- D)  $|E(x, y)| \leq 0.7872$

**Sketch the surface  $z = f(x,y)$ .**

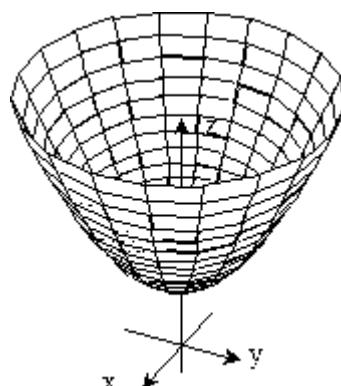
307)  $f(x, y) = 2 - x^2 - y^2$

307) \_\_\_\_\_

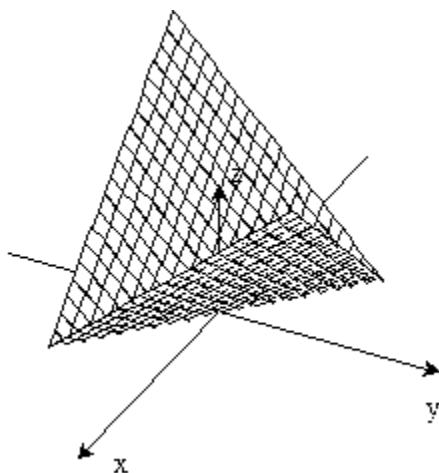
A)



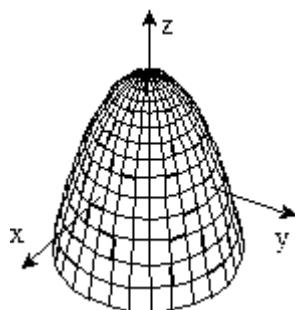
B)



C)



D)



**Find the domain and range and describe the level curves for the function  $f(x,y)$ .**

308)  $f(x, y) = 9x^2 - 3y^2$

308) \_\_\_\_\_

A) Domain: all points in the  $x$ - $y$  plane; range: all real numbers; level curves: hyperbolas

$$9x^2 - 3y^2 = c$$

B) Domain: all points in the  $x$ - $y$  plane; range: all real numbers; level curves: ellipses

$$9x^2 + 3y^2 = c$$

C) Domain: all points in the first quadrant of the  $x$ - $y$  plane; range: all real numbers; level curves:

$$9x^2 - 3y^2 = c$$

D) Domain: all points in the  $x$ - $y$  plane; range: real numbers  $z \geq 0$ ; level curves: ellipses

$$9x^2 + 3y^2 = c$$

Use Taylor's formula to find the requested approximation of  $f(x, y)$  near the origin.

- 309) Cubic approximation to  $f(x, y) = \ln(1 + 5x + y)$

309)

A)  $5x + y - \frac{25}{2}x^2 - 5xy - \frac{1}{2}y^2 + \frac{125}{3}x^3 + 25x^2y + 5xy^2 + \frac{1}{3}y^3$

B)  $1 + 5x + y - \frac{25}{2}x^2 - 5xy - \frac{1}{2}y^2 + \frac{125}{3}x^3 + 25x^2y + 5xy^2 + \frac{1}{3}y^3$

C)  $5x + y - \frac{25}{2}x^2 - 5xy - \frac{1}{2}y^2 + \frac{125}{3}x^3 + \frac{25}{3}x^2y + \frac{5}{3}xy^2 + \frac{1}{3}y^3$

D)  $1 + 5x + y - \frac{25}{2}x^2 - 5xy - \frac{1}{2}y^2 + \frac{125}{3}x^3 + \frac{25}{3}x^2y + \frac{5}{3}xy^2 + \frac{1}{3}y^3$

Find all the first order partial derivatives for the following function.

- 310)  $f(x, y, z) = xz\sqrt{x+y}$

310)

A)  $\frac{\partial f}{\partial x} = z\left(\sqrt{x+y} + \frac{x}{2\sqrt{x+y}}\right); \frac{\partial f}{\partial y} = \frac{xy}{2\sqrt{x+y}}; \frac{\partial f}{\partial z} = x\sqrt{x+y}$

B)  $\frac{\partial f}{\partial x} = z\left(\sqrt{x+y} - \frac{x}{2\sqrt{x+y}}\right); \frac{\partial f}{\partial y} = -\frac{xy}{2\sqrt{x+y}}; \frac{\partial f}{\partial z} = x\sqrt{x+y}$

C)  $\frac{\partial f}{\partial x} = z\left(\sqrt{x+y} - \frac{x}{\sqrt{x+y}}\right); \frac{\partial f}{\partial y} = -\frac{xy}{\sqrt{x+y}}; \frac{\partial f}{\partial z} = x\sqrt{x+y}$

D)  $\frac{\partial f}{\partial x} = z\left(\sqrt{x+y} + \frac{x}{\sqrt{x+y}}\right); \frac{\partial f}{\partial y} = \frac{xy}{\sqrt{x+y}}; \frac{\partial f}{\partial z} = x\sqrt{x+y}$

Solve the problem.

- 311) Find the derivative of the function  $f(x, y, z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$  at the point  $(8, -8, 8)$  in the direction in

311)

which the function increases most rapidly.

A)  $\frac{3}{8}\sqrt{2}$

B)  $\frac{1}{4}\sqrt{3}$

C)  $\frac{1}{4}\sqrt{2}$

D)  $\frac{3}{8}\sqrt{3}$

Answer the question.

- 312) Describe the results of applying the method of Lagrange multipliers to a function  $f(x, y)$  if the points  $(x, y)$  are constrained to follow a curve  $g(x, y) = c$  that is everywhere perpendicular to the level curves of  $f$ . Assume that both  $f(x, y)$  and  $g(x, y)$  satisfy all the requirements and conditions for the method to be applicable.

312)

- A) Since  $\nabla f$  is everywhere parallel to  $\nabla g$ , every point on  $g(x, y) = c$  is a local extremum. Applying the method of Lagrange multipliers should yield the equation  $g(x, y) = c$ .
- B) Since  $\nabla f$  is everywhere parallel to  $\nabla g$ , there will be a single local minimum and a single local maximum along  $g(x, y) = c$ . Applying the method of Lagrange multipliers should identify the locations of these two local extrema.
- C) The results cannot be generally predicted. Specific expressions for  $f(x, y)$  and  $g(x, y) = c$  are required.
- D) Generally, local extrema of  $f(x, y)$  occur at points on the curve  $g(x, y) = c$  where the curve becomes tangent to a level curve of  $f(x, y)$ . Since the curve defined by  $g(x, y) = c$  is everywhere perpendicular to the level curves of  $f(x, y)$  for this particular case, it is never tangent to a level curve, and there are no local extrema along  $g(x, y) = c$ . The method of Lagrange multipliers will fail to find any local extrema since there are none.

**Find the equation for the level surface of the function through the given point.**

313)  $f(x, y, z) = \frac{x^2y}{xz + y^2}$ , (4, 4, 1)

313) \_\_\_\_\_

A)  $20(xz + y^2) = 64x^2y$

B)  $\frac{64}{17} = \frac{x^2y}{xz + y^2}$

C)  $\frac{16}{5} = \frac{x^2y}{xz + y^2}$

D)  $\frac{4}{5} = \frac{x^2y}{xz + y^2}$

**Answer the question.**

314) Find the direction in which the function is increasing or decreasing most rapidly at the point  $P_0$ .

314) \_\_\_\_\_

$f(x, y, z) = xy - \ln(z)$ ,  $P_0(2, -2, 2)$

A)  $\frac{1}{\sqrt{33}}(-4i - 4j + k)$

B)  $\frac{33}{\sqrt{33}}(-4i + 4j - k)$

C)  $\frac{1}{33}(-4i + 4j - k)$

D)  $\frac{1}{\sqrt{33}}(-4i + 4j - k)$

**Find an upper bound for the magnitude  $|E|$  of the error in the linear approximation  $L$  to  $f$  at the given point over the given region.**

315)  $f(x, y, z) = 8x^2 + 6y^2 + 3z^2$  at (1, -2, 3)

315) \_\_\_\_\_

R:  $|x - 1| \leq 0.1$ ,  $|y + 2| \leq 0.1$ ,  $|z - 3| \leq 0.1$

A)  $|E| \leq 0.64$

B)  $|E| \leq 0.48$

C)  $|E| \leq 0.96$

D)  $|E| \leq 0.72$

**Find the absolute maxima and minima of the function on the given domain.**

316)  $f(x, y) = x^2 + 6x + y^2 + 14y + 10$  on the rectangular region  $-1 \leq x \leq 1$ ,  $-2 \leq y \leq 2$

316) \_\_\_\_\_

A) Absolute maximum: 58 at (2, 2); absolute minimum: -19 at (-1, -2)

B) Absolute maximum: 49 at (1, 2); absolute minimum: -19 at (-1, -2)

C) Absolute maximum: 49 at (1, 2); absolute minimum: 10 at (0, 0)

D) Absolute maximum: 58 at (2, 2); absolute minimum: 10 at (0, 0)

**Find the limit.**

317)  $\lim_{(x, y) \rightarrow (0, 1)} \frac{y^5 \sin x}{x}$

317) \_\_\_\_\_

A) 1

B)  $\infty$

C) 0

D) No limit

**Solve the problem.**

318) Find the least squares line for the points (1, -5), (1, -3).

318) \_\_\_\_\_

A)  $y = \frac{1}{3} - \frac{2}{3}x$

B)  $y = \frac{1}{3} - \frac{1}{3}x$

C)  $y = \frac{1}{3} + \frac{1}{3}x$

D)  $y = \frac{1}{3} + \frac{2}{3}x$

319) About how much will  $f(x, y, z) = -6x + 8y - 10z$  change if the point  $(x, y, z)$  moves from (-9, 4, -10) a distance of  $ds = \frac{1}{10}$  unit in the direction  $2i - 3j + 6k$ ?

319) \_\_\_\_\_

A)  $-\frac{8}{5}$

B)  $-\frac{52}{35}$

C)  $-\frac{44}{35}$

D)  $-\frac{48}{35}$

**Find all the first order partial derivatives for the following function.**

320)  $f(x, y, z) = e(\sin(x) + yz)$

320) \_\_\_\_\_

A)  $\frac{\partial f}{\partial x} = \cos(x)e(\sin(x) + yz); \frac{\partial f}{\partial y} = e(\sin(x) + yz); \frac{\partial f}{\partial z} = e(\sin(x) + yz)$

B)  $\frac{\partial f}{\partial x} = (\cos(x) + yz)e(\sin(x) + yz); \frac{\partial f}{\partial y} = ze(\sin(x) + yz); \frac{\partial f}{\partial z} = ye(\sin(x) + yz)$

C)  $\frac{\partial f}{\partial x} = e(\sin(x) + yz); \frac{\partial f}{\partial y} = ze(\sin(x) + yz); \frac{\partial f}{\partial z} = ye(\sin(x) + yz)$

D)  $\frac{\partial f}{\partial x} = \cos(x)e(\sin(x) + yz); \frac{\partial f}{\partial y} = ze(\sin(x) + yz); \frac{\partial f}{\partial z} = ye(\sin(x) + yz)$

**Solve the problem.**

- 321) Find the derivative of the function  $f(x, y) = x^2 + xy + y^2$  at the point  $(-6, -5)$  in the direction in which the function decreases most rapidly.

321) \_\_\_\_\_

A)  $-3\sqrt{62}$

B)  $-\sqrt{653}$

C)  $-\sqrt{545}$

D)  $-\sqrt{659}$

**Find the limit.**

322)  $\lim_{P \rightarrow (7, 7, 5)} \sec^2 x - \tan^2 y + z$

322) \_\_\_\_\_

A) 8

B) 7

C) 6

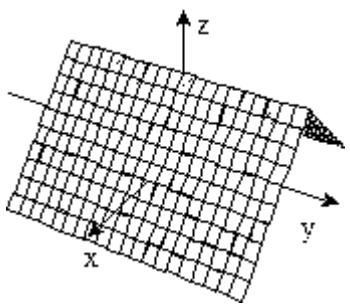
D) 5

Sketch the surface  $z = f(x,y)$ .

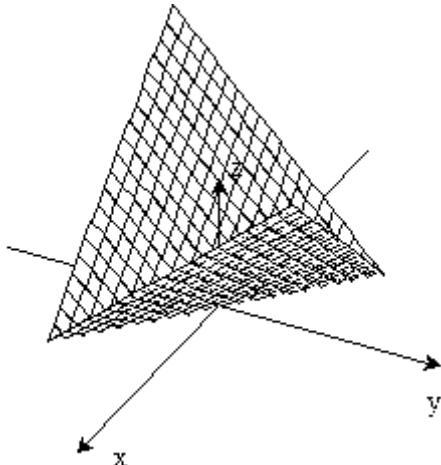
323)  $f(x, y) = x^2$

323) \_\_\_\_\_

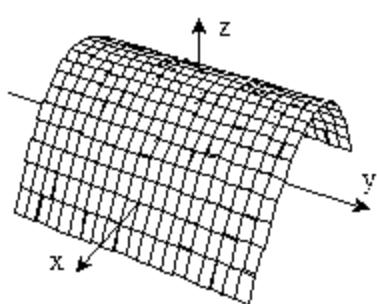
A)



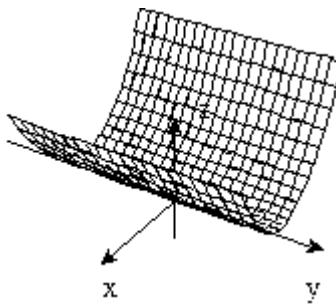
B)



C)



D)



Find an upper bound for the magnitude  $|E|$  of the error in the linear approximation  $L$  to  $f$  at the given point over the given region.

324)  $f(x, y) = -2x - 5y - 4$  at  $(4, -6)$

324) \_\_\_\_\_

R:  $|x - 4| \leq 0.1, |y + 6| \leq 0.1$

A)  $|E| \leq 0.02$

B)  $|E| \leq 0$

C)  $|E| \leq 0.01$

D)  $|E| \leq 0.04$

Solve the problem.

325) Write an equation for the tangent line to the curve  $\frac{x^2}{16} + \frac{y^2}{49} = 1$  at the point  $\left(\frac{4}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ .

325) \_\_\_\_\_

A)  $\frac{x}{4} + \frac{y}{7} = \sqrt{2}$

B)  $\frac{x}{7} + \frac{y}{4} = \sqrt{2}$

C)  $\frac{x}{4} + \frac{y}{7} = 1$

D)  $\frac{x}{7} + \frac{y}{4} = 1$

Find the linearization of the function at the given point.

326)  $f(x, y) = e^{5x} + 6y$  at  $(0, 0)$

326) \_\_\_\_\_

A)  $L(x, y) = 5x + 6y + 1$

B)  $L(x, y) = 5x + 6y$

C)  $L(x, y) = 6x + 5y + 1$

D)  $L(x, y) = 6x + 5y$

**Solve the problem.**327) Find the point on the plane  $x + 2y - z = 12$  that is nearest the origin.

327) \_\_\_\_\_

A)  $(4, 4, 0)$

B)  $(-2, 8, 2)$

C)  $(2, 4, 0)$

D)  $(2, 4, -2)$

**Estimate the error in the quadratic approximation of the given function at the origin over the given region.**328)  $f(x, y) = \ln(1 + 2x + 6y)$ ,  $-0.1 \leq x, y \leq 0.1$ 

328) \_\_\_\_\_

A)  $|E(x, y)| \leq 0.48$

B)  $|E(x, y)| \leq 0.7855$

C)  $|E(x, y)| \leq 0.0053$

D)  $|E(x, y)| \leq 0.576$

**Find the requested partial derivative.**329)  $\frac{\partial z}{\partial y}$  if  $z^3 = z + xy - 1$  and  $y^3 = x + y - 1$ 

329) \_\_\_\_\_

A)  $\frac{x - 3y^3 - y}{3z^2 + 1}$

B)  $\frac{x - 3y^3 - y}{3z^2 - 1}$

C)  $\frac{x + 3y^3 - y}{3z^2 - 1}$

D)  $\frac{x + 3y^3 - y}{3z^2 + 1}$

**Find all the first order partial derivatives for the following function.**330)  $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$ 

330) \_\_\_\_\_

A)  $\frac{\partial f}{\partial x} = \left( \frac{x}{2(x^2 + y^2)^{3/2}} \right); \frac{\partial f}{\partial y} = \left( \frac{y}{2(x^2 + y^2)^{3/2}} \right)$

B)  $\frac{\partial f}{\partial x} = -\left( \frac{1}{2(x^2 + y^2)^{3/2}} \right); \frac{\partial f}{\partial y} = -\left( \frac{1}{2(x^2 + y^2)^{3/2}} \right)$

C)  $\frac{\partial f}{\partial x} = -\left( \frac{x}{(x^2 + y^2)^{3/2}} \right); \frac{\partial f}{\partial y} = -\left( \frac{y}{(x^2 + y^2)^{3/2}} \right)$

D)  $\frac{\partial f}{\partial x} = -\left( \frac{x}{2(x^2 + y^2)^{3/2}} \right); \frac{\partial f}{\partial y} = -\left( \frac{y}{2(x^2 + y^2)^{3/2}} \right)$

**Find the extreme values of the function subject to the given constraint.**331)  $f(x, y, z) = x^2 + y^2 + z^2$ ,  $x + 2y + 3z = 6$ 

331) \_\_\_\_\_

A) Maximum: none; minimum:  $\frac{2}{7}$  at  $\left(\frac{1}{7}, \frac{2}{7}, \frac{3}{7}\right)$

B) Maximum: none; minimum:  $\frac{72}{7}$  at  $\left(\frac{6}{7}, \frac{12}{7}, \frac{18}{7}\right)$

C) Maximum: none; minimum:  $\frac{18}{7}$  at  $\left(\frac{3}{7}, \frac{6}{7}, \frac{9}{7}\right)$

D) Maximum: none; minimum: 2 at  $\left(\frac{1}{7}, \frac{4}{7}, \frac{9}{7}\right)$

**Find all local extreme values of the given function and identify each as a local maximum, local minimum, or saddle point.**

332)  $f(x, y) = (x^2 - 16)^2 - (y^2 - 4)^2$

332) \_\_\_\_\_

- A)  $f(0, 0) = 240$ , saddle point;  $f(0, 2) = 256$ , local maximum;  $f(4, 0) = -16$ , local minimum
- B)  $f(0, 0) = 240$ , saddle point;  $f(4, 2) = 0$ , saddle point;  $f(4, -2) = 0$ , saddle point;  $f(-4, 2) = 0$ , saddle point;  $f(-4, -2) = 0$ , saddle point
- C)  $f(0, 0) = 240$ , saddle point;  $f(0, 2) = 256$ , local maximum;  $f(0, -2) = 256$ , local maximum;  $f(4, 0) = -16$ , local minimum;  $f(4, 2) = 0$ , saddle point;  $f(4, -2) = 0$ , saddle point;  $f(-4, 0) = -16$ , local minimum;  $f(-4, 2) = 0$ , saddle point;  $f(-4, -2) = 0$ , saddle point
- D)  $f(0, 2) = 256$ , local maximum;  $f(0, -2) = 256$ , local maximum;  $f(4, 0) = -16$ , local minimum;  $f(-4, 0) = -16$ , local minimum

**Find the limit.**

333)  $\lim_{(x, y) \rightarrow \left(0, -\frac{\pi}{4}\right)} \frac{\sec x + 1}{7x - \tan y}$

333) \_\_\_\_\_

- A) -2
- B) 2
- C)  $\sqrt{2} + 1$
- D)  $-\sqrt{2} - 1$

**Solve the problem.**

334) Evaluate  $\frac{dw}{dt}$  at  $t = \frac{3}{2}\pi$  for the function  $w(x, y, z) = \frac{xy}{z}$ ;  $x = \sin t$ ,  $y = \cos t$ ,  $z = t^2$ .

334) \_\_\_\_\_

- A)  $-2\left(\frac{1}{\pi^2}\right)$
- B)  $2\left(\frac{1}{\pi^2}\right)$
- C)  $-\frac{4}{9}\left(\frac{1}{\pi^2}\right)$
- D)  $-2\left(\frac{1}{\pi}\right)$

335) Find the derivative of the function  $f(x, y) = e^{xy}$  at the point  $(0, 7)$  in the direction in which the function increases most rapidly.

335) \_\_\_\_\_

- A) 21
- B) 14
- C) 6
- D) 7

336) Find the equation for the tangent plane to the surface  $z = -8x^2 + 3y^2$  at the point  $(2, 1, -29)$ .

336) \_\_\_\_\_

- A)  $-32x + 6y - z = -29$
- B)  $2x + y - 29z = 1$
- C)  $2x + y - 29z = -26$
- D)  $-32x + 6y - z = -28$

**Find all the first order partial derivatives for the following function.**

337)  $f(x, y, z) = \frac{z}{\sqrt{x+y^2}}$

337) \_\_\_\_\_

- A)  $\frac{\partial f}{\partial x} = \frac{z}{2(x+y^2)^{3/2}}$ ;  $\frac{\partial f}{\partial y} = -\frac{yz}{(x+y^2)^{3/2}}$ ;  $\frac{\partial f}{\partial z} = \frac{1}{\sqrt{x+y^2}}$
- B)  $\frac{\partial f}{\partial x} = \frac{z}{2(x+y^2)^{3/2}}$ ;  $\frac{\partial f}{\partial y} = \frac{yz}{(x+y^2)^{3/2}}$ ;  $\frac{\partial f}{\partial z} = \frac{1}{\sqrt{x+y^2}}$
- C)  $\frac{\partial f}{\partial x} = -\frac{z}{2(x+y^2)^{3/2}}$ ;  $\frac{\partial f}{\partial y} = \frac{yz}{(x+y^2)^{3/2}}$ ;  $\frac{\partial f}{\partial z} = -\frac{1}{\sqrt{x+y^2}}$
- D)  $\frac{\partial f}{\partial x} = -\frac{z}{2(x+y^2)^{3/2}}$ ;  $\frac{\partial f}{\partial y} = -\frac{yz}{(x+y^2)^{3/2}}$ ;  $\frac{\partial f}{\partial z} = \frac{1}{\sqrt{x+y^2}}$

338)  $f(x, y) = x^3 + 6x^2y + 9xy^3$

338)

A)  $\frac{\partial f}{\partial x} = 3x^2 + 12xy + 9y^3; \frac{\partial f}{\partial y} = 6x^2 + 27xy^2$

B)  $\frac{\partial f}{\partial x} = 3x^2 + 2xy + 9y^3; \frac{\partial f}{\partial y} = 6x^2 + 3xy^2$

C)  $\frac{\partial f}{\partial x} = x^2 + 6xy + 9y^3; \frac{\partial f}{\partial y} = 6x^2 + 9xy^2$

D)  $\frac{\partial f}{\partial x} = 3x^2; \frac{\partial f}{\partial y} = 6x^2 + 27xy^2$

**Find all local extreme values of the given function and identify each as a local maximum, local minimum, or saddle point.**

339)  $f(x, y) = 5x^2y + 3xy^2$

339)

A)  $f\left(\frac{1}{3}, \frac{1}{5}\right) = \frac{34}{225}$ , local minimum

B)  $f\left(\frac{1}{5}, \frac{1}{3}\right) = \frac{2}{15}$ , local minimum

C)  $f(0, 0) = 0$ , saddle point

D)  $f(15, 15) = 27,000$ , local maximum

**Find an upper bound for the magnitude  $|E|$  of the error in the linear approximation  $L$  to  $f$  at the given point over the given region.**

340)  $f(x, y) = e^{4x} + 5y$  at  $(0, 0)$

340)

R:  $|x| \leq 0.1, |y| \leq 0.1$

A)  $|E| \leq 2.1522$

B)  $|E| \leq 1.2298$

C)  $|E| \leq 2.4596$

D)  $|E| \leq 4.3043$

**Solve the problem.**

- 341) Find the point on the line  $y = x - 3$  that is closest to the parabola  $y = 9x^2$ . Use the fact that the line connecting the closest points on each curve is normal to both curves.

341)

A)  $\left(\frac{23}{12}, -\frac{13}{12}\right)$

B)  $\left(\frac{37}{24}, -\frac{35}{24}\right)$

C)  $\left(\frac{7}{6}, -\frac{11}{6}\right)$

D)  $\left(\frac{19}{24}, -\frac{53}{24}\right)$

**Find all local extreme values of the given function and identify each as a local maximum, local minimum, or saddle point.**

342)  $f(x, y) = 100x^2 + 40xy + 16y^2$

342)

A)  $f(40, 40) = 96,000$ , local maximum

B)  $f(10, 4) = 256$ , saddle point;  $f(4, 10) = 4800$ , saddle point

C)  $f(40, 40) = 96,000$ , local maximum;  $f(0, 0) = 0$ , local minimum

D)  $f(0, 0) = 0$ , local minimum

**Find all the first order partial derivatives for the following function.**

343)  $f(x, y, z) = \ln(xy)^z$

343)

A)  $\frac{\partial f}{\partial x} = \frac{z}{x}; \frac{\partial f}{\partial y} = \frac{z}{y}; \frac{\partial f}{\partial z} = z \ln(xy)^{z-1}$

B)  $\frac{\partial f}{\partial x} = -\frac{z}{x}; \frac{\partial f}{\partial y} = -\frac{z}{y}; \frac{\partial f}{\partial z} = \ln xy$

C)  $\frac{\partial f}{\partial x} = \frac{z}{x}; \frac{\partial f}{\partial y} = \frac{z}{y}; \frac{\partial f}{\partial z} = \ln xy$

D)  $\frac{\partial f}{\partial x} = z \ln\left(\frac{z}{x}\right); \frac{\partial f}{\partial y} = z \ln\left(\frac{z}{y}\right); \frac{\partial f}{\partial z} = \ln xy$

**Use polar coordinates to find the limit of the function as  $(x, y)$  approaches  $(0, 0)$ .**

344)  $f(x, y) = \cos^{-1}\left(\frac{x^3 - xy^2}{x^2 + y^2}\right)$

344)

A) 1

B)  $\pi$

C)  $\frac{\pi}{2}$

D) No limit

**Solve the problem.**

- 345) Find any local extrema (maxima, minima, or saddle points) of  $f(x, y)$  given that  
 $f_x = 5x - 15$  and  $f_y = 6y - 12$ .

345) \_\_\_\_\_

- A) Saddle point at  $(2, 3)$   
B) Local maximum at  $(3, 2)$   
C) Local maximum at  $(2, 3)$   
D) Local minimum at  $(3, 2)$

- 346) Find the derivative of the function  $f(x, y, z) = \ln(xy + yz + zx)$  at the point  $(-8, -16, -24)$  in the direction in which the function increases most rapidly.

346) \_\_\_\_\_

- A)  $\frac{5}{56}\sqrt{2}$   
B)  $\frac{5}{104}\sqrt{2}$   
C)  $\frac{5}{88}\sqrt{2}$   
D)  $\frac{5}{136}\sqrt{2}$

**Answer the question.**

- 347) Consider a function  $f(x, y, z)$ , where the independent variables are constrained to lie on the curve  $\vec{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ . What mathematical fact forms the basis for the method of Lagrange multipliers?

347) \_\_\_\_\_

- A)  $\nabla f \cdot \nabla g = 0$   
B)  $\nabla f \cdot \vec{v} = 0$   
C)  $\nabla g = 0$   
D)  $f$  approaches a local extremum as  $\lambda \rightarrow 0$ .

**Solve the problem.**

- 348) The table below summarizes the construction cost of a set of homes (excluding the lot cost) along with the square footage of the home's floor space. Find a linear equation that relates the construction cost in thousands of dollars to the floor space in hundreds of square feet by finding the least squares line for the data.

348) \_\_\_\_\_

Floor Space (100's of sq. ft.)	Construction Cost (1000's of dollars)
11	76.6
12	86.5
14	101.2
15	96.3
18	127.9
22	159.0
26	198.5

- A)  $y = 7.97x - 13.54$   
B)  $y = 8.13x$   
C)  $y = 8.13x - 12.79$   
D)  $y = 7.87x - 13.08$

- 349) Find an equation for the level curve of the function  $f(x, y) = \int_x^y t dt$  that passes through the point  $(-8, -7)$ .

349) \_\_\_\_\_

- A)  $y^2 + x^2 = 113$   
B)  $y^2 - x^2 = 15$   
C)  $y^2 - x^2 = -15$   
D)  $y^2 - x^2 = 113$

**Find the requested partial derivative.**

- 350)  $(\partial z / \partial x)_y$  at  $(x, y, z) = (1, 1, 1)$  if  $z^3 + 18xyz = 19$

350) \_\_\_\_\_

- A)  $-\frac{6}{13}$   
B)  $-\frac{3}{4}$   
C)  $-\frac{6}{7}$   
D)  $-\frac{12}{7}$

**Determine whether the given function satisfies the wave equation.**

- 351)  $w(x, t) = \sin(5x + 5ct)$

351) \_\_\_\_\_

- A) Yes  
B) No

**Find the limit.**

$$352) \lim_{P \rightarrow (1, -1, 0)} \frac{-5xz - 2xy}{x^2 + y^2 - z^2}$$

352) \_\_\_\_\_

A) -2

B) 5

C) -5

D) 1

**Find the extreme values of the function subject to the given constraint.**

$$353) f(x, y, z) = x + 2y - 2z, \quad x^2 + y^2 + z^2 = 9$$

353) \_\_\_\_\_

- A) Maximum: 1 at (1, -2, -2); minimum: -1 at (-1, 2, 2)
- B) Maximum: 9 at (1, 2, -2); minimum: -9 at (-1, -2, 2)
- C) Maximum: 8 at (2, 1, -2); minimum: -8 at (-2, -1, 2)
- D) Maximum: 1 at (-1, -2, -3); minimum: -1 at (1, 2, 3)

**Use Taylor's formula to find the requested approximation of  $f(x, y)$  near the origin.**

$$354) \text{ Cubic approximation to } f(x, y) = \sin(7x + y)$$

354) \_\_\_\_\_

A)  $7x + y - \frac{343}{3}x^3 - \frac{1}{3}y^3$

B)  $7x + y - \frac{343}{6}x^3 - \frac{1}{6}y^3$

C)  $7x + y - \frac{343}{6}x^3 - \frac{49}{2}x^2y - \frac{7}{2}xy^2 - \frac{1}{6}y^3$

D)  $7x + y - \frac{343}{3}x^3 - \frac{49}{2}x^2y - \frac{7}{2}xy^2 - \frac{1}{3}y^3$

**Solve the problem.**

- 355) The resistance  $R$  produced by wiring resistors of  $R_1$  and  $R_2$  ohms in parallel can be calculated from the formula

355) \_\_\_\_\_

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

If  $R_1$  and  $R_2$  are measured to be 7 ohms and 9 ohms respectively and if these measurements are accurate to within 0.05 ohms, estimate the maximum percentage error in computing  $R$ .

A) 0.64%

B) 1.29%

C) 1.03%

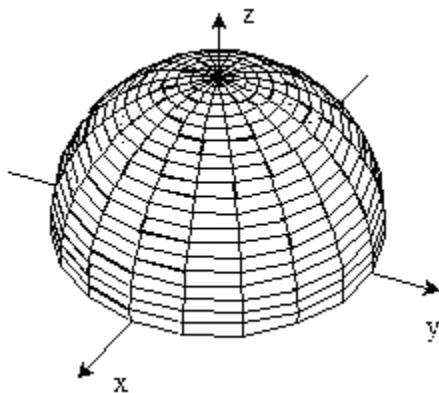
D) 0.77%

**Sketch the surface  $z = f(x,y)$ .**

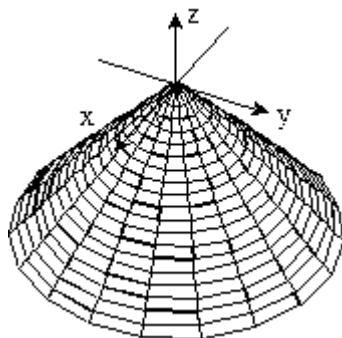
356)  $f(x, y) = 4x^2 + 4y^2 + 2$

356) \_\_\_\_\_

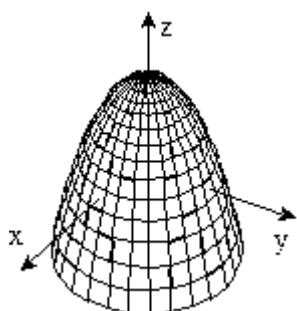
A)



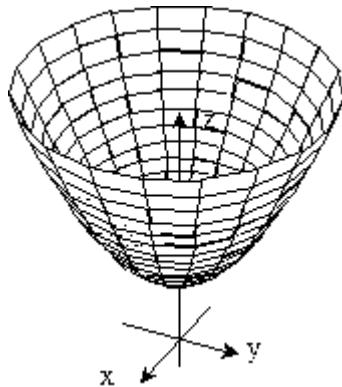
B)



C)



D)



**Find the domain and range and describe the level curves for the function  $f(x,y)$ .**

357)  $f(x, y) = 9x + 8y$

357) \_\_\_\_\_

- A) Domain: all points in the  $x-y$  plane; range: real numbers  $z \geq 0$ ; level curves: lines  $9x + 8y = c$ ,  $c \geq 0$
- B) Domain: all points in the  $x-y$  plane; range: real numbers  $z \geq 0$ ; level curves: lines  $9x + 8y = c$ ,  $c \leq 0$
- C) Domain: all points in the  $x-y$  plane; range: all real numbers; level curves: lines  $9x + 8y = c$
- D) Domain: all points in the  $x-y$  plane; range: all real numbers; level curves: lines  $9x + 8y = c$ ,  $c > 0$

**Find all the second order partial derivatives of the given function.**

358)  $f(x, y) = xy^2 + ye^{x^2} + 5$

358) \_\_\_\_\_

A)  $\frac{\partial^2 f}{\partial x^2} = 2ye^{x^2}; \frac{\partial^2 f}{\partial y^2} = 2x; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 2y + 2xe^{x^2}$

B)  $\frac{\partial^2 f}{\partial x^2} = 2ye^{x^2}(1 + 2x^2); \frac{\partial^2 f}{\partial y^2} = 2x; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 2y + 2xe^{x^2}$

C)  $\frac{\partial^2 f}{\partial x^2} = ye^{x^2}(1 + 2x^2); \frac{\partial^2 f}{\partial y^2} = x; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = y + xe^{x^2}$

D)  $\frac{\partial^2 f}{\partial x^2} = 2ye^{x^2}; \frac{\partial^2 f}{\partial y^2} = 2x; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 2xe^{x^2}$

**Use implicit differentiation to find the specified derivative at the given point.**

359) Find  $\frac{dy}{dx}$  at the point  $(1, 0)$  for  $\cos xy + ye^x = 0$ .

359) \_\_\_\_\_

A) 0

B) 1

C)  $\frac{1}{e}$

D) e

**Compute the gradient of the function at the given point.**

360)  $f(x, y) = \ln(-8x - 9y), (8, -5)$

360) \_\_\_\_\_

A)  $-\frac{1}{19}\mathbf{i} - \frac{1}{19}\mathbf{j}$

B)  $\frac{5}{19}\mathbf{i} - \frac{8}{19}\mathbf{j}$

C)  $-\frac{8}{19}\mathbf{i} + \frac{5}{19}\mathbf{j}$

D)  $\frac{8}{19}\mathbf{i} + \frac{9}{19}\mathbf{j}$

**At what points is the given function continuous?**

361)  $f(x, y) = e^{x+y}$

361) \_\_\_\_\_

A) All  $(x, y)$  satisfying  $x + y \geq 0$

B) All  $(x, y)$  in the first quadrant

C) All  $(x, y) \neq (0, 0)$

D) All  $(x, y)$

**Answer the question.**

362) Find the direction in which the function is increasing or decreasing most rapidly at the point  $P_0$ .

362) \_\_\_\_\_

$f(x, y) = xy^2 - yx^2, P_0(-2, 1)$

A)  $(5\sqrt{89})\mathbf{i} + (-8\sqrt{89})\mathbf{j}$

B)  $\left(\frac{5}{\sqrt{89}}\right)\mathbf{i} + \left(\frac{8}{\sqrt{89}}\right)\mathbf{j}$

C)  $\left(\frac{-8}{\sqrt{89}}\right)\mathbf{i} + \left(\frac{5}{\sqrt{89}}\right)\mathbf{j}$

D)  $\left(\frac{5}{\sqrt{89}}\right)\mathbf{i} + \left(\frac{-8}{\sqrt{89}}\right)\mathbf{j}$

**Write a chain rule formula for the following derivative.**

363)  $\frac{\partial w}{\partial t}$  for  $w = f(x, y, z); x = g(r, s, t), y = h(r, s, t), z = k(r, s, t)$

363) \_\_\_\_\_

A)  $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial t}$

B)  $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial s} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial t}$

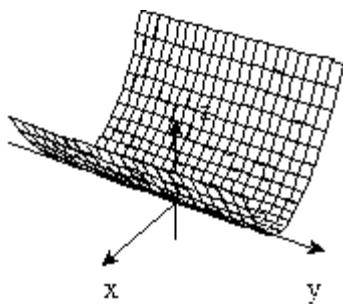
C)  $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial t}$

D)  $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$

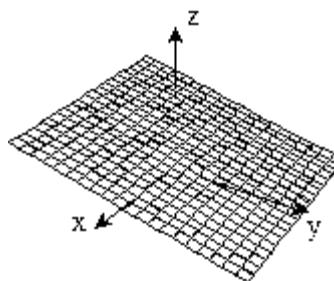
**Sketch the surface  $z = f(x,y)$ .**

364)  $f(x, y) = \sqrt{4 - x^2 - y^2}$

A)

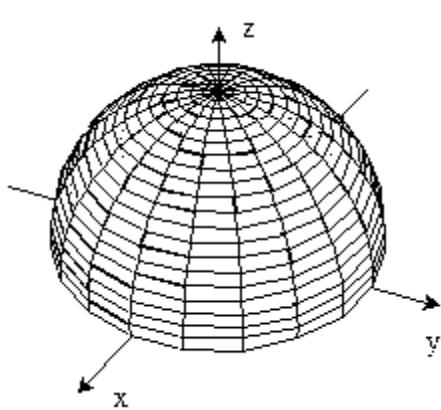


B)

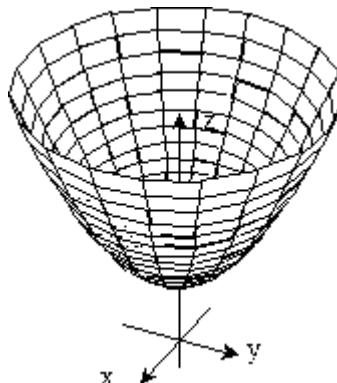


364) \_\_\_\_\_

C)



D)



**Find the absolute maximum and minimum values of the function on the given curve.**

365) Function:  $f(x, y) = x + y$ ; curve:  $x^2 + y^2 = 4$ ,  $y \geq 0$ . (Use the parametric equations  $x = 2 \cos t$ ,  $y = 2 \sin t$ .)

365) \_\_\_\_\_

A) Absolute maximum: 2 at  $t = \frac{\pi}{2}$ ; absolute minimum:  $-2\sqrt{2}$  at  $t = \frac{3\pi}{4}$

B) Absolute maximum:  $2\sqrt{2}$  at  $t = \frac{\pi}{4}$ ; absolute minimum: -2 at  $t = \pi$

C) Absolute maximum: 2 at  $t = \frac{\pi}{2}$ ; absolute minimum: -2 at  $t = \pi$

D) Absolute maximum:  $2\sqrt{2}$  at  $t = \frac{\pi}{4}$ ; absolute minimum:  $-2\sqrt{2}$  at  $t = \frac{3\pi}{4}$

**Answer the question.**

- 366) You are hiking on a mountainside, following a trail that slopes downward for a short distance and then begins to climb again. At the bottom of this local "dip", what can be said about the relationship between the trail's direction and the contour of the mountainside? [Hint - Think of the trail as a constrained path,  $g(x, y) = c$ , on the mountainside's surface, altitude =  $f(x, y)$ . Consider only infinitesimal displacements.]

366) \_\_\_\_\_

- A) The mountainside rises in all directions relative to the dip.
- B) At the bottom of the dip, the trail is headed along the mountain's contour line which passes through that point.
- C) At the bottom of the dip, the trail is headed perpendicular to the mountain's contour line which passes through that point.
- D) At the bottom of the dip, the trail is headed in the direction of the mountain's steepest ascent.

**Find the extreme values of the function subject to the given constraint.**

367)  $f(x, y, z) = x + y + z$ ,  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

367) \_\_\_\_\_

- A) Maximum: 9 at  $(3, 3, 3)$ ; minimum: 1 at  $(1, 1, -1)$ ,  $(1, -1, 1)$ ,  $(-1, 1, 1)$
- B) Maximum: 6 at  $(2, 2, 2)$ ; minimum: -3 at  $(-1, -1, -1)$
- C) Maximum: 9 at  $(3, 3, 3)$ ; minimum: 3 at  $(1, 1, 1)$
- D) Maximum: 6 at  $(2, 2, 2)$ ; minimum: 3 at  $(1, 1, 1)$

**Compute the gradient of the function at the given point.**

368)  $f(x, y) = \tan^{-1} \frac{-3x}{y}$ ,  $(-2, 9)$

368) \_\_\_\_\_

A)  $-\frac{3}{13}\mathbf{i} + \frac{2}{39}\mathbf{j}$       B)  $-\frac{3}{22}\mathbf{i} + \frac{1}{33}\mathbf{j}$       C)  $-\frac{3}{22}\mathbf{i} - \frac{1}{33}\mathbf{j}$       D)  $-\frac{3}{13}\mathbf{i} - \frac{2}{39}\mathbf{j}$

**Provide an appropriate answer.**

369) Suppose that  $x^2 + y^2 = r^2$  and  $x = r \cos \theta$ , as in polar coordinates. Find  $\left( \frac{\partial y}{\partial \theta} \right)$ .

369) \_\_\_\_\_

A)  $\left( \frac{\partial y}{\partial \theta} \right) = r(\cos \theta - \sin \theta)$

B)  $\left( \frac{\partial y}{\partial \theta} \right) = -r \sin \theta$

C)  $\left( \frac{\partial y}{\partial \theta} \right) = 0$

D)  $\left( \frac{\partial y}{\partial \theta} \right) = r \cos \theta$

**At what points is the given function continuous?**

370)  $f(x, y) = \frac{xy}{x + y}$

370) \_\_\_\_\_

- A) All  $(x, y)$  such that  $x \neq y$
- B) All  $(x, y) \neq (0, 0)$
- C) All  $(x, y)$
- D) All  $(x, y)$  such that  $x \neq -y$

**Provide an appropriate answer.**

371) According to the Gas Kinetic Theory, the average speed of a gas particle is given by  $\bar{v} = \sqrt{\frac{8kT}{\pi m}}$ ,

371) \_\_\_\_\_

where  $\bar{v}$  is the speed in m/s,  $k$  is the constant  $1.38 \times 10^{-23}$ ,  $T$  is the temperature of the gas in Kelvin, and  $m$  is the mass of the gas particle in kg. What is the average speed of an oxygen molecule with a mass of  $5.314 \times 10^{-26}$  kg at a temperature of 500 K?

- A) 330,700 m/s
- B) 174 m/s
- C) 575 m/s
- D) 1020 m/s

**Estimate the error in the quadratic approximation of the given function at the origin over the given region.**

372)  $f(x, y) = e^{6x} \sin y$ ,  $-0.1 \leq x, y \leq 0.1$

372)

- A)  $|E(x, y)| \leq 0.7872$   
C)  $|E(x, y)| \leq 0.5248$

- B)  $|E(x, y)| \leq 1.5743$   
D)  $|E(x, y)| \leq 0.3936$

**At what points is the given function continuous?**

373)  $f(x, y, z) = \frac{z}{x^2 + y^2 - 5}$

373)

- A) All  $(x, y, z)$  such that  $x^2 + y^2 \neq 5$   
B) All  $(x, y, z)$   
C) All  $(x, y, z)$  such that  $x^2 + y^2 \neq 0$   
D) All  $(x, y, z)$  such that  $x^2 + y^2 \neq 25$

**Find the derivative of the function at the given point in the direction of A.**

374)  $f(x, y, z) = \tan^{-1} \frac{5x}{8y - 8z}$ ,  $(-8, 0, 0)$ ,  $\mathbf{A} = 12\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$

374)

- A)  $-\frac{7}{65}$   
B)  $-\frac{9}{65}$   
C)  $-\frac{8}{65}$   
D)  $-\frac{2}{13}$

**Solve the problem.**

375) Find parametric equations for the normal line to the surface  $z = e^{8x^2 + 4y^2}$  at the point  $(0, 0, 1)$ .

375)

- A)  $x = t$ ,  $y = t$ ,  $z = t - 1$   
B)  $x = 0$ ,  $y = 0$ ,  $z = t + 1$   
C)  $x = t$ ,  $y = t$ ,  $z = -t - 1$   
D)  $x = 0$ ,  $y = 0$ ,  $z = t - 1$

**At what points is the given function continuous?**

376)  $f(x, y, z) = \ln(x + y + z - 9)$

376)

- A) All  $(x, y, z)$  such that  $x + y + z \geq 9$   
B) All  $(x, y, z)$  such that  $x + y + z > 9$   
C) All  $(x, y, z)$  in the first octant  
D) All  $(x, y, z)$

**Solve the problem.**

377) The surface area of a hollow cylinder (tube) is given by

377)

$$S = 2\pi(R_1 + R_2)(h + R_1 - R_2),$$

where  $h$  is the length of the cylinder and  $R_1$  and  $R_2$  are the outer and inner radii. If  $h$ ,  $R_1$ , and  $R_2$  are measured to be 8 inches, 6 inches, and 9 inches respectively, and if these measurements are accurate to within 0.1 inches, estimate the maximum possible error in computing  $S$ .

- A) 18.22  
B) 25.76  
C) 14.45  
D) 21.99

**Find an upper bound for the magnitude  $|E|$  of the error in the linear approximation  $L$  to  $f$  at the given point over the given region.**

378)  $f(x, y, z) = e^{9x + 4y + 2z}$  at  $(0, 0, 0)$

378)

$$\text{R: } |x| \leq 0.1, |y| \leq 0.1, |z| \leq 0.1$$

- A)  $|E| \leq 18.8769$   
B)  $|E| \leq 16.3358$   
C)  $|E| \leq 15.2467$   
D)  $|E| \leq 17.4248$

**Find the absolute maxima and minima of the function on the given domain.**

379)  $f(x, y) = 3x^2 + 7y^2$  on the disk bounded by the circle  $x^2 + y^2 = 9$

379)

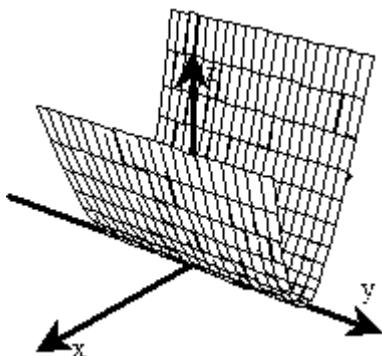
- A) Absolute maximum: 27 at  $(3, 0)$  and  $(-3, 0)$ ; absolute minimum: 0 at  $(0, 0)$   
B) Absolute maximum: 90 at  $(3, 3)$ ; absolute minimum: 0 at  $(0, 0)$   
C) Absolute maximum: 63 at  $(0, 3)$  and  $(0, -3)$ ; absolute minimum: 0 at  $(0, 0)$   
D) Absolute maximum: 63 at  $(0, 3)$  and  $(0, -3)$ ; absolute minimum: 27 at  $(3, 0)$  and  $(-3, 0)$

**Sketch a typical level surface for the function.**

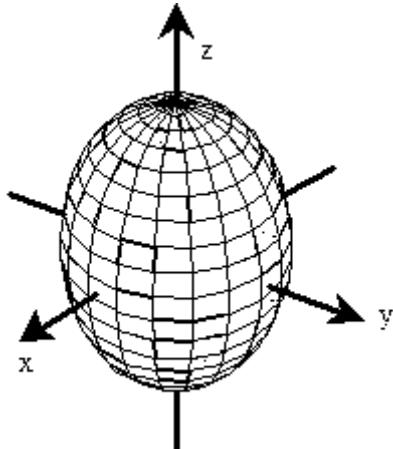
$$380) f(x, y, z) = \ln\left(\frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{50}\right)$$

380) \_\_\_\_\_

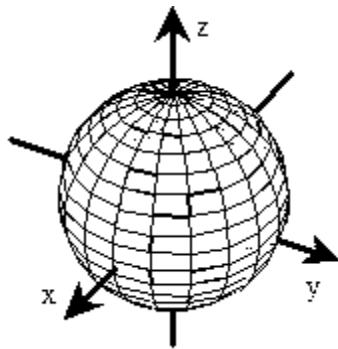
A)



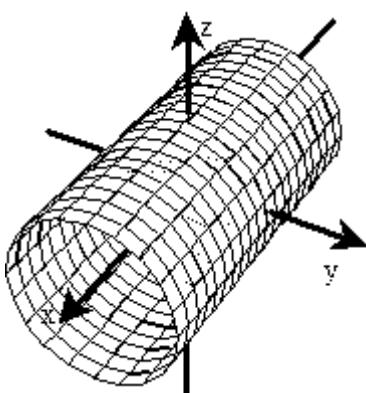
B)



C)



D)



**Solve the problem.**

$$381) \text{ Find } \frac{dw}{dt} \text{ at } t=0 \text{ for the function } w = \sin(x) \cos(y) \ln(z) \text{ where } x = -4t^2 + 5t + \frac{\pi}{2}, y = t, z = e^{-6t}.$$

381) \_\_\_\_\_

A)  $-6e^{-6}$

B) -12

C) 1

D) -6

$$382) \text{ Find the equation for the tangent plane to the surface } -6x - 7y - 8z = -15 \text{ at the point } (1, -1, 2).$$

382) \_\_\_\_\_

A)  $-6x + 7y - 16z = -21$

B)  $-6x - 7y - 8z = -15$

C)  $-6x - 7y - 8z = -21$

D)  $-6x + 7y - 16z = -15$

**Find the extreme values of the function subject to the given constraint.**

$$383) f(x, y, z) = x^3 + y^3 + z^3, \quad x^2 + y^2 + z^2 = 4$$

383) \_\_\_\_\_

A) Maximum: 8 at  $(2, 0, 0), (0, 2, 0), (0, 0, 2)$ ; minimum: 0 at  $(0, 0, 0)$

B) Maximum: 8 at  $(2, 0, 0)$ ; minimum: 0 at  $(0, 0, 0)$

C) Maximum: 8 at  $(2, 0, 0), (0, 2, 0), (0, 0, 2)$ ; minimum: -8 at  $(-2, 0, 0), (0, -2, 0), (0, 0, -2)$

D) Maximum: 8 at  $(2, 0, 0)$ ; minimum: -8 at  $(-2, 0, 0)$

**Solve the problem.**384) Write an equation for the tangent line to the curve  $xy = 70$  at the point  $(7, 10)$ .

384) \_\_\_\_\_

A)  $10x + 7y = 70$

B)  $10x + 7y = 140$

C)  $7x + 10y = 140$

D)  $7x + 10y = 70$

**Find the domain and range and describe the level curves for the function  $f(x,y)$ .**385)  $f(x, y) = \ln(5x + 3y)$ 

385) \_\_\_\_\_

A) Domain: all points in the  $x$ - $y$  plane; range: all real numbers; level curves: lines  $5x + 3y = c$ B) Domain: all points in the  $x$ - $y$  plane satisfying  $5x + 3y > 0$ ; range: real numbers  $z \geq 0$ ; level curves: lines  $5x + 3y = c$ C) Domain: all points in the  $x$ - $y$  plane satisfying  $5x + 3y > 0$ ; range: all real numbers; level curves: lines  $5x + 3y = c$ D) Domain: all points in the  $x$ - $y$  plane satisfying  $5x + 3y \geq 0$ ; range: all real numbers; level curves: lines  $5x + 3y = c$ **Solve the problem.**386) Maximize  $f(x, y, z) = e^{10x + 9y + 8z}$  subject to  $x + y + z = 0$ ,  $x + 2y + 3z = 0$ , and  $x + 4y + 9z = 0$ .

386) \_\_\_\_\_

A) 0

B) 27

C) e

D) 1

387) Find the extreme values of  $f(x, y, z) = 2x - 3y + z$  subject to  $x^2 + y^2 = 1$  and  $y^2 + z^2 = 1$ .

387) \_\_\_\_\_

A) Maximum:  $4\sqrt{2}$  at  $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ ; minimum:  $-4\sqrt{2}$  at  $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ B) Maximum:  $2\sqrt{2}$  at  $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ ; minimum:  $-2\sqrt{2}$  at  $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ C) Maximum:  $\sqrt{2}$  at  $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ ; minimum:  $-\sqrt{2}$  at  $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ D) Maximum:  $3\sqrt{2}$  at  $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ ; minimum:  $-3\sqrt{2}$  at  $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ 388) Find the point on the sphere  $x^2 + y^2 + z^2 = 4$  that is closest to the point  $(3, 1, -1)$ .

388) \_\_\_\_\_

A)  $\left(\frac{6}{\sqrt{11}}, \frac{2}{\sqrt{11}}, -\frac{2}{\sqrt{11}}\right)$

B)  $\left(\frac{6}{\sqrt{11}}, \frac{2}{\sqrt{11}}, \frac{2}{\sqrt{11}}\right)$

C)  $\left(-\frac{6}{\sqrt{11}}, \frac{2}{\sqrt{11}}, -\frac{2}{\sqrt{11}}\right)$

D)  $\left(-\frac{6}{\sqrt{11}}, \frac{2}{\sqrt{11}}, \frac{2}{\sqrt{11}}\right)$

389) Find the derivative of the function  $f(x, y) = x^2 + xy + y^2$  at the point  $(8, 9)$  in the direction in which the function increases most rapidly.

389) \_\_\_\_\_

A)  $\sqrt{1157}$

B)  $3\sqrt{146}$

C)  $\sqrt{1301}$

D)  $\sqrt{1163}$

**Find the extreme values of the function subject to the given constraint.**390)  $f(x, y) = 9x^2 + 3y^2$ ,  $x^2 + y^2 = 1$ 

390) \_\_\_\_\_

A) Maximum: 3 at  $(0, \pm 1)$ ; minimum: 0 at  $(0, 0)$ B) Maximum: 3 at  $(0, \pm 1)$ ; minimum: 9 at  $(\pm 1, 0)$ C) Maximum: 3 at  $(\pm 1, 0)$ ; minimum: 9 at  $(0, \pm 1)$ D) Maximum: 3 at  $(\pm 1, 0)$ ; minimum: 0 at  $(0, 0)$

**Find the limit.**

391)  $\lim_{\substack{(x,y) \rightarrow (3,3) \\ y \neq 3}} \frac{y-3}{x^2y+4y-3x^2-12}$

391) \_\_\_\_\_

A) 5

B)  $\frac{1}{13}$

C) 0

D) 13

**Solve the problem.**

- 392) The resistance  $R$  produced by wiring resistors of  $R_1$ ,  $R_2$ , and  $R_3$  ohms in parallel can be calculated from the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

392) \_\_\_\_\_

If  $R_1$ ,  $R_2$ , and  $R_3$  are measured to be 9 ohms, 10 ohms, and 8 ohms respectively, and if these measurements are accurate to within 0.05 ohms, estimate the maximum percentage error in computing  $R$ .

A) 0.68%

B) 0.34%

C) 0.56%

D) 0.45%

- 393) Write parametric equations for the tangent line to the curve of intersection of the surfaces  $x = y^2$  and  $y = 3z^2$  at the point  $(9, 3, 1)$ .

393) \_\_\_\_\_

A)  $x = 18t + 9$ ,  $y = 6t + 3$ ,  $z = t + 1$

B)  $x = 36t + 9$ ,  $y = 6t + 3$ ,  $z = t + 1$

C)  $x = 18t + 9$ ,  $y = 3t + 3$ ,  $z = t + 1$

D)  $x = 36t + 9$ ,  $y = 3t + 3$ ,  $z = t + 1$

**Find the linearization of the function at the given point.**

- 394)  $f(x, y) = 5x + 3y - 4$  at  $(3, 10)$

394) \_\_\_\_\_

A)  $L(x, y) = 3x + 10y - 4$

B)  $L(x, y) = 15x + 30y - 4$

C)  $L(x, y) = 15x + 30y + 16$

D)  $L(x, y) = 5x + 3y - 4$

**Answer the question.**

- 395) Which order of differentiation will calculate  $f_{xy}$  faster,  $x$  first or  $y$  first?

395) \_\_\_\_\_

$$f(x, y) = x^2y + \sqrt{y^2 + 1}$$

A)  $y$  first

B)  $x$  first

**Find the domain and range and describe the level curves for the function  $f(x, y)$ .**

396)  $f(x, y) = \frac{5x^2}{y}$

396) \_\_\_\_\_

A) Domain: all points in the  $x$ - $y$  plane except  $y = 0$ ; range: all real numbers; level curves: parabolas  $y = cx^2$

B) Domain: all points in the  $x$ - $y$  plane; range: all real numbers; level curves: parabolas  $y = cx^2$

C) Domain: all points in the  $x$ - $y$  plane; range: real numbers  $z \geq 0$ ; level curves: parabolas  $y = cx^2$

D) Domain: all points in the  $x$ - $y$  plane except  $y = 0$ ; range: real numbers  $z \geq 0$ ; level curves: parabolas  $y = cx^2$

**Solve the problem.**

- 397) Find an equation for the level curve of the function  $f(x, y) = \sqrt{x^2 + y^2}$  that passes through the point  $(3, 4)$ .

397) \_\_\_\_\_

A)  $x^2 + y^2 = 7$

B)  $x + y = 5$

C)  $x^2 + y^2 = 25$

D)  $x^2 + y^2 = 5$

- 398) Find the least squares line through the points  $(1, -24)$ ,  $(2, 18)$ , and  $(3, -60)$ .

398) \_\_\_\_\_

A)  $y = -18x + 14$

B)  $y = -18x - 66$

C)  $y = -42x - 66$

D)  $y = -42x + 14$

**Write a chain rule formula for the following derivative.**

399)  $\frac{\partial u}{\partial x}$  for  $u = f(p, q)$ ;  $p = g(x, y, z)$ ,  $q = h(x, y, z)$

399) \_\_\_\_\_

A)  $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial q} \frac{\partial q}{\partial x}$

B)  $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} \frac{\partial p}{\partial x} + \frac{\partial u}{\partial q} \frac{\partial q}{\partial y} + \frac{\partial u}{\partial q} \frac{\partial q}{\partial z}$

C)  $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} \frac{\partial x}{\partial p} + \frac{\partial u}{\partial q} \frac{\partial x}{\partial q}$

D)  $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} \frac{\partial p}{\partial x} + \frac{\partial u}{\partial q} \frac{\partial q}{\partial x}$

**Solve the problem.**

- 400) Write parametric equations for the tangent line to the curve of intersection of the surfaces

400) \_\_\_\_\_

$z = 4x^2 + 7y^2$  and  $z = x + y + 9$  at the point  $(1, 1, 11)$ .

A)  $x = -7t + 1$ ,  $y = 9t + 1$ ,  $z = -6t + 11$

B)  $x = -13t + 1$ ,  $y = 7t + 1$ ,  $z = -6t + 11$

C)  $x = -13t + 1$ ,  $y = 9t + 1$ ,  $z = -6t + 11$

D)  $x = -7t + 1$ ,  $y = 7t + 1$ ,  $z = -6t + 11$

**Find an upper bound for the magnitude  $|E|$  of the error in the linear approximation  $L$  to  $f$  at the given point over the given region.**

401)  $f(x, y) = 9x^2y^3$  at  $(2, 1)$

401) \_\_\_\_\_

R:  $|x - 2| \leq 0.2$ ,  $|y - 1| \leq 0.2$

A)  $|E| \leq 19.38816$

B)  $|E| \leq 25.09056$

C)  $|E| \leq 19.65492$

D)  $|E| \leq 13.68576$

**Solve the problem.**

- 402) If the length, width, and height of a rectangular solid are measured to be 9, 8, and 6 inches respectively and each measurement is accurate to within 0.1 inch, estimate the maximum percentage error in computing the volume of the solid.

402) \_\_\_\_\_

A) 4.83%

B) 3.62%

C) 3.22%

D) 4.03%

**Use polar coordinates to find the limit of the function as  $(x, y)$  approaches  $(0, 0)$ .**

403)  $f(x, y) = \frac{9xy}{\sqrt{x^2 + y^2}}$

403) \_\_\_\_\_

A) -1

B) 0

C) 1

D)  $\pi$

**Solve the problem.**

- 404) About how much will  $f(x, y, z) = \ln(-5x - 6y - 8z)$  change if the point  $(x, y, z)$  moves from  $(9, 3, -10)$  a distance of  $ds = \frac{1}{10}$  unit in the direction of  $12\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ ?

404) \_\_\_\_\_

A)  $-\frac{58}{1105}$

B)  $-\frac{4}{85}$

C)  $-\frac{11}{221}$

D)  $-\frac{61}{1105}$

- 405) Maximize  $f(x, y) = 7x^2 + 5xy + 9y^2$  subject to  $x + y = 1$  and  $x + 4y = 9$ .

405) \_\_\_\_\_

A)  $\frac{551}{9}$

B)  $\frac{217}{3}$

C)  $\frac{451}{9}$

D) 39

- 406) The van der Waals equation provides an approximate model for the behavior of real gases. The equation is

406)

$$P(V, T) = \frac{RT}{V - b} - \frac{a}{V^2}, \text{ where } P \text{ is pressure, } V \text{ is volume, } T \text{ is Kelvin temperature, and } a, b, \text{ and } R \text{ are}$$

constants. Find the derivative of the function with respect to each variable.

A)  $P_V = -\frac{2a}{V^3} + \frac{RT}{(V - b)^2}; P_T = \frac{R}{V - b}$

B)  $P_V = \frac{R}{V - b}; P_T = \frac{2a}{V^3} - \frac{RT}{(V - b)^2}$

C)  $P_V = \frac{2a}{V} - \frac{RT}{(V - b)^2}; P_T = \frac{R}{V - b}$

D)  $P_V = \frac{2a}{V^3} - \frac{RT}{(V - b)^2}; P_T = \frac{R}{V - b}$

**Find the absolute maxima and minima of the function on the given domain.**

- 407)  $f(x, y) = 2xy^2 + 9xy$  on the trapezoidal region with vertices  $(0, 0)$ ,  $(1, 0)$ ,  $(0, 2)$ , and  $(1, 1)$

407)

A) Absolute maximum: 11 at  $(1, 1)$ ; absolute minimum: 0 at  $(0, 0)$

B) Absolute maximum: 11 at  $(1, 1)$ ; absolute minimum: 2 at  $(1, 0)$

C) Absolute maximum: 18 at  $(0, 2)$ ; absolute minimum: 0 at  $(0, 0)$

D) Absolute maximum: 4 at  $(2, 0)$ ; absolute minimum: 2 at  $(1, 0)$

**Find the absolute maximum and minimum values of the function on the given curve.**

- 408) Function:  $f(x, y) = xy$ ; curve:  $\frac{x^2}{16} + \frac{y^2}{100} = 1, y \geq 0$ . (Use the parametric equations  $x = 4 \cos t$ ,

408)

$$y = 10 \sin t.)$$

A) Absolute maximum: 10 at  $t = \frac{\pi}{4}$ ; absolute minimum:  $-10$  at  $t = \frac{3\pi}{4}$

B) Absolute maximum: 20 at  $t = \frac{\pi}{4}$ ; absolute minimum:  $-10$  at  $t = \frac{3\pi}{4}$

C) Absolute maximum: 10 at  $t = \frac{\pi}{4}$ ; absolute minimum:  $-20$  at  $t = \frac{3\pi}{4}$

D) Absolute maximum: 20 at  $t = \frac{\pi}{4}$ ; absolute minimum:  $-20$  at  $t = \frac{3\pi}{4}$

**Compute the gradient of the function at the given point.**

- 409)  $f(x, y, z) = 3x + 10y + 2z, (-10, -7, -5)$

409)

A)  $-30\mathbf{i} - 70\mathbf{j} - 10\mathbf{k}$

B)  $3\mathbf{i} + 10\mathbf{j} + 2\mathbf{k}$

C)  $-30\mathbf{i} + 70\mathbf{j} - 10\mathbf{k}$

D)  $-10\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$

**Solve the problem.**

- 410) If the length, width, and height of a rectangular solid are measured to be 6, 2, and 2 inches respectively and each measurement is accurate to within 0.1 inch, estimate the maximum possible error in computing the volume of the solid.

410)

A) 350.00

B) 280.00

C) 311.11

D) 233.33

## Answer Key

Testname: TEST 5

1) C

2) C

3) C

4) A

5) A

6) A

7) B

8) B

9) A

10) Ellipses ;  $\frac{x^2}{5} + \frac{y^2}{3} + \frac{z^2}{2} = 300$

11) Let  $\delta = 0.04$ . Then if  $|x| < \delta$  and  $|y| < \delta$  and  $|z| < \delta$ ,  $|f(x, y, z) - f(0, 0, 0)| = |x + y + z| \leq |x| + |y| + |z| < 0.04 + 0.04 + 0.04 = 0.12 = \epsilon$ .

12)  $\frac{\partial f}{\partial y} = \lim_{h \rightarrow 0} \frac{324(-5+h) + 6(-5+h)^2 + 30 + 1440}{h}$

$$= \lim_{h \rightarrow 0} \frac{216h + 6h^2}{h} = \lim_{h \rightarrow 0} 216 + 6h = 216$$

13) The sphere centered at  $(x_0, y_0, z_0)$  of radius  $\delta$  circumscribes the cube centered at  $(x_0, y_0, z_0)$  and with sides  $\delta' = \sqrt{2}\delta$ .

Therefore,  $|x - x_0| < \frac{\sqrt{2}\delta}{2}$ ,  $|y - y_0| < \frac{\sqrt{2}\delta}{2}$ , and  $|z - z_0| < \frac{\sqrt{2}\delta}{2}$  imply  $\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} < \delta$ .

Likewise, the cube centered at  $(x_0, y_0, z_0)$  with sides  $2\delta$  circumscribes the sphere with radius  $\delta$ . Thus,

$\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} < \delta$  implies  $|x - x_0| < \delta$ ,  $|y - y_0| < \delta$ , and  $|z - z_0| < \delta$ . The requirements are equivalent.

14)  $\lim_{(x, y, z) \rightarrow (x_0, y_0, z_0)} f(x, y, z) = \lim_{(x, y, z) \rightarrow (x_0, y_0, z_0)} x^3 y^3 z^3 = x_0^3 y_0^3 z_0^3 = f(x_0, y_0, z_0)$ , which proves the assertion.

15)  $\left| \cos\left(\frac{1}{y}\right) \right| \leq 1$  implies  $-1 \leq \cos\left(\frac{1}{y}\right) \leq 1$ , which allows us to obtain  $-\sin x \leq \sin(x) \cos\left(\frac{1}{y}\right) \leq \sin x$  for  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ . As  $\lim_{(x, y) \rightarrow (0, 0)} -\sin x = 0$  and  $\lim_{(x, y) \rightarrow (0, 0)} \sin x = 0$ , the Sandwich Theorem implies  $\lim_{(x, y) \rightarrow (0, 0)} \sin(x) \cos\left(\frac{1}{y}\right) = 0$ .

16) Let  $\delta = 0.03$ . Then if  $|x| < \delta$  and  $|y| < \delta$ ,  $|f(x, y) - f(0, 0)| = \left| \frac{x+y}{x^2+y^2+1} \right| \leq |x+y| \leq |x| + |y| < 0.03 + 0.03 = 0.06 = \epsilon$ .

17) Let  $\delta = 0.04$ . Then if  $|x| < \delta$  and  $|y| < \delta$ ,  $|f(x, y) - f(0, 0)| = |x+y| \leq |x| + |y| < 0.04 + 0.04 = 0.08 = \epsilon$ .

18)  $\lim_{(x, y) \rightarrow (0, 0)} 4 - x^2 y^3 = 4$ . Hence, by the Sandwich Theorem,  $\lim_{(x, y) \rightarrow (0, 0)} \frac{4 \tan^{-1} xy}{xy} = 4$ .

19) Answers will vary. One possibility is Path 1:  $x = t$ ,  $y = t$ ; Path 2:  $x = 0$ ,  $y = t$

20)  $\frac{\partial f}{\partial x} = \lim_{h \rightarrow 0} \frac{5 - 27(-5+h) + 567(-5+h) + 3073}{h}$

$$= \lim_{h \rightarrow 0} \frac{540h}{h} = \lim_{h \rightarrow 0} -2862 = 540$$

## Answer Key

Testname: TEST 5

$$21) \frac{\partial f}{\partial y} = \lim_{h \rightarrow 0} \frac{4 + 30(7+h) - 6(7+h)^2 + 500}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-54h - 6h^2}{h} = \lim_{h \rightarrow 0} -54 - 6h = -54$$

22) Answers will vary. One possibility is Path 1:  $x = t$ ,  $y = t$ ; Path 2:  $x = t$ ,  $y = 2t$

$$23) \text{ Let } \delta = 0.03. \text{ Then if } |x| < \delta \text{ and } |y| < \delta, |f(x, y) - f(0, 0)| = \left| \frac{2x+y}{x^2y^2+1} \right| \leq |2x+y| \leq 2|x| + |y| < 0.06 + 0.03 = 0.09 = \varepsilon.$$

$$24) \frac{\partial f}{\partial y} = \lim_{h \rightarrow 0} \frac{441 - 28(-4+h) + 6(-4+h)^2 - 649}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-24h + 6h^2}{h} = \lim_{h \rightarrow 0} -24 + 6h = -76$$

$$25) \text{ Let } \delta = \sin^{-1}(\sqrt{0.04}). \text{ Then if } |x| < \delta \text{ and } |y| < \delta \text{ and } |z| < \delta, |f(x, y, z) - f(0, 0, 0)| = |\sin^2 x + \sin^2 y + \sin^2 z| \leq |\sin^2 x| + |\sin^2 y| + |\sin^2 z| < 0.04 + 0.04 + 0.04 = 0.12 = \varepsilon.$$

26) Answers will vary. One possibility is Path 1:  $x = t$ ,  $y = t$ ; Path 2:  $x = t$ ,  $y = t^2$

27) Answers will vary. One possibility is Path 1:  $x = t$ ,  $y = t$ ; Path 2:  $x = t$ ,  $y = -t$

28) Answers will vary. One possibility is Path 1:  $x = 0$ ,  $y = t$ ; Path 2:  $x = -t^2$ ,  $y = t$

$$29) \frac{\partial f}{\partial x} = \lim_{h \rightarrow 0} \frac{-35(3+h)^2 + 196 + 4 + 115}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-210h - 35h^2}{h} = \lim_{h \rightarrow 0} -210 - 35h = -210$$

30) Answers will vary. One possibility is Path 1:  $x = t$ ,  $y = 0$ ; Path 2:  $x = 0$ ,  $y = t$

$$31) \left| \sin\left(\frac{1}{y}\right) \right| \leq 1 \text{ implies } -1 \leq \sin\left(\frac{1}{y}\right) \leq 1, \text{ which allows us to obtain } -\sin x \leq \sin(x) \sin\left(\frac{1}{y}\right) \leq \sin x \text{ for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}. \text{ As } \lim_{(x, y) \rightarrow (0, 0)} -\sin x = 0 \text{ and } \lim_{(x, y) \rightarrow (0, 0)} \sin x = 0, \text{ the Sandwich Theorem implies } \lim_{(x, y) \rightarrow (0, 0)} \sin(x) \sin\left(\frac{1}{y}\right) = 0.$$

$$32) \frac{\partial f}{\partial y} = \lim_{h \rightarrow 0} \frac{15(-5+h) - 4(-5+h)^2 + 30 + 145}{h}$$

$$= \lim_{h \rightarrow 0} \frac{55h - 4h^2}{h} = \lim_{h \rightarrow 0} 55 - 4h = 55$$

$$33) \lim_{(x, y, z) \rightarrow (0, 0, 0)} f(x, y, z) = \lim_{(x, y, z) \rightarrow (0, 0, 0)} e^{x^2 + y^2 + z^2} = e^{(0^2 + 0^2 + 0^2)} = 1 = f(0, 0, 0), \text{ which proves the assertion.}$$

34) Answers will vary. One possibility is Path 1:  $x = t$ ,  $y = t$ ; Path 2:  $x = 0$ ,  $y = t$

35) This follows from applying the chain rule.

$$36) \frac{\partial f}{\partial z} = \lim_{h \rightarrow 0} \frac{-3(7+h) - 27 - 9(7+h) + 111}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-12h}{h} = \lim_{h \rightarrow 0} -12 = -12$$

$$37) \text{ Let } \delta = \langle a \rangle. \text{ Then if } \left| \sqrt{x^2 + y^2 + z^2} \right| < \delta, |f(x, y, z) - f(0, 0, 0)| = \left| \frac{\sqrt{x^2 + y^2 + z^2}}{x+1} \right| \leq \left| \sqrt{x^2 + y^2 + z^2} \right| < 0.01 = \varepsilon.$$

38) Answers will vary. One possibility is Path 1:  $x = t$ ,  $y = t$ ; Path 2:  $x = t$ ,  $y = t^3/2$

39) No local minimum

## Answer Key

### Testname: TEST 5

40) Let  $\delta = 0.02$ . Then if  $|x| < \delta$  and  $|y| < \delta$ ,  $|f(x, y) - f(0, 0)| = |(1 + \cos x)(x + y)| \leq 2|x + y| \leq 2(|x| + |y|) < 0.04 + 0.04 = 0.08 = \epsilon$ .

41) Let  $\delta = \epsilon$ . Then if  $|x| < \delta$ ,  $|y| < \delta$ , and  $|z| < \delta$ ,  $|f(x, y, z) - f(0, 0, 0)| = |x + y - z| \leq |x| + |y| + |z| < 0.01 + 0.01 + 0.01 = 0.03 = \epsilon$ .

$$\begin{aligned}42) \frac{\partial f}{\partial x} &= \lim_{h \rightarrow 0} \frac{7(4+h)^2 + 32(4+h) + 320 - 560}{h} \\&= \lim_{h \rightarrow 0} \frac{7(16+8h+h^2) + 32h - 112}{h} \\&= \lim_{h \rightarrow 0} \frac{102h+h^2}{h} = \lim_{h \rightarrow 0} 102+h = 102\end{aligned}$$

43) The function is not necessarily continuous at  $(x_0, y_0)$ . It is continuous only if  $\lim_{(x, y) \rightarrow (x_0, y_0)} f(x, y) = -2$ .

44) B

45) D

46) A

47) A

48) A

49) C

50) B

51) C

52) C

53) D

54) B

55) C

56) D

57) C

58) B

59) D

60) C

61) B

62) D

63) D

64) A

65) C

66) B

67) B

68) A

69) D

70) D

71) A

72) B

73) C

74) C

75) D

76) A

77) A

78) A

79) A

## Answer Key

Testname: TEST 5

- 80) B
- 81) C
- 82) C
- 83) C
- 84) B
- 85) C
- 86) B
- 87) C
- 88) D
- 89) B
- 90) C
- 91) B
- 92) D
- 93) A
- 94) C
- 95) D
- 96) D
- 97) A
- 98) A
- 99) D
- 100) C
- 101) C
- 102) B
- 103) C
- 104) A
- 105) A
- 106) C
- 107) D
- 108) C
- 109) B
- 110) A
- 111) D
- 112) B
- 113) B
- 114) C
- 115) C
- 116) A
- 117) C
- 118) D
- 119) A
- 120) B
- 121) A
- 122) D
- 123) B
- 124) A
- 125) D
- 126) C
- 127) B
- 128) B
- 129) B

## Answer Key

Testname: TEST 5

- 130) D
- 131) D
- 132) C
- 133) C
- 134) A
- 135) B
- 136) B
- 137) B
- 138) D
- 139) A
- 140) C
- 141) A
- 142) A
- 143) B
- 144) B
- 145) D
- 146) C
- 147) D
- 148) B
- 149) B
- 150) B
- 151) A
- 152) D
- 153) B
- 154) B
- 155) C
- 156) A
- 157) D
- 158) C
- 159) D
- 160) B
- 161) A
- 162) D
- 163) B
- 164) C
- 165) A
- 166) C
- 167) C
- 168) D
- 169) A
- 170) B
- 171) D
- 172) B
- 173) C
- 174) D
- 175) A
- 176) A
- 177) B
- 178) C
- 179) A

## Answer Key

Testname: TEST 5

- 180) B
- 181) A
- 182) B
- 183) B
- 184) D
- 185) D
- 186) B
- 187) A
- 188) A
- 189) D
- 190) C
- 191) D
- 192) D
- 193) A
- 194) B
- 195) B
- 196) B
- 197) B
- 198) B
- 199) B
- 200) A
- 201) C
- 202) A
- 203) D
- 204) B
- 205) C
- 206) C
- 207) A
- 208) A
- 209) A
- 210) C
- 211) B
- 212) D
- 213) A
- 214) A
- 215) C
- 216) B
- 217) B
- 218) B
- 219) D
- 220) D
- 221) A
- 222) B
- 223) A
- 224) A
- 225) B
- 226) A
- 227) D
- 228) B
- 229) D

## Answer Key

Testname: TEST 5

- 230) B
- 231) C
- 232) C
- 233) B
- 234) A
- 235) B
- 236) B
- 237) A
- 238) C
- 239) D
- 240) B
- 241) B
- 242) D
- 243) A
- 244) A
- 245) D
- 246) A
- 247) D
- 248) C
- 249) B
- 250) A
- 251) D
- 252) C
- 253) D
- 254) C
- 255) A
- 256) D
- 257) D
- 258) C
- 259) B
- 260) C
- 261) A
- 262) C
- 263) A
- 264) A
- 265) A
- 266) A
- 267) D
- 268) A
- 269) B
- 270) A
- 271) B
- 272) B
- 273) B
- 274) D
- 275) A
- 276) B
- 277) C
- 278) A
- 279) C

## Answer Key

Testname: TEST 5

- 280) A
- 281) D
- 282) D
- 283) A
- 284) D
- 285) B
- 286) C
- 287) A
- 288) A
- 289) A
- 290) B
- 291) A
- 292) A
- 293) C
- 294) B
- 295) B
- 296) A
- 297) A
- 298) A
- 299) C
- 300) B
- 301) D
- 302) B
- 303) B
- 304) A
- 305) C
- 306) A
- 307) D
- 308) A
- 309) A
- 310) A
- 311) C
- 312) D
- 313) C
- 314) D
- 315) D
- 316) B
- 317) A
- 318) D
- 319) D
- 320) D
- 321) C
- 322) C
- 323) D
- 324) B
- 325) A
- 326) A
- 327) D
- 328) D
- 329) C

## Answer Key

Testname: TEST 5

- 330) C
- 331) C
- 332) C
- 333) B
- 334) C
- 335) D
- 336) A
- 337) D
- 338) A
- 339) C
- 340) B
- 341) B
- 342) D
- 343) C
- 344) C
- 345) D
- 346) C
- 347) B
- 348) A
- 349) C
- 350) C
- 351) A
- 352) D
- 353) B
- 354) C
- 355) A
- 356) D
- 357) C
- 358) B
- 359) A
- 360) D
- 361) D
- 362) D
- 363) C
- 364) C
- 365) B
- 366) B
- 367) A
- 368) D
- 369) D
- 370) D
- 371) C
- 372) C
- 373) A
- 374) A
- 375) B
- 376) B
- 377) D
- 378) B
- 379) C

## Answer Key

Testname: TEST 5

- 380) B
- 381) D
- 382) B
- 383) C
- 384) B
- 385) C
- 386) D
- 387) D
- 388) A
- 389) C
- 390) B
- 391) B
- 392) C
- 393) B
- 394) D
- 395) B
- 396) A
- 397) C
- 398) A
- 399) D
- 400) B
- 401) B
- 402) D
- 403) B
- 404) C
- 405) A
- 406) D
- 407) A
- 408) D
- 409) B
- 410) B