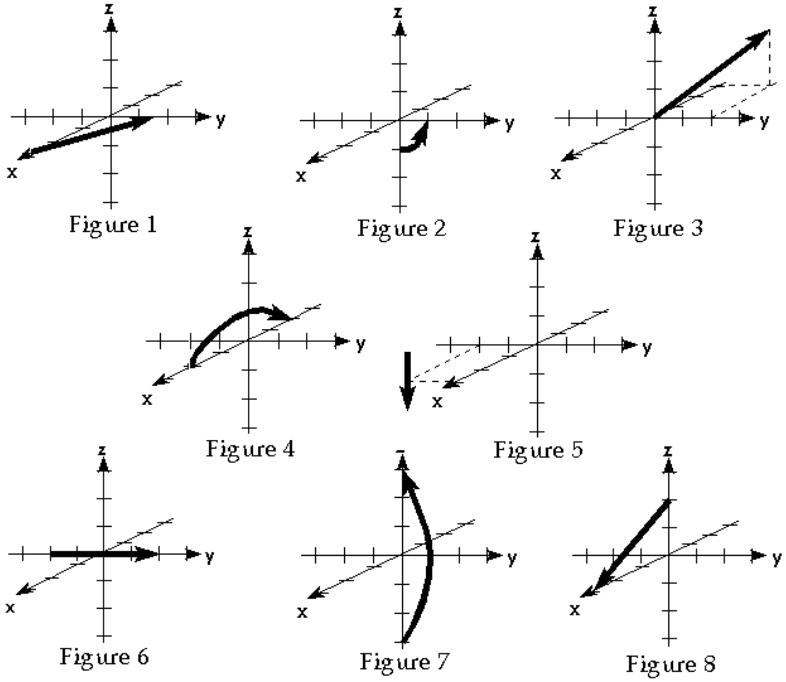
Exam

Name___________________________________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Match the vector equation with the correct graph.



1) $\mathbf{r}(t) = -3t\mathbf{i} + 2t\mathbf{j} + 2t\mathbf{k};\ 0 \le t \le 1$ 1) ________

A) Figure 8 B) Figure 3 C) Figure 5 D) Figure 1

2) $\mathbf{r}(t) = (3 - 2t)\mathbf{i} + t\mathbf{j};\ 0 \le t \le \frac{3}{2}$ 2) ________

A) Figure 5 B) Figure 1 C) Figure 8 D) Figure 3

3) $\mathbf{r}(t) = (1 - t^2)\mathbf{j} + 3t\mathbf{k};\ -1 \le t \le 1$ 3) ________

A) Figure 2 B) Figure 4 C) Figure 7 D) Figure 8

4) $\mathbf{r}(t) = 3\mathbf{i} - 2\mathbf{j} - t\mathbf{k};\ -1 \le t \le 1$ 4) ________

A) Figure 3 B) Figure 6 C) Figure 8 D) Figure 5

5) $\mathbf{r}(t) = \frac{3}{2}t\mathbf{i} + (2 - t)\mathbf{k};\ 0 \le t \le 2$ 5) ________

A) Figure 5 B) Figure 1 C) Figure 8 D) Figure 3

6) $\mathbf{r}(t) = 2\cos t\mathbf{i} + \sin t\mathbf{k};\ 0 \le t \le \pi$ 6) ________

A) Figure 6 B) Figure 2 C) Figure 7 D) Figure 4

7) $\mathbf{r}(t) = t\mathbf{j};\ -2 \le t \le 2$ 7) ________

A) Figure 1 B) Figure 6 C) Figure 5 D) Figure 3

8) $\mathbf{r}(t) = \sin t\mathbf{j} - \cos t\mathbf{k};\ 0 \le t \le \frac{\pi}{2}$ 8) ________

A) Figure 1 B) Figure 7 C) Figure 4 D) Figure 2

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

9) The curve $x = y^2$ is revolved about the x–axis. Find a parametrization of the surface of revolution and then find the equation of the tangent plane at the point $(x,y,z) = (9, 0, 3)$. 9) ____________

10) a). For the field $\mathbf{F}(x,y) = M\mathbf{i} + N\mathbf{j}$ and the closed counterclockwise plane curve C in the xy–plane, show that $\int_C \mathbf{F} \cdot n\, ds = \int_C M\,dy - N\,dx$. 10) ____________

b). How would the equality change if the closed path is followed clockwise?

11) Find a parametrization for the ellipsoid $\frac{x^2}{16} + \frac{y^2}{121} + \frac{z^2}{81} = 1$. (Recall that the parametrization of an ellipse $\frac{x^2}{16} + \frac{y^2}{121} = 1$ is $x = 4\cos\theta$, $y = 11\sin\theta$, $0 \le \theta < 2\pi$). 11) ____________

12) Suppose that the parametrized plane curve C: $(f(u), g(u))$ is revolved about the x–axis, where $g(u) > 0$ and $a \le u \le b$. Show that the surface area of the surface of revolution is $2\pi \int_a^b g(u)\sqrt{[g'(u)]^2 + [f'(u)]^2}\, du$ 12) ____________

Find the equation for the plane tangent to the parametrized surface S at the point P.

13) S is the parabolic cylinder $\mathbf{r}(x, z) = x\mathbf{i} + 2x^2\mathbf{j} + z\mathbf{k}$; P is the point corresponding to $(x, z) = (1, -6)$ 13) ____________

Solve the problem.

14) Assuming C is a simple closed path , what is special about the integral 14) ____________

$$\int_C \big(9x + 3\sin 3x \cos 3y\big)\,dx + \big(3x + 3\cos 3x \sin 3y\big)\,dy$$? Give reasons for your answer.

Parametrize the surface S.

15) S is the portion of the cylinder $x^2 + y^2 = 36$ that lies between $z = 4$ and $z = 6$ 15) ____________

Sketch the vector field in the plane along with its horizontal and vertical components at a representative assortment of points on the circle $x^2 + y^2 = 4$.

16) $\mathbf{F} = -\frac{y}{\sqrt{x^2+y^2}}\mathbf{i} - \frac{x}{\sqrt{x^2+y^2}}\mathbf{j}$ 16) ____________

Solve the problem.

17) Consider a fluid with a flow field $\mathbf{F} = x^2y^3\mathbf{i} + 2z^2\mathbf{j} + z\mathbf{k}$. A miniature paddlewheel (idealized) is to be inserted into the flow at the point (1,1,1). Find a vector describing the orientation of the paddlewheel axis which produces the maximum rotational speed. 17) ____________

18) Let $M = \frac{y}{x^2+y^2}$ and $N = \frac{-x}{x^2+y^2}$. Show that 18) ____________

$\int_C M\,dx + N\,dy \neq \int\int_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$, where R is the region bounded by the unit circle C centered at the origin. Why is Green's Theorem failing in this case?

19) Imagine a force field in which the force is always parallel to $d\mathbf{r}$. What is special about the work done in moving a particle in such a field? 19) ____________

Find the equation for the plane tangent to the parametrized surface S at the point P.

20) S is the cylinder $\mathbf{r}(\theta, z) = 12\cos^2\theta\mathbf{i} + 6\sin 2\theta\mathbf{j} + z\mathbf{k}$; P is the point corresponding to $(\theta, z) = \left(\frac{\pi}{4}, 4\right)$ 20) ____________

Solve the problem.

21) Imagine a force field in which the force is always perpendicular to $d\mathbf{r}$. What is special about the work done in moving a particle in such a field? 21) ____________

22) Find the values of b and c that make $\mathbf{F} = \frac{18x^2y^8}{z^4}\mathbf{i} + \frac{bx^3y^7}{z^4}\mathbf{j} + \frac{cx^3y^8}{z^5}\mathbf{k}$ a gradient field. 22) ____________

Sketch the vector field in the plane along with its horizontal and vertical components at a representative assortment of points on the circle $x^2 + y^2 = 4$.

23) $\mathbf{F} = \frac{y}{\sqrt{x^2+y^2}}\mathbf{i} + \frac{x}{\sqrt{x^2+y^2}}\mathbf{j}$ 23) ____________

Solve the problem.

24) Show that the value of the integral does not depend on the path taken from A to B. 24) ____________

$$\int_A^B z^6\,dx + 3y\,dy + 6xz\,dz$$

25) Assume the curl of a vector field **F** is zero. Can one automatically conclude that the circulation $\int_C \mathbf{F}\cdot d\mathbf{r} = 0$ for all closed paths C? Explain or justify your answer. 25) ____________

26) The velocity field **F** of a fluid has a constant magnitude k and always points towards the origin. Following the smooth curve y = f(x) from (a, f(a)) to (b, f(b)), show that the flow along the curve is 26) ____________

$$\int_C \mathbf{F}\cdot\mathbf{T}\,ds = k[(a^2 + (f(a))^2)^{1/2} - (b^2 + (f(b))^2)^{1/2}]$$

27) What can be said about the flux of $\mathbf{F} = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$ across a sphere centered at the origin? Will differing radii change the flux? 27) ____________

Parametrize the surface S.

28) S is the portion of the cone $\frac{x^2}{64} + \frac{y^2}{64} = \frac{z^2}{36}$ that lies between z = 3 and z = 4 28) ____________

29) S is the portion of the sphere $x^2 + y^2 + z^2 = 100$ between $z = -5\sqrt{2}$ and $z = 5\sqrt{2}$ 29) ____________

Find the equation for the plane tangent to the parametrized surface S at the point P.

30) S is the paraboloid $\mathbf{r}(\theta, r) = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j} + 4r^2\mathbf{k}$; P is the point corresponding to $(\theta, r) = \left(\frac{\pi}{4}, 2\right)$ 30) ____________

Solve the problem.

31) For a surface parametrized in the parameters u and v and a force **F**, show that $\int \mathbf{F}\cdot\mathbf{n}\,d\sigma = \int \mathbf{F}\cdot(\mathbf{r_u}\times\mathbf{r_v})\,dudv$. 31) ____________

Sketch the vector field in the plane along with its horizontal and vertical components at a representative assortment of points on the circle $x^2 + y^2 = 4$.

32) $\mathbf{F} = x\mathbf{i} - y\mathbf{j}$ 32) ____________

Parametrize the surface S.

33) S is the portion of the paraboloid $z = 2x^2 + 2y^2$ that lies between z = 5 and z = 7 33) ____________

Find the equation for the plane tangent to the parametrized surface S at the point P.

34) S is the cone $\mathbf{r}(r, \theta) = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j} + 3r\mathbf{k}$; $P = \left(-\frac{9}{2}\sqrt{2}, \frac{9}{2}\sqrt{2}, 27\right)$ 34) ____________

35) S is the sphere $\mathbf{r}(\theta, \phi) = 7\cos\theta\sin\phi\mathbf{i} + 7\sin\theta\sin\phi\mathbf{j} + 7\cos\phi\mathbf{k}$; P is the point corresponding to $(\theta, \phi) = \left(\frac{\pi}{3}, \frac{\pi}{4}\right)$ 35) ____________

Solve the problem.

36) Assuming all the necessary derivatives exist, show that if $\int_C \frac{\partial f}{\partial y}\,dx - \frac{\partial f}{\partial x}\,dy = 0$ closed curves C to which Green's Theorem applies, then f satisfies the Laplace equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ for all regions bounded by closed curves C to which Green's Theorem applies. 36) ____________

Sketch the vector field in the plane along with its horizontal and vertical components at a representative assortment of points on the circle $x^2 + y^2 = 4$.

37) $\mathbf{F} = \frac{y}{\sqrt{x^2+y^2}}\mathbf{i} - \frac{x}{\sqrt{x^2+y^2}}\mathbf{j}$ 37) ____________

Solve the problem.

38) For the surface $z = f(x,y)$, show that the surface integral $\iint g(x,y,z)\,d\sigma = \iint g(x, y, f(x,y))\sqrt{[f_x(x,y)]^2 + [f_y(x,y)]^2 + 1}\,dxdy$. 38) ____________

Parametrize the surface S.

39) S is the lower portion of the sphere $x^2 + y^2 + z^2 = 1$ cut by the cone $z = \sqrt{x^2 + y^2}$ 39) ____________

40) S is the portion of the plane $8x + 8y - 8z = 5$ that lies within the cylinder $x^2 + y^2 = 1$ 40) ____________

Solve the problem.

41) Assuming C is a simple closed path , what is special about the integral $\int_C \left(8x + 5e^{8x}\cos 8y\right)dx + \left(7x + 5e^{8x}\sin 8y\right)dy$? Give reasons for your answer. 41) ____________

42) Show that $df = -\frac{4(y^8 + z)}{x^5z^7}\,dx + \frac{8y^7}{x^4z^7}\,dy - \left(\frac{7y^8}{x^4z^8} + \frac{6}{x^4z^7}\right)dz$ is exact. 42) ____________

Parametrize the surface S.

43) S is the cap cut from the paraboloid $z = \frac{5}{16} - 4x^2 - 4y^2$ by the cone $z = \sqrt{x^2 + y^2}$ 43) ____________

Sketch the vector field in the plane along with its horizontal and vertical components at a representative assortment of points on the circle $x^2 + y^2 = 4$.

44) $\mathbf{F} = -x\mathbf{i} - y\mathbf{j}$ 44) ____________

Solve the problem.

45) Consider a small region inside an elastic material such as gelatin. As the material "jiggles", this small region oscillates about its equilibrium position (x_o, y_o, z_o). The force that tends to restore the small region to its equilibrium position can be approximated as $\mathbf{F} = -k(x - x_o)\mathbf{i} - k(y - y_o)\mathbf{j} - k(z - z_o)\mathbf{k}$ Find a potential function f for this force field. 45) ____________

46) The velocity field **F** of a fluid is the spin field $\mathbf{F} = -\frac{ky}{\sqrt{x^2+y^2}}\mathbf{i} + \frac{kx}{\sqrt{x^2+y^2}}\mathbf{j}$. Following the smooth curve $y = f(x)$ from $(a, f(a))$ to $(b, f(b))$, show that the flux across the curve is $\int_C \mathbf{F}\cdot\mathbf{n}\,ds = k[(a^2 + (f(a))^2)^{1/2} - (b^2 + (f(b))^2)^{1/2}]$ 46) ____________

47) In thermodynamics, the differential form of the internal energy of a system is $dU = T\,dS - P\,dV$, where U is the internal energy, T is the temperature, S is the entropy, P is the pressure, and V is the volume of the system. The First Law of Thermodynamics asserts that dU is an exact differential. Using this information, justify the thermodynamic relation $\frac{\partial T}{\partial V} = -\frac{\partial P}{\partial S}$. 47) ____________

Find the equation for the plane tangent to the parametrized surface S at the point P.

48) S is the cylinder $\mathbf{r}(\theta, z) = 2\cos\theta\,\mathbf{i} + 2\sin\theta\,\mathbf{j} + z\mathbf{k}$; P is the point corresponding to $(\theta, z) = \left(\frac{\pi}{2}, 6\right)$ 48) ____________

Solve the problem.

49) For some inexact differential forms df, a function g(x, y, z) can be found such that dh = g(x, y, z) df is exact. When it exists, the function g(x,y,z) is called an "integrating factor". Show that $g(x, y, z) = \frac{yz}{x}$ is an integrating factor for the inexact differential $df = -\frac{1}{x}\,dx + \frac{1}{y}\,dy + \frac{1}{z}\,dz$. 49) ____________

Sketch the vector field in the plane along with its horizontal and vertical components at a representative assortment of points on the circle $x^2 + y^2 = 4$.

50) $\mathbf{F} = -x\mathbf{i} + y\mathbf{j}$ 50) ____________

Solve the problem.

51) Assuming C is a closed path , what is special about the integral $\int_C 7x^6y^5\,dx + 5x^7y^4\,dy$? Give reasons for your answer. 51) ____________

52) Find the values of a, b and c that make $\mathbf{F} = 8y^6z^4(ax^3 + bx^9)\mathbf{i} + 48y^5z^4(-9x^4 - 2x^{10})\mathbf{j} + cy^6z^3(-9x^4 - 2x^{10})\mathbf{k}$ a gradient field. 52) ____________

53) Consider the counter-clockwise integral $\int_C f(x,y)\,dx + g(x,y)\,dy$ where C is a closed path in a region where Green's Theorem applies. To evaluate the integral, should one use the flux-divergence form or the circulation-flow form of Green's theorem? Explain. 53) ____________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Calculate the work done by the force F along the path C.

54) $\mathbf{F} = xe^{8x^2}\mathbf{i} + e^{7y}\mathbf{j} + e^{5z}\mathbf{k}$; the path is $C_1 \cup C_2$ where C_1 is the straight line from (0, 0, 0) to (1, 1, 0) and C_2 is the straight line from (1, 1, 0) to (1, 1, 1) 54) _______

A) $W = \frac{e^8 - 1}{16} + \frac{e^7 - 1}{7} + \frac{e^5 - 1}{5}$ B) $W = \frac{e^8 - 1}{16} + \frac{e^7 - 1}{7}$

C) $W = \frac{e^8}{16} + \frac{e^7}{7} + \frac{e^5}{5}$ D) $W = \frac{e^8 - 1}{8} + \frac{2e^7 - 2}{7} + \frac{e^5 - 1}{5}$

Find the potential function f for the field F.

55) $\mathbf{F} = -\frac{1}{x}\mathbf{i} + \frac{1}{y}\mathbf{j} - \frac{1}{z}\mathbf{k}$ 55) _______

A) $f(x, y, z) = \frac{1}{x^2} - \frac{1}{y^2} + \frac{1}{z^2} + C$ B) $f(x, y, z) = \ln(y - x - z) + C$

C) $f(x, y, z) = \frac{1}{x^2y^2z^2} + C$ D) $f(x, y, z) = \ln\left(\frac{y}{xz}\right) + C$

Using Green's Theorem, compute the counterclockwise circulation of F around the closed curve C.

56) $\mathbf{F} = \sin 6y\mathbf{i} + \cos 10x\mathbf{j}$; C is the rectangle with vertices at (0, 0), $\left(\frac{\pi}{10}, 0\right)$, $\left(\frac{\pi}{10}, \frac{\pi}{6}\right)$, and $\left(0, \frac{\pi}{6}\right)$ 56) _______

A) $-\frac{2}{3}\pi$ B) $\frac{1}{3}\pi$ C) $-\frac{1}{3}\pi$ D) 0

Solve the problem.

57) The shape and density of a thin shell are indicated below. Find the radius of gyration about the z-axis. 57) _______

Shell: portion of the cone $x^2 + y^2 - z^2 = 0$ between z = 1 and z = 3
Density: $\delta = 3$

A) $R_z = \sqrt{5}$ B) $R_z = \sqrt{\frac{5}{2}}$ C) $R_z = 0$ D) $R_z = 120\sqrt{2}\pi$

Using Green's Theorem, calculate the area of the indicated region.

58) The circle $\mathbf{r}(t) = (7\cos t)\mathbf{i} + (7\sin t)\mathbf{j}$, $0 \le t \le 2\pi$ 58) _______

A) 7π B) 14π C) 49π D) 2π

Calculate the flux of the field F across the closed plane curve C.

59) $\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j}$; the curve C is the closed counterclockwise path around the triangle with vertices at (0, 0), (1, 0), and (0, 3) 59) _______

A) – 2 B) 0 C) 12 D) 4

Evaluate. The differential is exact.

60) $\int_{(0,0,0)}^{(1,1,1)} 9x^8y^7z^4\,dx + 7x^9y^6z^4\,dy + 4x^9y^7z^3\,dz$ 60) _______

A) 0 B) 1 C) 3 D) $\frac{1}{3}$

Evaluate the surface integral of g over the surface S.

61) S is the cylinder $y^2 + z^2 = 9$, $z \ge 0$ and $4 \le x \le 7$; $g(x, y, z) = z$ 61) _______

A) 54 B) 198 C) 18 D) 27

Evaluate the line integral of f(x,y) along the curve C.

62) $f(x, y) = y + x$, C: $x^2 + y^2 = 36$ in the first quadrant from (6, 0) to (0, 6) 62) _______

A) 72 B) 144 C) 36 D) 0

Solve the problem.

63) The shape and density of a thin shell are indicated below. Find the moment of inertia about the z-axis. 63) _______

Shell: upper hemisphere of $x^2 + y^2 + z^2 = 25$ cut by the plane z = 0
Density: $\delta = 5$

A) $I_z = 125\pi$ B) $I_z = \frac{12500}{3}\pi$ C) $I_z = \frac{625}{2}\pi$ D) $I_z = 1250\pi$

64) The shape and density of a thin shell are indicated below. Find the radius of gyration about the z-axis. 64) _______

Shell: upper hemisphere of $x^2 + y^2 + z^2 = 25$ cut by the plane z = 0
Density: $\delta = 1$

A) $R_z = \sqrt{10\pi}$ B) $R_z = \sqrt{\frac{50}{3}}$ C) $R_z = \sqrt{\frac{100}{3}}$ D) $R_z = 125\pi$

Use Stokes' Theorem to calculate the circulation of the field F around the curve C in the indicated direction.

65) $\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j} + y\mathbf{k}$; C: the counter-clockwise path around the perimeter of the rectangle in the x-y plane formed from the x-axis, y-axis , x = 1 and y = 4 65) _______

A) 16 B) 8 C) –16 D) – 8

Solve the problem.

66) The shape and density of a thin shell are indicated below. Find the moment of inertia about the z-axis. 66) _______

Shell: portion of the cone $x^2 + y^2 - z^2 = 0$ between z = 1 and z = 3
Density: $\delta = 2$

A) $I_z = 160$ B) $I_z = 80\sqrt{2}$ C) $I_z = 160\pi$ D) $I_z = 80\sqrt{2}\pi$

Evaluate the line integral along the curve C.

67) $\int_C \left(6x^2 + 9e^{9y} + \frac{1}{z+1}\right) ds$, C is the path from (0, 0, 0) to (1, 1, 1) given by: 67) _______

C_1: $\mathbf{r}(t) = t\mathbf{i}$, $0 \le t \le 1$
C_2: $\mathbf{r}(t) = \mathbf{i} + t\mathbf{j}$, $0 \le t \le 1$
C_3: $\mathbf{r}(t) = \mathbf{i} + \mathbf{j} + t\mathbf{k}$, $0 \le t \le 1$

A) $15 + 9e^9 + \ln 2$ B) $24 + 10e^9 + \ln 2$ C) $7 + e^9$ D) $5 + e^9$

Evaluate the surface integral of g over the surface S.

68) S is the plane $x + y + z = 3$ above the rectangle $0 \le x \le 5$ and $0 \le y \le 2$; $g(x,y,z) = 3z$ 68) _______

A) 240 B) $-15\sqrt{3}$ C) –60 D) $60\sqrt{3}$

Solve the problem.

69) The shape and density of a thin shell are indicated below. Find the moment of inertia about the z-axis. 69) _______

Shell: portion of the cone $x^2 + y^2 - z^2 = 0$ between $z = 3$ and $z = 4$

Density: $\delta = 3$

A) $I_z = 525\pi$ B) $I_z = \frac{525}{2}\sqrt{2}$ C) $I_z = \frac{525}{2}\sqrt{2}\pi$ D) $I_z = 525$

Find the divergence of the field F.

70) $\mathbf{F} = -2x^8\mathbf{i} + 7xy\mathbf{j} + 7xz\mathbf{k}$ 70) _______

A) $-16x^7 + 7y + 7z$ B) -2 C) $-16x^7 + 14x - 2$ D) $-16x^7 + 14x$

Find the flux of the vector field F across the surface S in the indicated direction.

71) $\mathbf{F} = xz\mathbf{i} + yz\mathbf{j} + \mathbf{k}$, S is the cap cut from the sphere $x^2 + y^2 + z^2 = 4$ by the plane $z = 1$, direction is outward 71) _______

A) $\frac{15}{2}\pi$ B) $-\frac{15}{2}\pi$ C) $\frac{3}{2}\pi$ D) 15π

Calculate the area of the surface S.

72) S is the cap cut from the paraboloid $z = \frac{9}{20} - 5x^2 - 5y^2$ by the cone $z = \sqrt{x^2 + y^2}$ 72) _______

A) $\frac{1}{150}[2\sqrt{2} + 1]$ B) $\frac{\pi}{75}[2\sqrt{2} - 1]$ C) $\frac{\pi}{150}[2\sqrt{2} - 1]$ D) $\frac{\pi}{75}[82\sqrt{82} - 1]$

Calculate the circulation of the field F around the closed curve C.

73) $\mathbf{F} = x^2y^3\mathbf{i} + x^2y^3\mathbf{j}$; curve C is the counterclockwise path around the rectangle with vertices at (0, 0), (3, 0), (3, 2), and (0, 2) 73) _______

A) 108 B) – 36 C) 0 D) –72

Find the flux of the curl of field F through the shell S.

74) $\mathbf{F} = -8z\mathbf{i} + 3x\mathbf{j} + 7y\mathbf{k}$; S is the portion of the cone $z = 3\sqrt{x^2 + y^2}$ below the plane $z = 2$ 74) _______

A) $\frac{8}{3}\pi$ B) $-\frac{4}{3}\pi$ C) $-\frac{8}{3}\pi$ D) $-\frac{2}{3}\pi$

Evaluate. The differential is exact.

75) $\int_{(0,0,0)}^{(1,1,1)} 8xe^{4x^2+8y^2+7z^2}\,dx + 16ye^{4x^2+8y^2+7z^2}\,dy + 14ze^{4x^2+8y^2+7z^2}\,dz$ 75) _______

A) 0 B) $e^{19} - 1$ C) $e^{19} - 3$ D) $e^4 + e^8 + e^7 - 1$

Use Stokes' Theorem to calculate the circulation of the field F around the curve C in the indicated direction.

76) $\mathbf{F} = 5y\mathbf{i} + 4x\mathbf{j} - 2z^3\mathbf{k}$; C: the portion of the plane $2x + 3y + 9z = 4$ in the first quadrant 76) _______

A) $\frac{4}{3}$ B) $-\frac{4}{3}$ C) 0 D) –1

Evaluate the line integral along the curve C.

77) $\int_C \frac{1}{x^2 + y^2 + z^2}\,ds$, C is the path given by: 77) _______

C_1: $\mathbf{r}(t) = (3 \cos t)\mathbf{i} + (3 \sin t)\mathbf{j}$ from (3, 0, 0) to (0, 3, 0)

C_2: $\mathbf{r}(t) = (3 \sin t)\mathbf{j} + (3 \cos t)\mathbf{k}$ from (0, 3, 0) to (0, 0, 3)

C_3: $\mathbf{r}(t) = (3 \sin t)\mathbf{i} + (3 \cos t)\mathbf{k}$ from (0, 0, 3) to (3, 0, 0)

A) 0 B) $-\frac{1}{2}\pi$ C) $\frac{\pi}{6}$ D) $\frac{1}{2}\pi$

Solve the problem.

78) The shape and density of a thin shell are indicated below. Find the radius of gyration about the z-axis. 78) _______

Shell: "nose" of the paraboloid $x^2 + y^2 = 2z$ cut by the plane $z = 3$

Density: $\delta = \frac{1}{\sqrt{x^2 + y^2 + 1}}$

A) $R_z = \frac{1}{3}\sqrt{18}$ B) $R_z = 144\pi$ C) $R_z = 3$ D) $R_z = \sqrt{3}$

Evaluate. The differential is exact.

79) $\int_{(0,0,0)}^{(5,2,8)} (2xy^2 - 2xz^2)\,dx + 2x^2y\,dy - 2x^2z\,dz$ 79) _______

A) –3000 B) 0 C) 1700 D) –1500

Evaluate the line integral along the curve C.

80) $\int_C \left(\frac{x^2 + y^2}{z^2}\right) ds$, C is the curve $\mathbf{r}(t) = (2 \sin 6t)\mathbf{i} + (2 \cos 6t)\mathbf{j} + 5t\mathbf{k}$, $2 \le t \le 4$ 80) _______

A) $\frac{169}{25}$ B) $\frac{1}{25}$ C) $\frac{13}{25}$ D) $-\frac{91}{400}$

Using Green's Theorem, compute the counterclockwise circulation of F around the closed curve C.

81) $\mathbf{F} = (-y - e^y \cos x)\mathbf{i} + (y - e^y \sin x)\mathbf{j}$; C is the right lobe of the lemniscate $r^2 = \cos 2\theta$ that lies in the first quadrant. 81) _______

A) 1 B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 0

Find the flux of the vector field F across the surface S in the indicated direction.

82) $\mathbf{F} = z\mathbf{k}$, S is the surface of the sphere $x^2 + y^2 + z^2 = 16$ in the first octant , direction away from the origin 82) _______

A) 16π B) 0 C) $\frac{64}{3}\pi$ D) $\frac{32}{3}\pi$

Find the gradient field F of the function f.

83) $f(x, y, z) = x^3e^{6x} + y^3z^6$ 83) ______

A) $\mathbf{F} = (3 + 6x)x^2e^{6x}\mathbf{i} + (x^3e^{6x} + 3y^2z^6)\mathbf{j} + (x^3e^{6x} + 6y^3z^5)\mathbf{k}$

B) $\mathbf{F} = (3 + 6x)x^2e^{6x}\mathbf{i} + 3y^2z^6\mathbf{j} + 6y^3z^5\mathbf{k}$

C) $\mathbf{F} = (3 + 6x)x^2e^{6x}\mathbf{i} + 3y^2\mathbf{j} + 6z^5\mathbf{k}$

D) $\mathbf{F} = (1 + x)x^2e^{6x}\mathbf{i} + y^2z^6\mathbf{j} + y^3z^5\mathbf{k}$

Calculate the circulation of the field F around the closed curve C.

84) $\mathbf{F} = xy^2\mathbf{i} + x^2y\mathbf{j}$; curve C is the counterclockwise path around $C_1 \cup C_2$: C_1: $\mathbf{r}(t)$ 9 cos t$\mathbf{i}$ + 9 sin t$\mathbf{j}$, $0 \le t \le \pi$ 84) ______
C_2: $\mathbf{r}(t) = t\mathbf{i}$, $-9 \le t \le 9$

A) 0 B) 162 C) 81 D) 9

Find the flux of the vector field F across the surface S in the indicated direction.

85) $\mathbf{F} = x^5y\mathbf{i} - z\mathbf{k}$; S is portion of the cone $z = 2\sqrt{x^2 + y^2}$ between z = 0 and z = 1; direction is outward 85) ______

A) $\frac{1}{6}\pi$ B) $-\frac{1}{12}$ C) $-\frac{1}{2}\pi$ D) $-\frac{1}{6}\pi$

86) $\mathbf{F} = 4x\mathbf{i} + 4y\mathbf{j} + 2\mathbf{k}$, S is the surface cut from the bottom of the paraboloid $z = x^2 + y^2$ by the plane z = 2, direction is outward 86) ______

A) 12π B) 56π C) -112π D) 72π

87) $\mathbf{F} = \frac{z^2}{25}\mathbf{k}$; S is the upper hemisphere of $x^2 + y^2 + z^2 = 25$; direction is outward 87) ______

A) $\frac{25}{2}\pi$ B) $-\frac{25}{2}$ C) $-\frac{25}{2}\pi$ D) $\frac{25}{2}$

Using Green's Theorem, compute the counterclockwise circulation of F around the closed curve C.

88) $\mathbf{F} = -\sqrt{x^2 + y^2}\mathbf{i} + \sqrt{x^2 + y^2}\mathbf{j}$; C is the region defined by the polar coordinate inequalities $4 \le r \le 9$ and $0 \le \theta \le \pi$ 88) ______

A) 97 B) 0 C) 45 D) 130

Find the required quantity given the wire that lies along the curve r and has density δ.

89) Moment of inertia I_z about the z-axis, where $\mathbf{r}(t) = (3 \sin 4t)\mathbf{i} + (3 \cos 4t)\mathbf{j} + e^{6t}\mathbf{k}$, $0 \le t \le 1$; $\delta(x, y, z) = z^2$ 89) ______

A) $I_z = \frac{1}{36}$ B) $I_z = \frac{1}{72}$

C) $I_z = \frac{1}{2}(e^{12} - 1)$ D) $I_z = \frac{1}{72}[(144 + 36e^{12})^{3/2} - (144 + 36)^{3/2}]$

Find the flux of the curl of field F through the shell S.

90) $\mathbf{F} = (z - y)\mathbf{i} + (x - z)\mathbf{j} + (z - x)\mathbf{k}$; S: $\mathbf{r}(r,\theta) = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j} + (1 - r^2)\mathbf{k}$, $0 \le r \le 1$ and $0 \le \theta \le 2\pi$ 90) ______

A) -4π B) 2π C) -2π D) 4π

Test the vector field F to determine if it is conservative.

91) $\mathbf{F} = \left(\frac{e^x + e^{-x}}{yz}\right)\mathbf{i} + \left(\frac{e^{-x} + e^x}{y^2z}\right)\mathbf{j} + \left(\frac{e^x + e^{-x}}{yz^2}\right)\mathbf{k}$ 91) ______

A) Not conservative B) Conservative

Calculate the work done by the force F along the path C.

92) $\mathbf{F} = -5z\mathbf{i} + 6x\mathbf{j} + 7y\mathbf{k}$; C: $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t\mathbf{k}$, $0 \le t \le 1$ 92) ______

A) W = 16 B) $W = \frac{8}{3}$ C) W = 4 D) W = 8

Find the flux of the vector field F across the surface S in the indicated direction.

93) $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z^3\mathbf{k}$; S is portion of the cone $z = 3\sqrt{x^2 + y^2}$ between z = 3 and z = 6; direction is outward 93) ______

A) $-\frac{802}{5}$ B) $-\frac{1604}{5}\pi$ C) $\frac{1604}{5}\pi$ D) $\frac{802}{5}\pi$

Calculate the area of the surface S.

94) S is the portion of the paraboloid $z = 3x^2 + 3y^2$ that lies between z = 2 and z = 4 94) ______

A) $\frac{\pi}{54}[65\sqrt{65} - 17\sqrt{17}]$ B) $\frac{1}{27}[65\sqrt{65} - 17\sqrt{17}]$

C) $\frac{\pi}{27}[65\sqrt{65} - 17\sqrt{17}]$ D) $\frac{\pi}{9}[65\sqrt{65} - 17\sqrt{17}]$

Using Green's Theorem, find the outward flux of F across the closed curve C.

95) $\mathbf{F} = -\frac{1}{4(x^2 + y^2)^2}\mathbf{i}$; C is the region defined by the polar coordinate inequalities $1 \le r \le 3$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ 95) ______

A) $\frac{26}{81}$ B) $\frac{52}{81}$ C) 0 D) $-\frac{52}{81}$

Using the Divergence Theorem, find the outward flux of F across the boundary of the region D.

96) $\mathbf{F} = 3xy^2\mathbf{i} + 3x^2y\mathbf{j} + 7xy\mathbf{k}$; D: the region cut from the solid cylinder $x^2 + y^2 \le 9$ by the planes z = 0 and z = 7 96) ______

A) $\frac{1701}{2}\pi$ B) 3402π C) 1701π D) $\frac{1701}{4}\pi$

Find the flux of the curl of field F through the shell S.

97) $\mathbf{F} = (x-y)\mathbf{i} + (x-z)\mathbf{j} + (y-z)\mathbf{k}$; S is the portion of the cone $z = 4\sqrt{x^2 + y^2}$ below the plane z = 3 97) ______

A) $-\frac{9}{8}\pi$ B) $\frac{9}{4}$ C) $\frac{9}{8}\pi$ D) $-\frac{9}{4}\pi$

Evaluate. The differential is exact.

98) $\int_{(0,0,0)}^{(\pi/7,\, \pi/24,\, \pi/8)} 7 \sin 7x\, dx + 6 \sec^2 6y\, dy - 4 \cos 4z\, dz$ 98) ______

A) 0 B) 4 C) 2 D) 1

Find the gradient field F of the function f.

99) $f(x, y, z) = x^7y^{10} + \frac{x^4}{z^8}$ 99) ______

A) $\mathbf{F} = \left(7x^6y^{10} + \frac{4x^3}{z^8}\right)\mathbf{i} + 10x^7y^9\mathbf{j} - \frac{8x^4}{z^9}\mathbf{k}$

B) $\mathbf{F} = 7x^6y^{10}\mathbf{i} + 10x^7y^9\mathbf{j} - \frac{8x^7}{z^9}\mathbf{k}$

C) $\mathbf{F} = (7x^6 + 4x^3)\mathbf{i} + 10y^9\mathbf{j} + \frac{8}{z^9}\mathbf{k}$

D) $\mathbf{F} = (7x^6 + 4x^3)\mathbf{i} + 10y^9\mathbf{j} - \frac{8}{z^9}\mathbf{k}$

Use Stokes' Theorem to calculate the circulation of the field F around the curve C in the indicated direction.

100) $\mathbf{F} = -4y^3\mathbf{i} + 4x^3\mathbf{j} + 7z^3\mathbf{k}$; C: the portion of the paraboloid $x^2 + y^2 = z$ cut by the cylinder $x^2 + y^2 = 4$ 100) ______

A) -96π B) -192π C) 192π D) 96π

Using the Divergence Theorem, find the outward flux of F across the boundary of the region D.

101) $\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j} + z\mathbf{k}$; D: the solid cube cut by the coordinate planes and the planes $x = 1$, $y = 1$, and $z = 1$ 101) ______

A) 3 B) 1 C) 2 D) 4

Evaluate the line integral of f(x,y) along the curve C.

102) $f(x, y) = x^2 + y^2$, C: $y = 4x + 2$, $0 \le x \le 3$ 102) ______

A) $79\sqrt{17}$ B) 237 C) $237\sqrt{17}$ D) $543\sqrt{17}$

Find the divergence of the field F.

103) $\mathbf{F} = \frac{y\mathbf{j} - x\mathbf{k}}{(y^2 + x^2)^{1/2}}$ 103) ______

A) $\frac{x^2}{(y^2 + x^2)^{3/2}}$ B) $\frac{x^2 - y^2}{(y^2 + x^2)^{3/2}}$ C) $\frac{-x^2}{(y^2 + x^2)^{3/2}}$ D) 0

Find the flux of the vector field F across the surface S in the indicated direction.

104) $\mathbf{F} = 7x\mathbf{i} + 7y\mathbf{j} + 7z\mathbf{k}$, S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant, direction away from the origin 104) ______

A) 0 B) $\frac{7}{2}\pi$ C) $\frac{7}{4}\pi$ D) $\frac{7}{3}\pi$

Solve the problem.

105) Find a field $\mathbf{G} = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$ in the xy-plane with the property that at any point $(a, b) \ne (0, 0)$, $\mathbf{G}$ is a vector of magnitude $\sqrt{a^2 + b^2}$ tangent to the circle $x^2 + y^2 = a^2 + b^2$ and pointing in the clockwise direction. 105) ______

A) $y\mathbf{i} - x\mathbf{j}$ B) $-y\mathbf{i} + x\mathbf{j}$ C) $x\mathbf{i} + y\mathbf{j}$ D) $x\mathbf{i} - y\mathbf{j}$

Using Green's Theorem, compute the counterclockwise circulation of F around the closed curve C.

106) $\mathbf{F} = (-4x + 4y)\mathbf{i} + (2x - 4y)\mathbf{j}$; C is the region bounded above by $y = -2x^2 + 112$ and below by $y = 5x^2$ in the first quadrant 106) ______

A) -872 B) 888 C) $\frac{3488}{3}$ D) $-\frac{1792}{3}$

Evaluate the line integral of f(x,y) along the curve C.

107) $f(x, y) = x$, C: $y = x^2$, $0 \le x \le \frac{\sqrt{15}}{2}$ 107) ______

A) $\frac{63}{8}$ B) $\frac{21}{4}$ C) 63 D) 21

Find the potential function f for the field F.

108) $\mathbf{F} = (y - z)\mathbf{i} + (x + 2y - z)\mathbf{j} - (x + y)\mathbf{k}$ 108) ______

A) $f(x, y, z) = xy + y^2 - xz - yz + C$

B) $f(x, y, z) = x + y^2 - xz - yz + C$

C) $f(x, y, z) = x(y + y^2) - xz - yz + C$

D) $f(x, y, z) = xy + y^2 - x - y + C$

Using Green's Theorem, find the outward flux of F across the closed curve C.

109) $\mathbf{F} = (x - y)\mathbf{i} + (x + y)\mathbf{j}$; C is the triangle with vertices at (0, 0), (7, 0), and (0, 3) 109) ______

A) 63 B) 0 C) 42 D) 21

Using the Divergence Theorem, find the outward flux of F across the boundary of the region D.

110) $\mathbf{F} = e^{yz}\mathbf{i} + 6y\mathbf{j} + 4z^2\mathbf{k}$; D: the solid sphere $x^2 + y^2 + z^2 \le 16$ 110) ______

A) 512π B) 1536π C) $\frac{256}{3}\pi$ D) 1536π

Evaluate the surface integral of the function g over the surface S.

111) $g(x,y,z) = x^2 + y^2 + z^2$; S is the surface of the cube formed from the coordinate planes and the planes $x = 1$, $y = 1$, and $z = 1$ 111) ______

A) 7 B) $\frac{7}{3}$ C) $\frac{5}{3}$ D) 5

Calculate the work done by the force F along the path C.

112) $\mathbf{F} = -\frac{1}{25x^2}\mathbf{i} + \frac{2z}{5x}\mathbf{j} + \frac{1}{50x^2}\mathbf{k}$; C: $\mathbf{r}(t) = \frac{\cos 5t}{5}\mathbf{i} + \frac{\sin 5t}{5}\mathbf{j} + 2t\mathbf{k}$, $0 \le t \le \frac{\pi}{20}$ 112) ______

A) $W = \frac{\sqrt{2}}{5} + \frac{1}{100}\pi^2$

B) $W = \frac{\sqrt{2} + 2}{5} + \frac{1}{200}\pi^2$

C) $W = \frac{\sqrt{2}}{5} + \frac{1}{200}\pi^2$

D) $W = \frac{\sqrt{2}}{10} + \frac{1}{200}\pi^2$

Using the Divergence Theorem, find the outward flux of F across the boundary of the region D.

113) $\mathbf{F} = (y-x)\mathbf{i} + (z-y)\mathbf{j} + (z-x)\mathbf{k}$; D: the region cut from the solid cylinder $x^2 + y^2 \le 4$ by the planes $z = 0$ and $z = 2$ 113) ______

A) -8π B) 8π C) -8 D) 0

114) $\mathbf{F} = z\mathbf{i} + xy\mathbf{j} + zy\mathbf{k}$; D: the solid cube cut by the coordinate planes and the planes $x = 1$, $y = 1$, and $z = 1$ 114) ______

A) $\frac{1}{2}$ B) 2 C) 1 D) $\frac{1}{4}$

Find the center of mass of the wire that lies along the curve r and has density δ.

115) $\mathbf{r}(t) = (-3 + 3t)\mathbf{i} + \mathbf{j} + 5t\mathbf{k}$, $0 \le t \le 1$; $\delta(x, y, z) = x + z^2$ 115) ______

A) $(-39, 0, 345)$ B) $\left(-\frac{39}{82}, 0, \frac{345}{82}\right)$ C) $\left(-\frac{39}{82}, 1, \frac{345}{82}\right)$ D) $\left(-\frac{13}{4}, \frac{41}{6}, 345\right)$

Find the potential function f for the field F.

116) $\mathbf{F} = (e^{y^2} - \sin x)\mathbf{i} + 2xye^{y^2}\mathbf{j} + \mathbf{k}$ 116) ______

A) $f(x, y, z) = \cos x + e^{y^2} + z + C$
B) $f(x, y, z) = \cos x + e^{y^2}(x + z) + C$
C) $f(x, y, z) = \cos x + xze^{y^2} + C$
D) $f(x, y, z) = \cos x + xe^{y^2} + z + C$

Calculate the work done by the force F along the path C.

117) $\mathbf{F} = \frac{y}{z}\mathbf{i} + \frac{x}{z}\mathbf{j} + \frac{x}{y}\mathbf{k}$; C: $\mathbf{r}(t) = t^8\mathbf{i} + t^7\mathbf{j} + t^5\mathbf{k}$, $0 \le t \le 1$ 117) ______

A) W = 20
B) W = 1
C) $W = \frac{7}{3}$
D) W = 0

Calculate the circulation of the field F around the closed curve C.

118) $\mathbf{F} = -\frac{3}{5}x^2y\mathbf{i} - \frac{3}{5}xy^2\mathbf{j}$; curve C is $\mathbf{r}(t) = 5\cos t\mathbf{i} + 5\sin t\mathbf{j}$, $0 \le t \le 2\pi$ 118) ______

A) – 3
B) 0
C) $-\frac{6}{5}$
D) – 6

Find the flux of the curl of field F through the shell S.

119) $\mathbf{F} = 2y\mathbf{i} + 3x\mathbf{j} + \cos(z)\mathbf{k}$; S: $\mathbf{r}(r,\theta) = 5\sin\phi\cos\theta\mathbf{i} + 5\sin\phi\sin\theta\mathbf{j} + 5\cos\phi\mathbf{k}$, $0 \le \theta \le 2\pi$ and $0 \le \phi \le \frac{\pi}{2}$ 119) ______

A) 25
B) -50π
C) 25π
D) 50π

Find the flux of the vector field F across the surface S in the indicated direction.

120) $\mathbf{F} = 8x\mathbf{i} + 8y\mathbf{j} + z\mathbf{k}$; S is portion of the plane $x + y + z = 5$ for which $0 \le x \le 1$ and $0 \le y \le 4$; direction is outward (away from origin) 120) ______

A) 80
B) 90
C) 180
D) – 50

Find the flux of the curl of field F through the shell S.

121) $\mathbf{F} = -2x^2y\mathbf{i} + 2xy^2\mathbf{j} + z^2\mathbf{k}$; S is the portion of the paraboloid $2 - x^2 - y^2 = z$ that lies above the x–y plane 121) ______

A) $\frac{16}{3}\pi$
B) 32
C) 32π
D) 4π

Using Green's Theorem, calculate the area of the indicated region.

122) The astroid $\mathbf{r}(t) = (9\cos^3 t)\mathbf{i} + (9\sin^3 t)\mathbf{j}$, $0 \le t \le 2\pi$ 122) ______

A) $\frac{243}{8}\pi$
B) 2π
C) $\frac{243}{16}\pi$
D) $\frac{243}{4}\pi$

Find the required quantity given the wire that lies along the curve r and has density δ.

123) Radius of gyration R_y about the y–axis, where $\mathbf{r}(t) = (5t\cos t)\mathbf{i} + \frac{10}{3}\sqrt{2}t^{3/2}\mathbf{j} + (5t\sin t)\mathbf{k}$, $-1 \le t \le 1$; 123) ______

$\delta(x, y, z) = 2$

A) $R_y = 125$
B) $R_y = 0$
C) $R_y = \frac{25}{2}$
D) $R_y = \frac{5}{3}\sqrt{3}$

Evaluate the line integral along the curve C.

124) $\int_C (y + z)\, ds$, C is the path from (0, 0, 0) to (5, –5, 1) given by: 124) ______

C_1: $\mathbf{r}(t) = 5t^2\mathbf{i} - 5t\mathbf{j}$, $0 \le t \le 1$
C_2: $\mathbf{r}(t) = 5\mathbf{i} - 5\mathbf{j} + (t-1)\mathbf{k}$, $1 \le t \le 2$

A) $-\frac{125}{12}\sqrt{5} - \frac{29}{12}$
B) $-\frac{13}{4}$
C) $-\frac{125}{12}\sqrt{5} + \frac{29}{12}$
D) $-\frac{59}{2}$

Find the surface area of the surface S.

125) S is the upper cap cut from the sphere $x^2 + y^2 + z^2 = 25$ by the cylinder $x^2 + y^2 = 16$ 125) ______

A) 10
B) -20π
C) 20π
D) 160π

Calculate the work done by the force F along the path C.

126) $\mathbf{F} = -7y\mathbf{i} + 7x\mathbf{j} + 2z^8\mathbf{k}$; C: $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j}$, $0 \le t \le 8$ 126) ______

A) W = 224
B) W = 56
C) W = 112
D) W = 0

Find the gradient field F of the function f.

127) $f(x, y, z) = \ln(x^4 + y^8 + z^8)$ 127) ______

A) $\mathbf{F} = \frac{4}{x^4}\mathbf{i} + \frac{8}{y^8}\mathbf{j} + \frac{8}{z^8}\mathbf{k}$

B) $\mathbf{F} = \frac{1}{x^4}\mathbf{i} + \frac{1}{y^8}\mathbf{j} + \frac{1}{z^8}\mathbf{k}$

C) $\mathbf{F} = \frac{4}{x}\ln(y^8 + z^8)\mathbf{i} + \frac{8}{y}\ln(x^4 + z^8)\mathbf{j} + \frac{8}{z}\ln(x^4 + y^8)\mathbf{k}$

D) $\mathbf{F} = \frac{4x^3}{x^4 + y^8 + z^8}\mathbf{i} + \frac{8x^7}{x^4 + y^8 + z^8}\mathbf{j} + \frac{8x^7}{x^4 + y^8 + z^8}\mathbf{k}$

Using the Divergence Theorem, find the outward flux of F across the boundary of the region D.

128) $\mathbf{F} = xy\mathbf{i} + y^2\mathbf{j} - 2yz\mathbf{k}$; D: the solid wedge cut from the first quadrant by the plane $y + z = 8$ and the parabolic cylinder $x = 16 - 25y^2$ 128) ______

A) $\frac{37376}{1875}$
B) $\frac{40448}{1875}$
C) $\frac{36352}{1875}$
D) $\frac{72704}{1875}$

Find the center of mass of the wire that lies along the curve r and has density δ.

129) $\mathbf{r}(t) = (2\cos 3t)\mathbf{i} + (2\sin 3t)\mathbf{j} + 6t\mathbf{k}$, $0 \le t \le 2\pi$; $\delta(x, y, z) = 1(1 + \sin 3t\cos 3t)$ 129) ______

A) $\left(0, 0, \frac{1}{2}(12\pi - 1)\right)$
B) $\left(0, 0, \frac{3}{2}(\pi + 1)\right)$
C) $\left(0, 0, \frac{3}{2}\pi\right)$
D) $(0, 0, 3\pi)$

Calculate the flow in the field F along the path C.

130) $\mathbf{F} = -\frac{zy}{4\sqrt{x^2 + y^2 + z^2}}\mathbf{i} + \frac{zx}{4\sqrt{x^2 + y^2 + z^2}}\mathbf{j} + \frac{z^3}{3\sqrt{x^2 + y^2 + z^2}}\mathbf{k}$, C is the curve 130) ______

$\mathbf{r}(t) = 4\cos 4t\mathbf{i} + 4\sin 4t\mathbf{j} + 3t\mathbf{k}$, $0 \le t \le 1$

A) 21
B) $\frac{61}{9}$
C) $\frac{61}{3}$
D) 0

Evaluate the work done between point 1 and point 2 for the conservative field F.

131) $\mathbf{F} = 4 \sin 4x \cos 9y \cos 5z\mathbf{i} + 9 \cos 4x \sin 9y \cos 5z\mathbf{j} + 5 \cos 4x \cos 9y \sin 5z\mathbf{k}$; $P_1(0, 0, 0)$, $P_2\left(\frac{1}{2}\pi, \frac{2}{9}\pi, \frac{\pi}{5}\right)$ 131) ______

A) W = –2 B) W = 2 C) W = 1 D) W = 0

Evaluate the line integral along the curve C.

132) $\int_C (y + z)\, ds$, C is the straight–line segment x = 0, y = 4 – t, z = t from (0, 4, 0) to (0, 0, 4) 132) ______

A) $16\sqrt{2}$ B) 16 C) 8 D) 0

Using Green's Theorem, compute the counterclockwise circulation of F around the closed curve C.

133) $\mathbf{F} = xy\mathbf{i} + x\mathbf{j}$; C is the triangle with vertices at (0, 0), (8, 0), and (0, 2) 133) ______

A) $-\frac{40}{3}$ B) $\frac{40}{3}$ C) $\frac{16}{3}$ D) 0

Find the flux of the vector field F across the surface S in the indicated direction.

134) $\mathbf{F} = -3\mathbf{i} + 10\mathbf{j} + 7\mathbf{k}$, S is the rectangular surface z = 0, $0 \le x \le 4$, and $0 \le y \le 7$, direction $\mathbf{k}$ 134) ______

A) 280 B) 196 C) 0 D) –84

Apply Green's Theorem to evaluate the integral.

135) $\oint_C (8y + x)\, dx + (y + 3x)\, dy$ 135) ______

C: The circle $(x - 4)^2 + (y - 6)^2 = 16$

A) –40 B) –400 C) -80π D) 80π

Calculate the area of the surface S.

136) S is the portion of the plane 2x + 3y + 6z = 3 that lies within the cylinder $x^2 + y^2 = 4$ 136) ______

A) $\frac{14}{3}\pi$ B) $\frac{14}{3}$ C) $\frac{28}{3}\pi$ D) $\frac{2}{3}\pi$

Find the center of mass of the wire that lies along the curve r and has density δ.

137) $\mathbf{r}(t) = (6t^2 - 2)\mathbf{i} + 5t\mathbf{k}$, $-1 \le t \le 1$; $\delta(x, y, z) = 4\sqrt{24x + 73}$ 137) ______

A) $\left(\frac{384}{365}, 0, 0\right)$ B) $\left(\frac{3072}{5}, 2, 0\right)$ C) $\left(\frac{3072}{5}, 0, 0\right)$ D) $\left(\frac{384}{365}, 2, 0\right)$

Solve the problem.

138) The shape and density of a thin shell are indicated below. Find the coordinates of the center of mass. 138) ______

Shell: portion of the sphere $x^2 + y^2 + z^2 = 100$ that lies in the first octant

Density: constant

A) $\left(\frac{10}{3}, \frac{10}{3}, \frac{10}{3}\right)$ B) $(5, 5, 5)$ C) $(10, 10, 10)$ D) $\left(\frac{5}{2}, \frac{5}{2}, \frac{5}{2}\right)$

Apply Green's Theorem to evaluate the integral.

139) $\oint_C (y^2 + 5)\, dx + (x^2 + 1)\, dy$ 139) ______

C: The triangle bounded by x = 0, x + y = 1, y = 0

A) 0 B) $\frac{2}{3}$ C) 4 D) –3

Solve the problem.

140) The shape and density of a thin shell are indicated below. Find the coordinates of the center of mass. 140) ______

Shell: cylinder $x^2 + z^2 = 49$ bounded by y = 0 and y = 2

Density: constant

A) $\left(0, 0, \frac{14}{\pi}\right)$ B) $(0, 1, 7)$ C) $(0, 1, 14)$ D) $\left(0, 1, \frac{14}{\pi}\right)$

Evaluate the line integral along the curve C.

141) $\int_C \left(\frac{2}{z}\right)^{1/3} ds$, C is the curve $\mathbf{r}(t) = (2t^3 \cos t)\mathbf{i} + (2t^3 \sin t)\mathbf{j} + 2t^3\mathbf{k}$, $0 \le t \le 3\sqrt{2}$ 141) ______

A) $36(4 - \sqrt{2})$ B) 0 C) $18(4 - \sqrt{2})$ D) $36(4 + \sqrt{2})$

Using Green's Theorem, compute the counterclockwise circulation of F around the closed curve C.

142) $\mathbf{F} = (x^2 + y^2)\mathbf{i} + (x - y)\mathbf{j}$; C is the rectangle with vertices at (0, 0), (4, 0), (4, 2), and (0, 2) 142) ______

A) 0 B) 24 C) –8 D) 8

143) $\mathbf{F} = (x - e^x \cos y)\mathbf{i} + (x + e^x \sin y)\mathbf{j}$; C is the lobe of the lemniscate $r^2 = \sin 2\theta$ that lies in the first quadrant. 143) ______

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) 1 D) 0

Evaluate the work done between point 1 and point 2 for the conservative field F.

144) $\mathbf{F} = 10xe^{5x^2-8y^2-2z^2}\mathbf{i} - 16ye^{5x^2-8y^2-2z^2}\mathbf{j} - 4ze^{5x^2-8y^2-2z^2}\mathbf{k}$; $P_1(0, 0, 0)$, $P_2(1, 1, 1)$ 144) ______

A) $W = e^5 + e^{-8} + e^{-2} - 1$ B) W = 0

C) $W = e^{-5}$ D) $W = e^{-5} - 1$

Find the surface area of the surface S.

145) S is the portion of the surface 3x + 4z = 4 that lies above the rectangle $7 \le x \le 10$ and $4 \le y \le 8$ in the x–y plane 145) ______

A) 102 B) 15 C) 75 D) 60

Solve the problem.

146) The shape and density of a thin shell are indicated below. Find the coordinates of the center of mass. 146) ______

Shell: cone $x^2 + y^2 - z^2 = 0$ between z = 3 and z = 4

Density: constant

A) $\left(0, 0, \frac{74}{21}\pi\right)$ B) $\left(0, 0, \frac{101}{39}\pi\right)$ C) $\left(0, 0, \frac{74}{21}\right)$ D) $\left(0, 0, \frac{101}{39}\right)$

Find the mass of the wire that lies along the curve r and has density δ.

147) $\mathbf{r}(t) = \left(\frac{\sqrt{45}}{2}t^2 - 9\right)\mathbf{i} + 6t\mathbf{j},\ 0 \le t \le 1;\ \delta = 3t$ 147) ______

A) 21 units B) $\frac{171}{10}$ units C) $\frac{57}{5}$ units D) $\frac{3}{2}$ units

Evaluate the line integral along the curve C.

148) $\int_C (xy + yz)\ ds$, C is the path from (1, 1, 0) to $(e^9, e^9, 1)$ given by: 148) ______

C_1: $\mathbf{r}(t) = e^{9t}\mathbf{i} + e^{9t}\mathbf{j},\ 0 \le t \le 1$

C_2: $\mathbf{r}(t) = e^9\mathbf{i} + e^9\mathbf{j} + 2t\mathbf{k},\ 0 \le t \le 1$

A) $\frac{\sqrt{2}}{3}(e^{27} - 1) + 2e^{18} + 4e^9$ B) $\frac{\sqrt{2}}{3}(e^{27} - 1) + 2e^{18} + 2e^9$

C) $\frac{\sqrt{2}}{3}(e^{27} - 1) + e^{18} + \frac{1}{2}e^9$ D) $\frac{\sqrt{2}}{3}(e^{27} - 1)$

Using Green's Theorem, calculate the area of the indicated region.

149) The area bounded above by $y = 3x^2$ and below by $y = 2x^3$ 149) ______

A) $\frac{27}{8}$ B) $\frac{27}{16}$ C) $\frac{27}{32}$ D) $\frac{27}{64}$

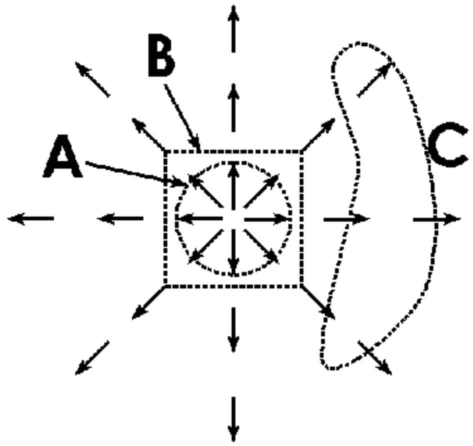
Find the flux of the vector field F across the surface S in the indicated direction.

150) $\mathbf{F} = 4\mathbf{i} + y^8z^7\mathbf{j} - y^7z^8\mathbf{k}$, S is the rectangular surface x = 0, $-6 \le y \le 6$, and $-8 \le z \le 8$, direction **i** 150) ______

A) 768 B) 192 C) 384 D) 0

Solve the problem.

151) The radial flow field of an incompressible fluid is shown below. Which of the closed paths would exhibit a non-zero flux? 151) ______



A) A, B, and C B) C

C) A and B D) None of the above

Find the potential function f for the field F.

152) $\mathbf{F} = \sec^2 x \sin^2 y\mathbf{i} + 2 \tan x \sin y \cos y\mathbf{j} + 5z^4\mathbf{k}$ 152) ______

A) $f(x, y, z) = \sec x \sin^2 y + z^5 + C$ B) $f(x, y, z) = \sec x \sin^2 y + z + C$

C) $f(x, y, z) = -\sec x \sin^2 y + z^5 + C$ D) $f(x, y, z) = \tan x \sin^2 y + z^5 + C$

Evaluate the surface integral of g over the surface S.

153) S is the dome $z = 2 - 5x^2 - 5y^2$ above z = 0; $g(x, y, z) = \frac{1}{\sqrt{100(x^2 + y^2) + 1}}$ 153) ______

A) $\frac{2}{5}\pi$ B) 8 C) 2π D) 8π

Use Stokes' Theorem to calculate the circulation of the field F around the curve C in the indicated direction.

154) $\mathbf{F} = 2y\mathbf{i} + y\mathbf{j} + z\mathbf{k}$; C: the counter-clockwise path around the boundary of the ellipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$ 154) ______

A) -20 B) 10π C) 20π D) -20π

Test the vector field F to determine if it is conservative.

155) $\mathbf{F} = \left(\frac{6x^5y^6}{z^7}\right)\mathbf{i} + \left(\frac{6x^6y^5}{z^7}\right)\mathbf{j} - \left(\frac{7x^6y^6}{z^8}\right)\mathbf{k}$ 155) ______

A) Not conservative B) Conservative

Find the divergence of the field F.

156) $\mathbf{F} = -6x^7\mathbf{i} + 3y^7\mathbf{j} + 8z^7\mathbf{k}$ 156) ______

A) 0
B) $-42x^6 + 21y^6 + 56z^6$
C) 35
D) $-6x^6 + 3y^6 + 8z^6$

Find the potential function f for the field F.

157) $\mathbf{F} = 2xe^{x^2+y^2}\mathbf{i} + 2ye^{x^2+y^2}\mathbf{j}$ 157) ______

A) $f(x, y, z) = e^{x^2} + e^{y^2} + C$
B) $f(x, y, z) = 2e^{x^2+y^2} + C$
C) $f(x, y, z) = e^{x^2+y^2} + C$
D) $f(x, y, z) = \frac{e^{x^2+y^2}}{2} + C$

Evaluate the line integral of f(x,y) along the curve C.

158) $f(x, y) = \frac{x^2}{\sqrt{1+4y}}$, C: $y = x^2$, $0 \le x \le 2$ 158) ______

A) 4
B) $\frac{8}{3}$
C) 8
D) 0

Evaluate the surface integral of the function g over the surface S.

159) $g(x, y, z) = \frac{y}{\sqrt{16y^2+1}}$; S is the surface of the parabolic cylinder $12y^2 + 6z = 48$ bounded by the planes $x = 0$, $x = 1$, $y = 0$, and $z = 0$ 159) ______

A) 12
B) 8
C) $\frac{1}{3}$
D) 2

Calculate the work done by the force F along the path C.

160) $\mathbf{F} = 4y\mathbf{i} + \sqrt{z}\mathbf{j} + (6x + 2z)\mathbf{k}$; C: $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$, $0 \le t \le 2$ 160) ______

A) $W = \frac{80}{3} + 20\sqrt{2}$
B) $W = 40 + 20\sqrt{2}$
C) $W = 0$
D) $W = \frac{80}{3} + \frac{16\sqrt{2}}{5}$

Evaluate the surface integral of the function g over the surface S.

161) $g(x, y, z) = x + z$; S is the surface of the wedge formed from the coordinate planes and the planes $x + z = 4$ and $y = 2$ 161) ______

A) $\frac{224}{3} + 32\sqrt{2}$
B) $\frac{176}{3} + 32\sqrt{2}$
C) $96 + 32\sqrt{2}$
D) $\frac{224}{3} + 8\sqrt{2}$

Find the flux of the vector field F across the surface S in the indicated direction.

162) $\mathbf{F} = 8x\mathbf{i} + 8y\mathbf{j} + 3\mathbf{k}$; S is "nose" of the paraboloid $z = 7x^2 + 7y^2$ cut by the plane $z = 4$; direction is outward 162) ______

A) $\frac{58}{7}$
B) 20π
C) -20π
D) $\frac{116}{7}\pi$

Find the surface area of the surface S.

163) S is the intersection of the plane $3x + 4y + 12z = 7$ and the cylinder with sides $y = 4x^2$ and $y = 8 - 4x^2$ 163) ______

A) $\frac{104}{3}$
B) $\frac{104}{9}$
C) $\frac{13}{9}$
D) $\frac{13}{18}$

Evaluate. The differential is exact.

164) $\int_{(1,1,1)}^{(2,4,2)} \frac{1}{x}\,dx + \frac{1}{y}\,dy + \frac{1}{z}\,dz$ 164) ______

A) $\ln \frac{16}{3}$
B) 0
C) ln 8
D) ln 16

Find the divergence of the field F.

165) $\mathbf{F} = \frac{y\mathbf{j} - z\mathbf{k}}{(y^2 + z^2)^{1/2}}$ 165) ______

A) $\frac{3(y^2 + z^2)}{(y^2 + z^2)^{3/2}}$
B) 0
C) $\frac{z^2 - y^2}{(y^2 + z^2)^{3/2}}$
D) $\frac{y^2 - z^2}{(y^2 + z^2)^{3/2}}$

Find the mass of the wire that lies along the curve r and has density δ.

166) C_1: $\mathbf{r}(t) = (6\cos t)\mathbf{i} + (6\sin t)\mathbf{j}$, $0 \le t \le \frac{\pi}{2}$; 166) ______

C_2: $\mathbf{r}(t) = 6\mathbf{j} + t\mathbf{k}$, $0 \le t \le 1$; $\delta = 7t^5$

A) $\frac{21}{160}\pi^5$ units
B) $\frac{7}{6}\left(\frac{1}{64}\pi^5 + 1\right)$ units
C) $\frac{21}{5}\pi^5$ units
D) $\frac{7}{6}\left(\frac{3}{32}\pi^6 + 1\right)$ units

Find the flux of the vector field F across the surface S in the indicated direction.

167) $\mathbf{F} = 4x\mathbf{i} + 4y\mathbf{j} + z^5\mathbf{k}$; S is portion of the cylinder $x^2 + y^2 = 16$ between $z = 0$ and $z = 4$; direction is outward 167) ______

A) 512π
B) 512
C) 256
D) -512π

168) $\mathbf{F} = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$, S is the surface of the rectangular prism formed from the coordinate planes and the planes $x = 5$, $y = 1$, and $z = 3$, direction is outward 168) ______

A) 440
B) 100
C) 220
D) 110

Find the mass of the wire that lies along the curve r and has density δ.

169) $\mathbf{r}(t) = (7\cos t)\mathbf{i} + (7\sin t)\mathbf{j} + 7t\mathbf{k}$, $0 \le t \le 2\pi$; $\delta = 8$ 169) ______

A) $784\pi\sqrt{2}$ units
B) $14\pi\sqrt{2}$ units
C) $112\pi\sqrt{2}$ units
D) 16π units

Find the gradient field F of the function f.

170) $f(x, y, z) = e^{x^7 + y^{10} + z^3}$ 170) ______

A) $\mathbf{F} = x^6e^{x^7 + y^{10} + z^3}\mathbf{i} + y^9e^{x^7 + y^{10} + z^3}\mathbf{j} + z^2e^{x^7 + y^{10} + z^3}\mathbf{k}$
B) $\mathbf{F} = 7x^6e^{x^7 + y^{10} + z^3}\mathbf{i} + 10y^9e^{x^7 + y^{10} + z^3}\mathbf{j} + 3z^2e^{x^7 + y^{10} + z^3}\mathbf{k}$
C) $\mathbf{F} = x^7e^{x^7 + y^{10} + z^3}\mathbf{i} + y^{10}e^{x^7 + y^{10} + z^3}\mathbf{j} + z^3e^{x^7 + y^{10} + z^3}\mathbf{k}$
D) $\mathbf{F} = 7x^6e^{x^7}\mathbf{i} + 10y^9e^{y^{10}}\mathbf{j} + 3z^2e^{z^3}\mathbf{k}$

171) $f(x, y, z) = \frac{xz + xy + yz}{xyz}$ 171) ______

A) $\mathbf{F} = -\frac{1}{x^2}\mathbf{i} - \frac{1}{y^2}\mathbf{j} - \frac{1}{z^2}\mathbf{k}$
B) $\mathbf{F} = -\frac{1}{x^2yz}\mathbf{i} - \frac{1}{xy^2z}\mathbf{j} - \frac{1}{xyz^2}\mathbf{k}$
C) $\mathbf{F} = \frac{1}{x^2yz}\mathbf{i} + \frac{1}{xy^2z}\mathbf{j} + \frac{1}{xyz^2}\mathbf{k}$
D) $\mathbf{F} = \frac{1}{x^2}\mathbf{i} + \frac{1}{y^2}\mathbf{j} + \frac{1}{z^2}\mathbf{k}$

Evaluate the surface integral of g over the surface S.

172) S is the portion of the cone $z = 3\sqrt{x^2 + y^2}$ between z = 0 and z = 1; g(x, y, z) = z − y 172) ______

A) $\frac{2}{27}\sqrt{10}\pi$ B) $\frac{2}{243}\sqrt{10}\pi$ C) $\frac{2}{27}\sqrt{10}$ D) $\frac{2}{81}\sqrt{10}\pi$

Find the flux of the curl of field F through the shell S.

173) $\mathbf{F} = e^x\mathbf{i} + e^y\mathbf{j} + 5xy\mathbf{k}$; S is the portion of the paraboloid $2 - x^2 - y^2 = z$ that lies above the x−y plane 173) ______

A) −80 B) 0 C) 80 D) −80π

Calculate the area of the surface S.

174) S is the portion of the sphere $x^2 + y^2 + z^2 = 81$ between $z = -\frac{9}{2}\sqrt{2}$ and $z = \frac{9}{2}\sqrt{2}$ 174) ______

A) $9\sqrt{2}\pi$ B) 162π C) $81\sqrt{2}\pi$ D) $162\sqrt{2}\pi$

Test the vector field F to determine if it is conservative.

175) $\mathbf{F} = \left(ze^{x+y} - \frac{1}{x}\right)\mathbf{i} + ze^{x+y}\mathbf{j} + e^{x+y}\mathbf{k}$ 175) ______

A) Not conservative B) Conservative

Solve the problem.

176) The base of the closed cubelike surface is the unit square in the xy−plane. The four sides lie in the planes x = 0, x = 1, y = 0, and y = 1. The top is an arbitrary smooth surface whose identity is unknown. Let $\mathbf{F} = x\mathbf{i} - 4y\mathbf{j} + (z + 7)\mathbf{k}$ and suppose the outward flux through the side parallel to the yz−plane is 1 and through the side parallel to the xz−plane is −5. What is the outward flux through the top? 176) ______

A) −4 B) 4
C) 0 D) Not enough information to determine

Find the potential function f for the field F.

177) $\mathbf{F} = \frac{1}{z}\mathbf{i} - 3\mathbf{j} - \frac{x}{z^2}\mathbf{k}$ 177) ______

A) $f(x, y, z) = \frac{x}{z} + C$
B) $f(x, y, z) = \frac{x}{z} - 3 + C$
C) $f(x, y, z) = \frac{x}{z} - 3y + C$
D) $f(x, y, z) = \frac{2x}{z} - 3y + C$

Calculate the flow in the field F along the path C.

178) $\mathbf{F} = e^{y^2}\mathbf{i} + \frac{1}{y}\mathbf{j} + 4\mathbf{k}$, C is the curve $\mathbf{r}(t) = 6t^2\mathbf{i} + 3t\mathbf{j} + (-2 - 5t)\mathbf{k}$, $1 \le t \le 4$ 178) ______

A) $\frac{2}{3}(e^{144} - e^9) + \ln 4 + 60$
B) $\frac{4}{3}(e^{144} - e^9) + \ln 4 - 60$
C) $\frac{2}{3}(e^{144} - e^9) + \ln 4 - 60$
D) $6(e^{144} - e^9) + \ln 4 - 60$

Find the gradient field F of the function f.

179) f(x, y, z) = z sin (x + y + z) 179) ______

A) $\mathbf{F} = -\cos x\mathbf{i} - \cos y\mathbf{j} + (\sin z - z\cos z)\mathbf{k}$
B) $\mathbf{F} = \cos x\mathbf{i} + \cos y\mathbf{j} + (\sin z + z\cos z)\mathbf{k}$
C) $\mathbf{F} = z\cos(x + y + z)\mathbf{i} + z\cos(x + y + z)\mathbf{j} + (\sin(x + y + z) + z\cos(x + y + z))\mathbf{k}$
D) $\mathbf{F} = -z\cos(x + y + z)\mathbf{i} - z\cos(x + y + z)\mathbf{j} + (\sin(x + y + z) - z\cos(x + y + z))\mathbf{k}$

Using Green's Theorem, find the outward flux of F across the closed curve C.

180) $\mathbf{F} = (-10x + 2y)\mathbf{i} + (9x - 5y)\mathbf{j}$; C is the region bounded above by $y = -5x^2 + 200$ and below by $y = 3x^2$ in the first quadrant 180) ______

A) 14,925 B) − 10000 C) − 10835 D) 14775

Find the mass of the wire that lies along the curve r and has density δ.

181) $\mathbf{r}(t) = 8\mathbf{i} + (5 - 3t)\mathbf{j} + 4t\mathbf{k}$, $0 \le t \le 2\pi$; $\delta = 8(1 + \sin 3t)$ 181) ______

A) $\frac{160}{3} + 80\pi$ units B) $\frac{80}{3} + 80\pi$ units C) 80π units D) 16π units

Find the surface area of the surface S.

182) S is the paraboloid $x^2 + y^2 - z = 0$ between the planes z = 0 and z = 6 182) ______

A) $\frac{31}{3}\pi$ B) $\frac{62}{3}\pi$ C) $\frac{124}{3}\pi$ D) 31π

Using Green's Theorem, find the outward flux of F across the closed curve C.

183) $\mathbf{F} = (-y - e^y\cos x)\mathbf{i} + (y - e^y\sin x)\mathbf{j}$; C is the right lobe of the lemniscate $r^2 = \cos 2\theta$ that lies in the first quadrant. 183) ______

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) 0

Calculate the work done by the force F along the path C.

184) $\mathbf{F} = t\mathbf{i} + \frac{1}{7}\mathbf{j} + \mathbf{k}$; C: $\mathbf{r}(t) = e^{7t}\mathbf{i} + e^{7t}\mathbf{j} + (-2t^2 + t)\mathbf{k}$, $-1 \le t \le 1$ 184) ______

A) $W = e^7 + e^{-7} + 2$ B) W = 2 C) $W = e^7 + e^{-7} - 2$ D) W = −2

Find the flux of the curl of field F through the shell S.

185) $\mathbf{F} = 4z\mathbf{i} - 7x\mathbf{j} + 3y\mathbf{k}$; S: $\mathbf{r}(r, \theta) = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j} + 6r\mathbf{k}$, $0 \le r \le 6$ and $0 \le \theta \le 2\pi$ 185) ______

A) 252π B) 0 C) −252 D) −252π

Find the potential function f for the field F.

186) $\mathbf{F} = -\left(\dfrac{xyz}{(1+x^2)^{3/2}}\right)\mathbf{i} + \left(\dfrac{z}{(1+x^2)^{1/2}}\right)\mathbf{j} + \left(\dfrac{y}{(1+x^2)^{1/2}}\right)\mathbf{k}$ 186) ______

A) $f(x, y, z) = \dfrac{1}{\sqrt{1+x^2}} + C$
B) $f(x, y, z) = -\dfrac{yz}{2\sqrt{1+x^2}} + C$
C) $f(x, y, z) = \dfrac{yz}{\sqrt{1+x^2}} + C$
D) $f(x, y, z) = \dfrac{yz}{2\sqrt{1+x^2}} + C$

Evaluate. The differential is exact.

187) $\displaystyle\int_{(0,0,0)}^{(\pi,\pi,\pi)} -2\sin x\cos x\,dx - \sin y\cos z\,dy - \cos y\sin z\,dz$ 187) ______

A) 1 B) –2 C) 0 D) 2

Evaluate the work done between point 1 and point 2 for the conservative field F.

188) $\mathbf{F} = 6x\mathbf{i} + 6y\mathbf{j} + 6z\mathbf{k}$; $P_1(5, 1, 5)$, $P_2(8, 5, 9)$ 188) ______

A) W = – 357 B) W = 0 C) W = 663 D) W = 357

Solve the problem.

189) Find a field $\mathbf{G} = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$ in the xy–plane with the property that at any point $(a, b) \neq (0, 0)$, **G** is a unit vector pointing away from the origin. 189) ______

A) $-\dfrac{x\mathbf{i} + y\mathbf{j}}{\sqrt{x^2+y^2}}$ B) $x\mathbf{i} + y\mathbf{j}$ C) $\dfrac{x\mathbf{i} - y\mathbf{j}}{\sqrt{x^2-y^2}}$ D) $\dfrac{x\mathbf{i} + y\mathbf{j}}{\sqrt{x^2+y^2}}$

Find the required quantity given the wire that lies along the curve r and has density δ.

190) Moment of inertia I_x about the x–axis, where $\mathbf{r}(t) = \dfrac{20}{3}\sqrt{2}t^{3/2}\mathbf{i} + (10t\cos t)\mathbf{j} + (10t\sin t)\mathbf{k}$, $0 \le t \le 1$; 190) ______

$\delta(x, y, z) = \dfrac{5}{y^2 + z^2}$

A) $I_x = 75$ B) $I_x = 100$ C) $I_x = 0$ D) $I_x = 50$

Find the surface area of the surface S.

191) S is the area cut from the plane $z = 6y$ by the cylinder $x^2 + y^2 = 36$ 191) ______

A) $36\sqrt{37}\pi$ B) $12\sqrt{37}\pi$ C) $36\sqrt{37}$ D) $6\sqrt{37}\pi$

Find the center of mass of the wire that lies along the curve r and has density δ.

192) $\mathbf{r}(t) = 4t\mathbf{i} + 8t\mathbf{j} + 6t^2\mathbf{k}$, $0 \le t \le 1$; $\delta(x, y, z) = \dfrac{x}{\sqrt{80 + 24z}}$ 192) ______

A) $(8, 16, 12)$ B) $\left(\dfrac{16}{3}, \dfrac{32}{3}, 8\right)$ C) $\left(\dfrac{8}{3}, \dfrac{16}{3}, 3\right)$ D) $(4, 8, 6)$

Solve the problem.

193) The shape and density of a thin shell are indicated below. Find the moment of inertia about the z–axis. 193) ______

Shell: "nose" of the paraboloid $x^2 + y^2 = 2z$ cut by the plane $z = 1$

Density: $\delta = \dfrac{1}{\sqrt{x^2 + y^2 + 1}}$

A) $I_z = \dfrac{8}{3}\pi$ B) $I_z = 2\pi$ C) $I_z = \dfrac{2}{3}\pi$ D) $I_z = 16\pi$

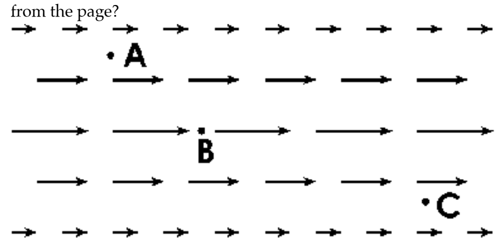
194) The shape and density of a thin shell are indicated below. Find the coordinates of the center of mass. 194) ______

Shell: "nose" of the paraboloid $x^2 + y^2 = 2z$ cut by the plane $z = 1$

Density: $\delta = \sqrt{x^2 + y^2 + 1}$

A) $\left(0, 0, \dfrac{4}{3}\right)$ B) $\left(0, 0, \dfrac{8}{3}\right)$ C) $\left(0, 0, \dfrac{7}{15}\right)$ D) $\left(0, 0, \dfrac{7}{12}\right)$

195) The flow **F** of a fluid in a plane is illustrated below. At which point or points would $\nabla \times \mathbf{F}$ point out from the page? 195) ______



A) A B) A and C C) B D) C

196) The shape and density of a thin shell are indicated below. Find the coordinates of the center of mass. 196) ______

Shell: "nose" of the paraboloid $x^2 + y^2 = 2z$ cut by the plane $z = 3$

Density: $\delta = \sqrt{x^2 + y^2 + 1}$

A) $\left(0, 0, \dfrac{96}{11}\right)$ B) $\left(0, 0, \dfrac{15}{8}\right)$ C) $(0, 0, 8)$ D) $\left(0, 0, \dfrac{9}{11}\right)$

Test the vector field F to determine if it is conservative.

197) $\mathbf{F} = -\cos x\cos y\mathbf{i} + \sin x\sin y\mathbf{j} - \sec^2 z\mathbf{k}$ 197) ______

A) Not conservative B) Conservative

Using the Divergence Theorem, find the outward flux of F across the boundary of the region D.

198) $\mathbf{F} = -16xz^7\mathbf{i} + 12yj + 2z^8\mathbf{k}$; D: the solid wedge cut from the first quadrant by the plane $z = 3y$ and the elliptic cylinder $x^2 + 16y^2 = 169$ 198) ______

A) $\dfrac{6591}{8}$ B) $\dfrac{6591}{4}$ C) $\dfrac{768}{169}$ D) $\dfrac{507}{4}$

Calculate the work done by the force F along the path C.

199) $\mathbf{F} = xy\mathbf{i} + 9\mathbf{j} + 6x\mathbf{k}$; C: $\mathbf{r}(t) = \cos 5t\mathbf{i} + \sin 5t\mathbf{j} + t\mathbf{k}$, $0 \le t \le \frac{\pi}{10}$ 199) ______

A) $W = 0$ B) $W = \frac{158}{15}$ C) $W = \frac{152}{15}$ D) $W = \frac{148}{15}$

Evaluate. The differential is exact.

200) $\int_{(1, 1, 1)}^{(5, 4, 9)} \frac{3x^2}{y^4}\,dx - \frac{4x^3}{y^5}\,dy + \frac{1}{z}\,dz$ 200) ______

A) $\frac{381}{256} + \ln 9$ B) $-\frac{131}{256} + \ln 9$ C) 0 D) $\frac{125}{256} + \ln 9$

Solve the problem.

201) Let $f(x,y) = \ln(x^2 + y^2)$. Which one of the following curves is a simple closed path in the domain of the function f? 201) ______

A) The rectangle with vertices at (–3, 2), (3, 2), (3, –2), and (–3, –2)
B) The unit circle $x^2 + y^2 = 1$
C) The unit circle $(x - 5)^2 + y^2 = 1$
D) The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Using the Divergence Theorem, find the outward flux of F across the boundary of the region D.

202) $\mathbf{F} = \sin y\mathbf{i} + xz\mathbf{j} + 4z\mathbf{k}$; D: the thick sphere $1 \le x^2 + y^2 + z^2 \le 4$ 202) ______

A) 112π B) $\frac{112}{3}\pi$ C) $\frac{28}{3}\pi$ D) 32π

Evaluate. The differential is exact.

203) $\int_{(1, 1, 1)}^{(2, 2, 2)} -\frac{3z^8}{x^4y^2}\,dx - \frac{2z^8}{x^3y^3}\,dy + \frac{8z^7}{x^3y^2}\,dz$ 203) ______

A) 1 B) 8 C) 5 D) 7

Find the gradient field F of the function f.

204) $f(x, y, z) = (x^{10}y^2 - y^6z^3)e^{-z^2}$ 204) ______

A) $\mathbf{F} = 10x^9y^2\mathbf{i} + (6y^5z^3 - 2x^{10}y)\mathbf{j} + (2z^2 - 3)y^6z^2\mathbf{k}$
B) $\mathbf{F} = 10x^9y^2e^{-z^2}\mathbf{i} + (2x^{10}y - 6y^5z^3)e^{-z^2}\mathbf{j} + (3 - 2z^2)y^6z^2e^{-z^2}\mathbf{k}$
C) $\mathbf{F} = 10x^9y^2e^{-z^2}\mathbf{i} + (2x^{10}y - 6y^5z^3)e^{-z^2}\mathbf{j} + y^2ze^{-z^2}[y^4z(2z^2 - 3) - 2x^{10}]\mathbf{k}$
D) $\mathbf{F} = 10x^9\mathbf{i} + (6y^5 - 2y)\mathbf{j} + 2z^4e^{-z^2}\mathbf{k}$

205) $f(x, y, z) = \ln\left(\frac{(x + y)^5}{z^3}\right) + z^{10}$ 205) ______

A) $\mathbf{F} = \frac{5}{x + y}\mathbf{i} + \frac{5}{x + y}\mathbf{j} + \left(10z^9 - \frac{3}{z}\right)\mathbf{k}$ B) $\mathbf{F} = \frac{5}{x + y}\mathbf{i} + \frac{5}{x + y}\mathbf{j} + \left(10z^9 + \frac{3}{z}\right)\mathbf{k}$
C) $\mathbf{F} = \frac{5}{x}\mathbf{i} + \frac{5}{y}\mathbf{j} + \left(10z^9 + \frac{3}{z}\right)\mathbf{k}$ D) $\mathbf{F} = \frac{5}{x}\mathbf{i} + \frac{5}{y}\mathbf{j} + \left(10z^9 - \frac{3}{z}\right)\mathbf{k}$

Evaluate the surface integral of g over the surface S.

206) S is the cap cut from the sphere $x^2 + y^2 + z^2 = 25$ by the cone $z = \sqrt{x^2 + y^2}$; $g(x, y, z) = \frac{xz^2}{125}$ 206) ______

A) 20π B) 0 C) 4π D) 10π

Using the Divergence Theorem, find the outward flux of F across the boundary of the region D.

207) $\mathbf{F} = 3x^3\mathbf{i} + 3y^3\mathbf{j} + 3z^3\mathbf{k}$; D: the thick sphere $9 \le x^2 + y^2 + z^2 \le 25$ 207) ______

A) $103{,}752\pi$ B) 2176π C) $\frac{103752}{5}\pi$ D) 882π

Test the vector field F to determine if it is conservative.

208) $\mathbf{F} = \left(\frac{7x^6(y - x^2)^7}{z}\right)\mathbf{i} + \left(\frac{4x^7(y - x^2)^6}{z}\right)\mathbf{j} - \left(\frac{x^7(y - x^2)^4}{z^2}\right)\mathbf{k}$ 208) ______

A) Conservative B) Not conservative

Solve the problem.

209) The shape and density of a thin shell are indicated below. Find the radius of gyration about the z–axis. 209) ______

Shell: portion of the cone $x^2 + y^2 - z^2 = 0$ between $z = 2$ and $z = 4$
Density: $\delta = 5$

A) $R_z = \sqrt{5}$ B) $R_z = \sqrt{10}$ C) $R_z = 0$ D) $R_z = 600\sqrt{2}\pi$

Using Green's Theorem, find the outward flux of F across the closed curve C.

210) $\mathbf{F} = \sin 5y\mathbf{i} + \cos 5x\mathbf{j}$; C is the rectangle with vertices at (0, 0), $\left(\frac{\pi}{5}, 0\right)$, $\left(\frac{\pi}{5}, \frac{\pi}{5}\right)$, and $\left(0, \frac{\pi}{5}\right)$ 210) ______

A) $\frac{2}{5}\pi$ B) $-\frac{2}{5}\pi$ C) 0 D) $-\frac{4}{5}\pi$

Using Green's Theorem, compute the counterclockwise circulation of F around the closed curve C.

211) $\mathbf{F} = \ln(x^2 + y^2)\mathbf{i} + \tan^{-1}\left(\frac{x}{y}\right)\mathbf{j}$; C is the region defined by the polar coordinate inequalities $4 \le r \le 6$ and $0 \le \theta \le \pi$ 211) ______

A) –4 B) 40 C) 0 D) 52

Find the required quantity given the wire that lies along the curve r and has density δ.

212) Radius of gyration I_x about the x–axis, where $\mathbf{r}(t) = (5 - 5t)\mathbf{i} + 5t\mathbf{j}$, $0 \le t \le 1$; $\delta(x, y, z) = 4$ 212) ______

A) $R_x = \frac{5}{3}\sqrt{3}$ B) $R_x = 0$ C) $R_x = \frac{500}{3}\sqrt{2}$ D) $R_x = 5$

Using Green's Theorem, find the outward flux of F across the closed curve C.

213) $\mathbf{F} = -\sqrt{x^2 + y^2}\mathbf{i} + \sqrt{x^2 + y^2}\mathbf{j}$; C is the region defined by the polar coordinate inequalities $5 \le r \le 6$ and $0 \le \theta \le \pi$ 213) ______

A) 0 B) 22 C) 61 D) 11

Use Stokes' Theorem to calculate the circulation of the field F around the curve C in the indicated direction.

214) $\mathbf{F} = 3y\mathbf{i} + 6x\mathbf{j} + z^3\mathbf{k}$; C: the counter–clockwise path around the perimeter of the triangle in the x–y plane formed from the x–axis, y–axis , and the line $y = 5 - 5x$ 214) ______

A) $\frac{3}{2}$ B) 15 C) – 15 D) $\frac{15}{2}$

Calculate the circulation of the field F around the closed curve C.

215) $\mathbf{F} = y^3\mathbf{i} + x^2\mathbf{j}$; curve C is the counterclockwise path around the triangle with vertices at (0, 0), (2, 0), and (0, 1) 215) ______

A) $\frac{11}{6}$ B) 0 C) $\frac{5}{6}$ D) $\frac{1}{2}$

Calculate the work done by the force F along the path C.

216) $\mathbf{F} = \frac{1}{x+8}\mathbf{i} + \mathbf{j} + 3\mathbf{k}$; C: $\mathbf{r}(t) = t^9\mathbf{i} + t^9\mathbf{j} + t\mathbf{k}$, $0 \le t \le 1$ 216) ______

A) $W = \ln\left(\frac{9}{72}\right) + 4$ B) $W = \ln\left(\frac{9}{8}\right)$ C) $W = \ln\left(\frac{9}{8}\right) + 12$ D) $W = \ln\left(\frac{9}{8}\right) + 4$

Evaluate. The differential is exact.

217) $\int_{(0,0,0)}^{(1,1,1)} (-8x - 5x^4y^6)\,dx - 6x^5y^5\,dy - 40z^4\,dz$ 217) ______

A) –11 B) 0 C) –12 D) –13

Find the surface area of the surface S.

218) S is the paraboloid $x^2 + y^2 - z = 0$ below the plane $z = 12$ 218) ______

A) $\frac{57}{2}\pi$ B) $\frac{171}{2}\pi$ C) $\frac{343}{6}\pi$ D) 57π

Find the gradient field F of the function f.

219) $f(x, y, z) = \frac{x^2 + y^2 + z^2}{x^7}$ 219) ______

A) $\mathbf{F} = \frac{9x^2 + 7y^2 + 7z^2}{x^8}\mathbf{i} + \frac{2y}{x^7}\mathbf{j} + \frac{2z}{x^7}\mathbf{k}$ B) $\mathbf{F} = \frac{7}{x^8}(x^2 + y^2 + z^2)\mathbf{i} + \frac{2y}{x^7}\mathbf{j} + \frac{2z}{x^7}\mathbf{k}$

C) $\mathbf{F} = \frac{-5}{x^6}\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$ D) $\mathbf{F} = \frac{-5x^2 - 7y^2 - 7z^2}{x^8}\mathbf{i} + \frac{2y}{x^7}\mathbf{j} + \frac{2z}{x^7}\mathbf{k}$

Solve the problem.

220) The shape and density of a thin shell are indicated below. Find the moment of inertia about the z–axis. 220) ______

Shell: upper hemisphere of $x^2 + y^2 + z^2 = 25$ cut by the plane $z = 0$

Density: $\delta = 5$

A) $I_z = \frac{625}{2}\pi$ B) $I_z = 125\pi$ C) $I_z = \frac{12500}{3}\pi$ D) $I_z = 1250\pi$

Calculate the flux of the field F across the closed plane curve C.

221) $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$; the curve C is the closed counterclockwise path around the rectangle with vertices at (0, 0), (9, 0), (9, 2), and (0, 2) 221) ______

A) 85 B) 0 C) 36 D) 77

Find the potential function f for the field F.

222) $\mathbf{F} = -\left(\frac{x}{(x^2 + y^2 + z^2)^{3/2}}\right)\mathbf{i} - \left(\frac{y}{(x^2 + y^2 + z^2)^{3/2}}\right)\mathbf{j} - \left(\frac{z}{(x^2 + y^2 + z^2)^{3/2}}\right)\mathbf{k}$ 222) ______

A) $f(x, y, z) = \frac{2}{5(x^2 + y^2 + z^2)^{5/2}} + C$ B) $f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}} + C$

C) $f(x, y, z) = -\frac{1}{\sqrt{x^2 + y^2 + z^2}} + C$ D) $f(x, y, z) = \frac{3}{\sqrt{x^2 + y^2 + z^2}} + C$

Using Green's Theorem, calculate the area of the indicated region.

223) The area bounded above by $y = 2x$ and below by $y = 4x^2$ 223) ______

A) $\frac{1}{6}$ B) $\frac{1}{12}$ C) $\frac{5}{24}$ D) $\frac{1}{24}$

Calculate the flow in the field F along the path C.

224) $\mathbf{F} = 3\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}$; C is the curve $\mathbf{r}(t) = 3\cos 8t\,\mathbf{i} + 3\sin 8t\,\mathbf{j} + 5t\mathbf{k}$, $0 \le t \le \frac{1}{2}\pi$ 224) ______

A) $60 + 120\pi$ B) 15π C) $18 + 120\pi$ D) $24 + 120\pi$

Solve the problem.

225) Find values for a, b, and c so that $\nabla \times \mathbf{F} = \mathbf{0}$ for $\mathbf{F} = ax\mathbf{i} + by\mathbf{j} + cz\mathbf{k}$. 225) ______

A) $\nabla \times \mathbf{F} = \mathbf{0}$ for any a, b, and c.

B) a = 1, b = 1, c = 1

C) a = 2, b = –1, c = –1

D) There is no possible way to make $\nabla \times \mathbf{F} = \mathbf{0}$.

Calculate the area of the surface S.

226) S is the portion of the cylinder $x^2 + y^2 = 4$ that lies between $z = 4$ and $z = 5$ 226) ______

A) 18π B) 36π C) 4π D) 2π

Calculate the circulation of the field F around the closed curve C.

227) $\mathbf{F} = (-x - y)\mathbf{i} + (x + y)\mathbf{j}$, curve C is the counterclockwise path around the circle with radius 2 centered at (5, 6) 227) ______

A) 16π B) 8π C) $8(1 + \pi)$ D) $8(1 + \pi) + 88$

Find the divergence of the field F.

228) $\mathbf{F} = 30xz^5\mathbf{i} + 9y\mathbf{j} - 5z^6\mathbf{k}$ 228) ______

A) $30z^5 + 9$ B) 9 C) $60z^5 + 9$ D) $60z^5$

Calculate the flux of the field F across the closed plane curve C.

229) $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$; the curve C is the counterclockwise path around the circle $x^2 + y^2 = 16$ 229) ______

A) 0 B) 8π C) 32π D) 64π

Find the flux of the vector field F across the surface S in the indicated direction.

230) $\mathbf{F} = x^5y^4\mathbf{i} + y\mathbf{j} - z\mathbf{k}$; S is the portion of the parabolic cylinder $z = 1 - y^2$ for which $z \geq 0$ and $2 \leq x \leq 3$; direction is outward (away from the x–y plane) 230) ______

A) –2 B) 2 C) 0 D) 1

Find the required quantity given the wire that lies along the curve r and has density δ.

231) Radius of gyration R_z about the z–axis, where $\mathbf{r}(t) = 4e^{10t}\mathbf{i} + 2e^{10t}\mathbf{j} + 6e^{10t}\mathbf{k}$, $0 \leq t \leq 1$; $\delta = e^{-20t}$ 231) ______

A) $R_z = \sqrt{\dfrac{20\left(e^{10} - 1\right)}{e^{-10} - 1}}$ B) $R_z = \sqrt{\dfrac{20\left(e^{10} - 1\right)}{1 - e^{-10}}}$

C) $R_z = 2\sqrt{5}$ D) $R_z = 0$

Find the surface area of the surface S.

232) S is the portion of the surface $6\sqrt{12}x + 6y^2 - 12 \ln y - 12z = 0$ that lies above the rectangle $0 \leq x \leq 8$ and $1 \leq y \leq 7$ in the x–y plane 232) ______

A) 8[24 + ln 7] B) $\frac{4}{3}$[24 + ln 7] C) 16[24 + ln 7] D) 48[24 + ln 7]

Test the vector field F to determine if it is conservative.

233) $\mathbf{F} = x^4y^4z^4\mathbf{i} + x^4y^4z^4\mathbf{j} + z^4\mathbf{k}$ 233) ______

A) Not conservative B) Conservative

Calculate the circulation of the field F around the closed curve C.

234) $\mathbf{F} = xy\mathbf{i} + 6\mathbf{j}$, curve C is $\mathbf{r}(t) = 3 \cos t\mathbf{i} + 3 \sin t\mathbf{j}$, $0 \leq t \leq 2\pi$ 234) ______

A) 18 B) 12 C) 0 D) 6

Evaluate the surface integral of g over the surface S.

235) S is the hemisphere $x^2 + y^2 + z^2 = 5$, $z \geq 0$; $g(x,y,z) = z^2$ 235) ______

A) $\frac{50}{3}\pi$ B) 100π C) $\frac{20}{3}\pi$ D) $\frac{25}{3}\pi$

Evaluate the line integral of f(x,y) along the curve C.

236) $f(x, y) = y^2 + x^2$, C: the perimeter of the circle $x^2 + y^2 = 16$ 236) ______

A) 64π B) 8π C) 128π D) 32π

Using Green's Theorem, find the outward flux of F across the closed curve C.

237) $\mathbf{F} = \ln(x^2 + y^2)\,\mathbf{i} + \tan^{-1}\left(\dfrac{x}{y}\right)\mathbf{j}$; C is the region defined by the polar coordinate inequalities $1 \leq r \leq 8$ and $0 \leq \theta \leq \pi$ 237) ______

A) 65 B) 63 C) 0 D) 126

Find the flux of the curl of field F through the shell S.

238) $\mathbf{F} = 4x^2y\mathbf{i} - 4xy^2\mathbf{j} + \ln z\mathbf{k}$; S: $\mathbf{r}(r, \theta) = r \cos \theta\mathbf{i} + r \sin \theta\mathbf{j} + 3r\mathbf{k}$, $0 \leq r \leq 3$ and $0 \leq \theta \leq 2\pi$ 238) ______

A) – 72 B) 162π C) -72π D) 72

Solve the problem.

239) Find the outward flux of a constant vector field $\mathbf{F} = \mathbf{C}$ across any closed surface to which the Divergence Theorem applies. 239) ______

A) 0 B) 1

C) $|\mathbf{F}|$ D) Not enough information to determine

Find the gradient field F of the function f.

240) $f(x, y, z) = \left(\dfrac{x + y}{y + z}\right)^3$ 240) ______

A) $\mathbf{F} = 3\left(\dfrac{x + y}{y + z}\right)^2\mathbf{i} + \dfrac{3(x + y)^2(z + x)}{(y + z)^4}\mathbf{j} - 3\dfrac{(x + y)^3}{(y + z)^4}\mathbf{k}$

B) $\mathbf{F} = \dfrac{3(x + y)^2}{(y + z)^3}\mathbf{i} + \dfrac{3(x + y)^2(z - x)}{(y + z)^4}\mathbf{j} - 3\dfrac{(x + y)^3}{(y + z)^4}\mathbf{k}$

C) $\mathbf{F} = 3\left(\dfrac{x + y}{y + z}\right)^2\mathbf{i} + \dfrac{3(x + y)^2(z + x)}{(y + z)^4}\mathbf{j} + 3\dfrac{(x + y)^3}{(y + z)^4}\mathbf{k}$

D) $\mathbf{F} = 3\left(\dfrac{x + y}{y + z}\right)^2\mathbf{i} + \dfrac{3(x + y)^2(z - x)}{(y + z)^4}\mathbf{j} + 3\dfrac{(x + y)^3}{(y + z)^4}\mathbf{k}$

Find the surface area of the surface S.

241) S is the cap cut from the sphere $x^2 + y^2 + z^2 = 49$ by the cone $z = 4\sqrt{x^2 + y^2}$ 241) ______

A) $49\left(1 - \dfrac{4\sqrt{17}}{17}\right)$ B) $49\left(\dfrac{4\sqrt{17}}{17} - 1\right)\pi$ C) $98\left(1 - \dfrac{4\sqrt{17}}{17}\right)\pi$ D) $49\left(1 - \dfrac{4\sqrt{17}}{17}\right)\pi$

Find the flux of the curl of field F through the shell S.

242) $\mathbf{F} = 8y\mathbf{i} + 5z\mathbf{j} + -2x\mathbf{k}$; S: $\mathbf{r}(r, \theta) = r \cos \theta\mathbf{i} + r \sin \theta\mathbf{j} + (4 - r^2)\mathbf{k}$, $0 \leq r \leq 2$ and $0 \leq \theta \leq 2\pi$ 242) ______

A) –32 B) 32π C) -32π D) -16π

Calculate the flow in the field F along the path C.

243) $\mathbf{F} = y^2\mathbf{i} + z\mathbf{j} + x\mathbf{k}$; C is the curve $\mathbf{r}(t) = (3 + 2t)\mathbf{i} + 4t\mathbf{j} - 4t\mathbf{k}$, $0 \leq t \leq 1$ 243) ______

A) $\frac{32}{3}$ B) 68 C) $-\frac{40}{3}$ D) –4

Solve the problem.

244) The shape and density of a thin shell are indicated below. Find the coordinates of the center of mass. 244) ______

Shell: cylinder $x^2 + z^2 = 25$ bounded by y = 0 and y = 3

Density: constant

A) $\left(0, \frac{3}{2}, 10\right)$ B) $\left(0, 0, \frac{10}{\pi}\right)$ C) $\left(0, \frac{3}{2}, \frac{10}{\pi}\right)$ D) $\left(0, \frac{3}{2}, 5\right)$

Find the potential function f for the field F.

245) $\mathbf{F} = 4x^3y^{10}z^8\mathbf{i} + 10x^4y^9z^8\mathbf{j} + 8x^4y^{10}z^7\mathbf{k}$ 245) ______

A) $f(x, y, z) = x^4y^{10}z^8 + 10x^4y^9z^8 + 8x^4y^{10}z^7 + C$

B) $f(x, y, z) = \frac{x^4y^{10}z^8}{320}$

C) $f(x, y, z) = x^4y^{10}z^8 + C$

D) $f(x, y, z) = x^{12}y^{30}z^{24} + C$

Use Stokes' Theorem to calculate the circulation of the field F around the curve C in the indicated direction.

246) $\mathbf{F} = 3x\mathbf{i} + 2x\mathbf{j} + 7z\mathbf{k}$; C: the cap cut from the upper hemisphere $x^2 + y^2 + z^2 = 16$ $(z \ge 0)$ by the cylinder $x^2 + y^2 = 4$ 246) ______

A) 2π B) 8π C) 3π D) 4π

Evaluate. The differential is exact.

247) $\int_{(0, 0, 0)}^{(3, 8, 9)} (10x + 1)e^{10x}\, dx + z\, dy + y\, dz$ 247) ______

A) $3e^{30} + 73$ B) $3e^{30} + 17$ C) $3e^{30} + 72$ D) 0

Solve the problem.

248) The shape and density of a thin shell are indicated below. Find the radius of gyration about the z-axis. 248) ______

Shell: "nose" of the paraboloid $x^2 + y^2 = 2z$ cut by the plane $z = 3$

Density: $\delta = \frac{1}{\sqrt{x^2 + y^2 + 1}}$

A) $R_z = 3$ B) $R_z = \frac{1}{3}\sqrt{18}$ C) $R_z = 144\pi$ D) $R_z = \sqrt{3}$

Using Green's Theorem, calculate the area of the indicated region.

249) The area bounded above by $y = 7$ and below by $y = \frac{7}{81}x^2$ 249) ______

A) 84 B) 0 C) 42 D) 168

Calculate the area of the surface S.

250) S is the lower portion of the sphere $x^2 + y^2 + z^2 = 64$ cut by the cone $z = \sqrt{x^2 + y^2}$ 250) ______

A) $64\left(\frac{\sqrt{2}}{2} + 1\right)$ B) $16\left(\frac{\sqrt{2}}{2} + 1\right)\pi$ C) $64\left(\frac{\sqrt{2}}{2} + 1\right)\pi$ D) $128\left(\frac{\sqrt{2}}{2} + 1\right)\pi$

Evaluate the surface integral of the function g over the surface S.

251) $g(x, y, z) = x^2y^2z^2$; S is the surface of the rectangular prism formed from the planes $x = \pm 1$, $y = \pm 2$, and $z = \pm 1$ 251) ______

A) $\frac{20}{3}$ B) $\frac{4}{9}$ C) $\frac{20}{9}$ D) $\frac{40}{9}$

Find the flux of the curl of field F through the shell S.

252) $\mathbf{F} = x^3\mathbf{i} + 3x\mathbf{j} + 6\mathbf{k}$; S is the upper hemisphere of $x^2 + y^2 + z^2 = 100$ 252) ______

A) 100π B) 300π C) π D) 100

Apply Green's Theorem to evaluate the integral.

253) $\oint_C (3y\, dx + 4y\, dy)$ 253) ______

C: The boundary of $0 \le x \le \pi$, $0 \le y \le \sin x$

A) -2 B) 1 C) 0 D) 2

Find the flux of the curl of field F through the shell S.

254) $\mathbf{F} = (3 - y)\mathbf{i} + (1 + x)\mathbf{j} + z^2\mathbf{k}$; S is the upper hemisphere of $x^2 + y^2 + z^2 = 64$ 254) ______

A) 128π B) 2π C) -4π D) 256π

Calculate the flow in the field F along the path C.

255) $\mathbf{F} = (x - y)\mathbf{i} - (x^2 + y^2)\mathbf{j}$; C is curve from (3, 0) to (–3, 0) on the upper half of the circle $x^2 + y^2 = 9$ 255) ______

A) $-\frac{9}{2}\pi$ B) $\frac{9\pi - 1}{4}$ C) 9π D) $\frac{9}{2}\pi$

Evaluate the line integral along the curve C.

256) $\int_C (xz + y^2)\, ds$, C is the curve $\mathbf{r}(t) = (-7 - 2t)\mathbf{i} + 2t\mathbf{j} - 1t\mathbf{k}$, $0 \le t \le 1$ 256) ______

A) $-\frac{9}{2}$ B) $\frac{11}{2}$ C) $\frac{33}{2}$ D) $-\frac{3}{2}$

Find the required quantity given the wire that lies along the curve r and has density δ.

257) Radius of gyration R_x about the x–axis, where $\mathbf{r}(t) = (3 \sin t)\mathbf{j} + (3 \cos t)\mathbf{k}$, $0 \le t \le 1$; $\delta = 7e^{-2t}$ 257) ______

A) $R_x = \frac{189}{2}(1 - e^{-2})$ B) $R_x = -3$

C) $R_x = 3$ D) $R_x = \frac{189}{2}(e^{-2} - 1)$

Test the vector field F to determine if it is conservative.

258) $\mathbf{F} = 6x^6y^7\mathbf{i} + \left(7x^6y^6 + \frac{z^7}{y^2}\right)\mathbf{j} - \frac{7z^6}{y}\mathbf{k}$ 258) ______

A) Not conservative B) Conservative

259) $\mathbf{F} = xy\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ 259) ______

A) Not conservative B) Conservative

Calculate the flux of the field F across the closed plane curve C.

260) $\mathbf{F} = e^{5x}\mathbf{i} + e^{2y}\mathbf{j}$; the curve C is the closed counterclockwise path around the triangle with vertices at (0, 0), (4, 0), and (0, 3) 260) ______

A) $\frac{3}{20}e^{20}(1 - e^{-20}) + \frac{2}{3}(e^6 - 1) - 7$ B) $\frac{3}{4}e^{20}(e^{-4} - 1) + \frac{2}{3}(1 - e^6) - 7$

C) $\frac{3}{4}e^{20}(1 - e^{-4}) + \frac{2}{3}(e^6 - 1) + 7$ D) 0

Find the flux of the vector field F across the surface S in the indicated direction.

261) $\mathbf{F} = 5x^2\mathbf{j} - z^2\mathbf{k}$; S is the portion of the parabolic cylinder $y = 6x^2$ for which $0 \le z \le 4$ and $-1 \le x \le 1$; direction is outward (away from the y–z plane) 261) ______

A) -40 B) $-\frac{40}{3}$ C) 40 D) $\frac{40}{3}$

Using Green's Theorem, find the outward flux of F across the closed curve C.

262) $\mathbf{F} = (x^2 + y^2)\mathbf{i} + (x - y)\mathbf{j}$; C is the rectangle with vertices at (0, 0), (7, 0), (7, 2), and (0, 7) 262) ______

A) -14 B) 84 C) 112 D) 42

Calculate the flux of the field F across the closed plane curve C.

263) $\mathbf{F} = y^3\mathbf{i} + x^2\mathbf{j}$; the curve C is the closed counterclockwise path formed from the semicircle $\mathbf{r}(t) = 3\cos t\mathbf{i} + 3\sin t\mathbf{j}$, $0 \le t \le \pi$, and the straight line segment from (–3, 0) to (3, 0) 263) ______

A) 12 B) -6 C) 6 D) 0

Evaluate the surface integral of g over the surface S.

264) S is the parabolic cylinder $y = 5x^2$, $0 \le x \le 4$ and $0 \le z \le 3$; $g(x, y, z) = 3x$ 264) ______

A) $\frac{3}{100}\left(1601\sqrt{1601} + 1\right)$ B) $\frac{9}{100}\left(1601\sqrt{1601} - 1\right)$

C) $\frac{3}{100}\left(1601\sqrt{1601} - 1\right)$ D) $\frac{9}{100}\left(1601\sqrt{1601} + 1\right)$

Find the flux of the curl of field F through the shell S.

265) $\mathbf{F} = (4y + 4)\mathbf{i} - 3x\mathbf{j} + (e^z - 1)\mathbf{k}$; S: $\mathbf{r}(r, \theta) = 3\sin\phi\cos\theta\mathbf{i} + 3\sin\phi\sin\theta\mathbf{j} + 3\cos\phi\mathbf{k}$, $0 \le \theta \le 2\pi$ and $0 \le \phi \le \frac{\pi}{2}$ 265) ______

A) 63π B) -63π C) -126π D) 126

Find the surface area of the surface S.

266) S is the portion of the paraboloid $z = 4 - x^2 - y^2$ that lies above the ring $4 \le x^2 + y^2 \le 9$ in the x–y plane 266) ______

A) $\frac{\pi}{2}[37\sqrt{37} - 17\sqrt{17}]$ B) $\frac{\pi}{4}[37\sqrt{37} - 17\sqrt{17}]$

C) $\frac{\pi}{3}[37\sqrt{37} - 17\sqrt{17}]$ D) $\frac{\pi}{6}[37\sqrt{37} - 17\sqrt{17}]$

Evaluate the line integral along the curve C.

267) $\int_C \frac{x + y + z}{5}\, ds$, C is the curve $\mathbf{r}(t) = 3t\mathbf{i} + (8\cos\frac{1}{2}t)\mathbf{j} + (8\sin\frac{1}{2}t)\mathbf{k}$, $0 \le t \le 2\pi$ 267) ______

A) $6\pi^2 + 64$ B) 6π C) $6 + 32$ D) $6\pi^2 + 32$

Test the vector field F to determine if it is conservative.

268) $\mathbf{F} = 3x^2y^3z^3\mathbf{i} + 3x^3y^2z^3\mathbf{j} + 3x^3y^3z^2\mathbf{k}$ 268) ______

A) Not conservative B) Conservative

Using Green's Theorem, find the outward flux of F across the closed curve C.

269) $\mathbf{F} = (x - e^x\cos y)\mathbf{i} + (x + e^x\sin y)\mathbf{j}$; C is the lobe of the lemniscate $r^2 = \sin 2\theta$ that lies in the first quadrant 269) ______

A) 0 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{4}$

Calculate the work done by the force F along the path C.

270) $\mathbf{F} = -9x\mathbf{i} - 5x^3y^2\mathbf{j} + (-9z - 4y^2)\mathbf{k}$; the path is $C_1 \cup C_2 \cup C_3$ where C_1 is the straight line from (0, 0, 0) to (1, 0, 0), C_2 is the straight line from (1, 0, 0) to (1, 1, 0), and C_3 is the straight line from (1, 1, 0) to (1, 1, 1) 270) ______

A) $W = -\frac{44}{3}$ B) $W = -\frac{37}{6}$ C) $W = -\frac{41}{3}$ D) $W = -\frac{9}{2}$

Solve the problem.

271) The shape and density of a thin shell are indicated below. Find the moment of inertia about the z–axis. 271) ______

Shell: "nose" of the paraboloid $x^2 + y^2 = 2z$ cut by the plane $z = 1$

Density: $\delta = \frac{1}{\sqrt{x^2 + y^2 + 1}}$

A) $I_z = \frac{2}{3}\pi$ B) $I_z = 16\pi$ C) $I_z = \frac{8}{3}\pi$ D) $I_z = 2\pi$

Calculate the flux of the field F across the closed plane curve C.

272) $\mathbf{F} = (x+y)\mathbf{i} + xy\mathbf{j}$; the curve C is the closed counterclockwise path around the rectangle with vertices at (0, 0), (7, 0), (7, 6), and (0, 6) 272) ______

A) -69 B) 189 C) 336 D) 225

Evaluate the line integral of f(x,y) along the curve C.

273) $f(x,y) = 8y^2$, C: $y = e^{-x}$, $0 \le x \le 1$ 273) ______

A) $\frac{8}{3}\left[2\sqrt{2} - (1)^{3/2}\right]$ B) $4[2\sqrt{2} - (e^{-2} + 1)^{3/2}]$

C) $\frac{8}{3}\left[(1)^{3/2} - 2\sqrt{2}\right]$ D) $\frac{8}{3}[2\sqrt{2} - (e^{-2} + 1)^{3/2}]$

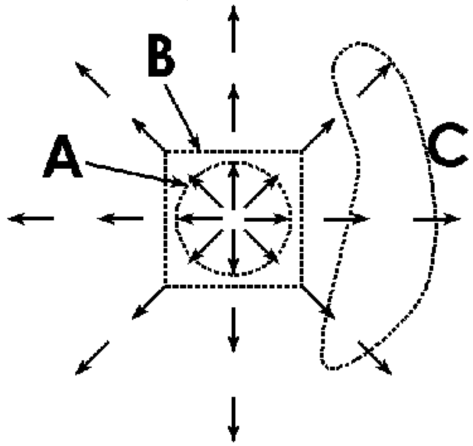
Find the required quantity given the wire that lies along the curve r and has density δ.

274) Moment of inertia I_y about the y–axis, where $\mathbf{r}(t) = (9 - 4t)\mathbf{i} + 3t\mathbf{j}$, $0 \le t \le 1$; $\delta(x, y, z) = 3$ 274) ______

A) $I_y = -3775$ B) $I_y = -755$ C) $I_y = 755$ D) $I_y = 3775$

Solve the problem.

275) The radial flow field of an incompressible fluid is shown below. For which of the closed paths is the circulation not necessarily zero? 275) ______



A) Paths A, B, and C B) Paths A and B only
C) Path C only D) None of the above

276) The shape and density of a thin shell are indicated below. Find the radius of gyration about the z-axis. 276) ______

Shell: upper hemisphere of $x^2 + y^2 + z^2 = 9$ cut by the plane $z = 0$
Density: $\delta = 1$

A) $R_z = \sqrt{6}\pi$ B) $R_z = \sqrt{6}$ C) $R_z = 27\pi$ D) $R_z = 2\sqrt{3}$

Evaluate the work done between point 1 and point 2 for the conservative field F.

277) $\mathbf{F} = (y + z)\mathbf{i} + x\mathbf{j} + x\mathbf{k}$; $P_1(0, 0, 0)$, $P_2(4, 10, 8)$ 277) ______

A) W = 40 B) W = 72 C) W = 0 D) W = 8

Test the vector field F to determine if it is conservative.

278) $\mathbf{F} = -\csc^2 x \csc y\,\mathbf{i} - \cot x \cot y \csc y\,\mathbf{j} - \cos x\,\mathbf{k}$ 278) ______

A) Conservative B) Not conservative

Find the potential function f for the field F.

279) $\mathbf{F} = \dfrac{x}{z^2\sqrt{x^2 + y^2}}\mathbf{i} + \dfrac{y}{z^2\sqrt{x^2 + y^2}}\mathbf{j} - \dfrac{2\sqrt{x^2 + y^2}}{z^3}\mathbf{k}$ 279) ______

A) $f(x, y, z) = \dfrac{x + y}{z^2\sqrt{x^2 + y^2}} + C$ B) $f(x, y, z) = \sqrt{x^2 + y^2} + \ln z + C$

C) $f(x, y, z) = \dfrac{1}{z^2\sqrt{x^2 + y^2}} + C$ D) $f(x, y, z) = \dfrac{\sqrt{x^2 + y^2}}{z^2} + C$

Solve the problem.

280) The shape and density of a thin shell are indicated below. Find the coordinates of the center of mass. 280) ______

Shell: cone $x^2 + y^2 - z^2 = 0$ between $z = 2$ and $z = 5$
Density: constant

A) $\left(0, 0, \dfrac{242}{71}\pi\right)$ B) $\left(0, 0, \dfrac{242}{71}\right)$ C) $\left(0, 0, \dfrac{26}{7}\pi\right)$ D) $\left(0, 0, \dfrac{26}{7}\right)$

Calculate the flux of the field F across the closed plane curve C.

281) $\mathbf{F} = y\mathbf{i} - x\mathbf{j}$; the curve C is the circle $(x + 9)^2 + (y + 4)^2 = 81$ 281) ______

A) 0 B) –324 C) 162 D) –162

Evaluate the line integral along the curve C.

282) $\displaystyle\int_C \left(\frac{x^2 + y^2}{z^2}\right) ds$, C is the curve $\mathbf{r}(t) = (-1 - t)\mathbf{i} - \mathbf{j} + (-1 - t)\mathbf{k}$, $0 \le t \le 1$ 282) ______

A) 3 B) $\dfrac{3}{2}\sqrt{2}$ C) $\dfrac{3}{2}$ D) $-\dfrac{1}{2}\sqrt{2}$

Evaluate the surface integral of the function g over the surface S.

283) $g(x, y, z) = x^3y^3z^3$; S is the surface of the rectangular prism formed from the coordinate planes and the planes $x = 3$, $y = 3$, and $z = 2$ 283) ______

A) $\dfrac{15309}{2}$ B) 30618 C) $\dfrac{729}{2}$ D) 2187

Using Green's Theorem, compute the counterclockwise circulation of F around the closed curve C.

284) $\mathbf{F} = (x - y)\mathbf{i} + (x + y)\mathbf{j}$; C is the triangle with vertices at (0, 0), (7, 0), and (0, 5) 284) ______

A) 70 B) 35 C) 175 D) 0

Calculate the flux of the field F across the closed plane curve C.

285) $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$; the curve C is the circle $(x + 3)^2 + (y + 4)^2 = 4$ 285) ______

A) 0 B) $8\pi - 6$ C) 8π D) 2π

Calculate the flow in the field F along the path C.

286) $\mathbf{F} = \nabla(xy^3z^3)$; C is the line segment from (7, 1, 1) to (8, 1, –1) 286) ______

A) 27 B) –10 C) 16 D) –15

Calculate the area of the surface S.

287) S is the portion of the cone $\dfrac{x^2}{9} + \dfrac{y^2}{9} = \dfrac{z^2}{16}$ that lies between $z = 1$ and $z = 3$ 287) ______

A) $\dfrac{15}{2}\pi$ B) $\dfrac{75}{8}\pi$ C) $\dfrac{225}{2}\pi$ D) $\dfrac{75}{8}$

Solve the problem.

288) The shape and density of a thin shell are indicated below. Find the coordinates of the center of mass. 288) ______

Shell: portion of the sphere $x^2 + y^2 + z^2 = 100$ that lies in the first octant
Density: constant

A) $(10, 10, 10)$ B) $\left(\frac{10}{3}, \frac{10}{3}, \frac{10}{3}\right)$ C) $\left(\frac{5}{2}, \frac{5}{2}, \frac{5}{2}\right)$ D) $(5, 5, 5)$

Using Green's Theorem, compute the counterclockwise circulation of F around the closed curve C.

289) $\mathbf{F} = -\frac{1}{4(x^2 + y^2)^2}\mathbf{i}$; C is the region defined by the polar coordinate inequalities $1 \le r \le 2$ and $0 \le \theta \le \pi$ 289) ______

A) $\frac{3}{4}$ B) $\frac{7}{12}$ C) 0 D) $-\frac{7}{12}$

Find the required quantity given the wire that lies along the curve r and has density δ.

290) Moment of inertia I_z about the z–axis, where $\mathbf{r}(t) = (5 \cos t)\mathbf{i} + (5 \sin t)\mathbf{j}$, $0 \le t \le \frac{\pi}{5}$; $\delta = 5(1 + \sin 5t)$ 290) ______

A) $I_z = 625\left(\frac{\pi}{5}\right)$ B) $I_z = 625\left(\frac{\pi + 2}{5}\right)$ C) $I_z = 25\left(\frac{\pi + 2}{5}\right)$ D) $I_z = 0$

Using Green's Theorem, find the outward flux of F across the closed curve C.

291) $\mathbf{F} = xy\mathbf{i} + x\mathbf{j}$; C is the triangle with vertices at (0, 0), (10, 0), and (0, 10) 291) ______

A) $-\frac{350}{3}$ B) $\frac{650}{3}$ C) $\frac{500}{3}$ D) 0

Evaluate the line integral of f(x,y) along the curve C.

292) $f(x, y) = \cos x + \sin y$, C: $y = x$, $0 \le x \le \frac{\pi}{2}$ 292) ______

A) 2 B) 0 C) $\sqrt{2}$ D) $2\sqrt{2}$

Find the surface area of the surface S.

293) S is the surface $\frac{5}{2}x^2 + 5z = 0$ that lies above the region bounded by the x–axis, $x = \sqrt{64 - 1}$, and y = x 293) ______

A) 511 B) $\frac{511}{2}$ C) $\frac{511}{6}$ D) $\frac{511}{3}$

Using the Divergence Theorem, find the outward flux of F across the boundary of the region D.

294) $\mathbf{F} = x\sqrt{x^2 + y^2}\mathbf{i} + y\sqrt{x^2 + y^2}\mathbf{j} + z\sqrt{x^2 + y^2}\mathbf{k}$; D: the thick cylinder $2 \le x^2 + y^2 \le 6$, $2 \le z \le 5$ 294) ______

A) 832π B) 4992π C) 1664π D) 3328π

Apply Green's Theorem to evaluate the integral.

295) $\oint_C (9x + y^3)\,dx + (3xy^2 + 5y)\,dy$ 295) ______

C: Any simple closed curve in the plane for which Green's Theorem holds

A) There is not enough information to evaluate the integral. [Lock in choice D]

B) 2

C) –2

D) 0

1) B
2) B
3) C
4) D
5) C
6) D
7) B
8) D
9) Answers will vary. One possibility: $\mathbf{r}(y,\theta) = y^2\mathbf{i} + y\cos\theta\mathbf{j} + y\sin\theta\mathbf{k}, -\infty < y < \infty, 0 \le \theta < 2\pi$; $x - 6z = -9$
10) Answers will vary.
11) Answers will vary. One possibility is $\mathbf{r}(\theta,\phi) = 4\cos\theta\sin\phi\mathbf{i} + 11\sin\theta\sin\phi\mathbf{j} + 9\cos\phi\mathbf{k}, 0 \le \theta < 2\pi, 0 \le \phi < \pi$
12) Answers will vary.
13) Answers will vary. One possibility is $4x - y = 2$
14) The integral is 3 times the area enclosed by C.
15) Answers will vary. One possibility is $\mathbf{r} = 6\cos\theta\mathbf{i} + 6\sin\theta\mathbf{j} + z\mathbf{k}$, $4 \le z \le 6, 0 \le \theta \le 2\pi$
16) Answers will vary.
17) The paddlewheel axis points along the vector $\mathbf{r} = -4\mathbf{i} - 3\mathbf{k}$.
18) Answers will vary. Notice that M and N are not defined at (0,0), which is contained inside the region R.
19) Work is proportional to the arc length s of the path.
20) $y = 6$
21) Work is identically zero.
22) $b = 48$ and $c = -24$
23) Answers will vary.
24) It passes the component test for exactness so the path taken does not matter
25) No, the region containing C may not be simply connected, in which case Stokes' Theorem does not apply.
26) Answers will vary.
27) The flux is independent of the radius. Flux is equal to 4π.
28) Answers will vary. One possibility is $\mathbf{r} = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j} + \frac{3}{4}r\mathbf{k}$, $4 \le r \le \frac{16}{3}, 0 \le \theta \le 2\pi$
29) Answers will vary. One possibility is $\mathbf{r} = 10\cos\phi\sin\theta\mathbf{i} + 10\sin\phi\sin\theta\mathbf{j} + 10\cos\theta\mathbf{k}, 0 \le \phi \le 2\pi, \frac{\pi}{4} \le \theta \le \frac{3\pi}{4}$
30) Answers will vary. One possibility is $8\sqrt{2}x + 8\sqrt{2}y - z = 16$
31) Answers will vary. Note that $\mathbf{n} = \dfrac{\mathbf{r_u}\times\mathbf{r_v}}{|\mathbf{r_u}\times\mathbf{r_v}|}$ and $d\sigma = |\mathbf{r_u}\times\mathbf{r_v}|\,dudv$. The assertion follows.
32) Answers will vary.
33) Answers will vary. One possibility is $\mathbf{r} = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j} + 2r^2\mathbf{k}$, $\frac{\sqrt{10}}{2} \le r \le \frac{\sqrt{14}}{2}, 0 \le \theta \le 2\pi$
34) Answers will vary. One possibility is $3\sqrt{2}x - 3\sqrt{2}y + 2z = 0$
35) Answers will vary. One possibility is $x + \sqrt{3}y + 2z = 14\sqrt{2}$

36) Let C be a closed curve for which Green's Theorem applies. Then

$$0 = \int_C \frac{\partial f}{\partial y}dx - \frac{\partial f}{\partial x}dy = \int\int_R -\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)dxdy.$$

If the Laplace equation does not hold for f on R, then there is a simple closed curve C' in the interior of R which bounds a simply connected region R' on which $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \ne 0$ for all (x, y) in R'. Moreover, without loss of generality, we can assume that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} > 0$. Hence

$$\int\int_{R'} -\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)dxdy > 0.$$

As Green's Theorem applies to C', we also have that

$$0 = \int_{C'} \frac{\partial f}{\partial y}dx - \frac{\partial f}{\partial x}dy = \int\int_R -\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)dxdy,$$

which is a contradiction.

37) Answers will vary.
38) Answers will vary. Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + f(x, y)\mathbf{k}$. Then $|\mathbf{r_u}\times\mathbf{r_v}| = \sqrt{[f_x(x,y)]^2 + [f_y(x,y)]^2 + 1}$. The assertion follows.
39) Answers will vary. One possibility is $\mathbf{r} = 1\cos\phi\sin\theta\mathbf{i} + 1\sin\phi\sin\theta\mathbf{j} + 1\cos\theta\mathbf{k}, 0 \le \phi \le 2\pi, \frac{\pi}{4} \le \theta \le \pi$
40) Answers will vary. One possibility is $\mathbf{r} = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j} + \left(\frac{5 - 8r\cos\theta - 8r\sin\theta}{-8}\right)\mathbf{k}, 0 \le r \le 1, 0 \le \theta \le 2\pi$
41) The integral is 7 times the area of C.
42) Answers will vary.
43) Answers will vary. One possibility is $\mathbf{r} = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j} + \left(\frac{5}{16} - 4r^2\right)\mathbf{k}$, $0 \le r \le \frac{1}{16}, 0 \le \theta \le 2\pi$
44) Answers will vary.
45) $f(x, y, z) = -\frac{k(x - x_0)^2}{2} - \frac{k(y - y_0)^2}{2} - \frac{k(z - z_0)^2}{2}$
46) Answers will vary.
47) Answers will vary.
48) $y = 2$
49) Answers will vary.
50) Answers will vary.
51) The integral is zero because the integrand is an exact differential form.
52) $a = -36$, $b = -20$, and $c = 32$
53) Both forms give the same result.
54) A
55) D
56) C
57) A
58) C
59) D
60) B
61) A
62) A
63) B
64) B
65) B

66) D
67) B
68) B
69) C
70) D
71) A
72) C
73) B
74) B
75) B
76) B
77) C
78) D
79) D
80) C
81) C
82) D
83) B
84) A
85) A
86) A
87) A
88) C
89) D
90) B
91) A
92) C
93) B
94) A
95) B
96) A
97) A
98) C
99) A
100) A
101) A
102) C
103) A
104) B
105) A
106) D
107) B
108) A
109) D
110) A
111) A
112) C
113) A
114) C
115) C

116) D
117) C
118) B
119) C
120) B
121) D
122) A
123) D
124) A
125) C
126) B
127) D
128) C
129) A
130) B
131) B
132) A
133) A
134) B
135) C
136) A
137) A
138) B
139) A
140) D
141) A
142) C
143) A
144) D
145) B
146) C
147) C
148) B
149) C
150) A
151) C
152) D
153) A
154) D
155) B
156) B
157) C
158) B
159) D
160) D
161) A
162) D
163) B
164) D
165) C

166) D
167) A
168) C
169) C
170) B
171) A
172) A
173) B
174) C
175) B
176) B
177) C
178) C
179) C
180) B
181) C
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207) C
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209) B
210) C
211) A
212) A
213) D
214) D
215) C

216) D
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219) D
220) C
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245) C
246) B
247) C
248) D
249) A
250) D
251) D
252) B
253) D
254) A
255) D
256) C
257) C
258) B
259) A
260) A
261) D
262) B
263) D
264) C
265) B

Answer Key
Testname: TEST 7

266) D
267) D
268) B
269) C
270) A
271) D
272) B
273) D
274) C
275) C
276) B
277) B
278) B
279) D
280) D
281) A
282) B
283) A
284) B
285) C
286) D
287) A
288) D
289) D
290) B
291) C
292) D
293) D
294) C
295) D