Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- This exam is closed book.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- Calculators, cell phones are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 4 pages, plus this cover sheet. Please make sure that your exam is complete.

Problem	Points	Score
1	15	
2	20	
3	10	
4	15	
5	15	
6	15	
7	10	
Total:	100	

Do not write in the table to the right.

1. (15 total points) Graph the rational function $f(x) = \dfrac{x^2}{x+1}$. p.212, pr.83

(a) 5 Points Find $L = \lim_{x \to -5} f(x)$. (DO NOT USE L'HÔPITAL's RULE)

$$\begin{aligned} \lim_{x\to -5} \frac{x^2+6x+5}{x+5} &= \lim_{x\to -5} \frac{\cancel{(x+5)}(+x+1)}{\cancel{(x+5)}} \\ &= \lim_{x\to -5}(x+1) \\ &= -4, \quad x \neq -5. \end{aligned}$$

(b) 10 Points Find a number δ such that for all x

$$0 < |x - x_0| < \delta \Rightarrow |f(x-L)| < \varepsilon.$$

Step 1

$$\begin{aligned} \left| \left(\frac{x^2+6x+5}{x+5} \right) - (-4) \right| &< 0.05 \Longrightarrow -0.05 < \frac{x^2+6x+5}{x+5} + 4 < 0.05 \\ &\Longrightarrow -4.05 < x+1 < -3.95,\ x \neq -5 \\ &\Longrightarrow -5.05 < x < -4.95,\ x \neq -5 \end{aligned}$$

Step 2

$$\begin{aligned} |x-(-5)| &< \delta \Longrightarrow -\delta < x+5 < \delta \Longrightarrow -\delta - 5 < x < \delta - 5. \\ \text{Then } -\delta - 5 &= -5.05 \Longrightarrow \delta = 0.05, \text{ or } \delta - 5 = -4.95 \Longrightarrow \delta = 0.05; \text{ thus } \delta = 0.05. \end{aligned}$$

2. A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?

(a) 10 Points $\lim_{x\to 1} \dfrac{x^{1/3}-1}{\sqrt{x}-1}$ (DO NOT USE L'HÔPITAL's RULE)

$$\begin{aligned} \lim_{x\to 1} \frac{x^{1/3}-1}{\sqrt{x}-1} &= \lim_{x\to 1} \frac{x^{1/3}-1}{\sqrt{x}-1} \frac{(x^{2/3}+x^{1/3}+1)(\sqrt{x}+1)}{(x^{2/3}+x^{1/3}+1)(\sqrt{x}+1)} \\ &= \lim_{x\to 1} \frac{\cancel{(x-1)}(\sqrt{x}+1)}{\cancel{(x-1)}(x^{2/3}+x^{1/3}+1)} \\ &= \lim_{x\to 1} \frac{\sqrt{x}+1}{x^{2/3}+x^{1/3}+1} \\ &= \frac{1+1}{1+1+1} \\ &= \frac{2}{3} \end{aligned}$$

(b) 10 Points $\lim_{x\to 0} \dfrac{x \sin x}{2 - 2\cos x}$ p.178 pr.101 (DO NOT USE L'HÔPITAL's RULE)

$$\begin{aligned}\lim_{x\to 0}\frac{x\sin x}{2-2\cos x} &= \lim_{x\to 0}\frac{x\sin x}{2(1-\cos x)} = \lim_{x\to 0}\frac{x\sin x}{2(2\sin^2(\frac{x}{2}))} = \lim_{x\to 0}\left[\frac{\frac{x}{2}\frac{x}{2}}{\sin^2(\frac{x}{2})}\cdot\frac{\sin x}{x}\right]\\ &= \lim_{x\to 0}\left[\frac{\frac{x}{2}}{\sin(\frac{x}{2})}\cdot\frac{\frac{x}{2}}{\sin(\frac{x}{2})}\frac{\sin x}{x}\right]\\ &= (1)(1)(1)=1.\end{aligned}$$

3. 10 Points fmd the total area between the region and the x-axis.

$$y=3x^2-3, -2\le x\le 2$$

p.282 pr.42

$$\begin{aligned}\frac{dy}{dt} &= \frac{d}{dt}\left(t^{-3/4}\sin t\right)^{4/3}\\ &= \frac{4}{3}\left(t^{-3/4}\sin t\right)^{1/3}[-\frac{3}{4}t^{-7/4}\sin t\\ &\quad +t^{-3/4}\cos t]\end{aligned}$$

4. 15 Points Find the equations of normals to the curve

$$xy+2x-y=0$$

that are parallel to the line $2x+y=0$. p.307, pr.29

$$\begin{aligned}xy+2x-y &= 0\Longrightarrow x\frac{dy}{dx}+y+2-\frac{dy}{dx}=0\\ &\Longrightarrow \frac{dy}{dx}=\frac{y+2}{1-x};\end{aligned}$$

the slope of the line $2x+y=0$ is -2. In order to be parallel, the normal lines must also have slope of -2. Since a normal is perpendicular to a tangent, the slope of tangent is $\frac{1}{2}$. Therefore

$$\begin{aligned}\frac{y+2}{1-x} &= \frac{1}{2}\Longrightarrow 2y+4=1-x\\ &\Longrightarrow x=-3-2y.\end{aligned}$$

Substituting in the original equation,

$$\begin{aligned}y(-3-2y)+2(-3-2y) &= 0\\ &\Longrightarrow y^2+4y+3=0\\ &\Longrightarrow y=-3 \text{ or } y=-1.\end{aligned}$$

$$\begin{aligned}\text{If } y &= -3, \text{then } x=3 \text{ and } y+3=-2(x-3)\\ &\Longrightarrow y=-2x+3.\\ \text{If } y &= -1, \text{ then } x=-1 \text{ and } y+1=-2(x+1)\\ &\Longrightarrow y=-2x-3.\end{aligned}$$

5. 15 Points For what values of a and b is

$$f(x) = \begin{cases} -2 & x \leq -1 \\ ax - b, & -1 < x < 1 \\ 3, & x \geq 1 \end{cases}$$

continuous at every x. p.83, pr.45

Clearly f is continuous if $x \neq -1$ and for $x \neq 1$ for if $x < -1$ or if $-1 < x < 1$ or if $x > 1$, f is a polynomial, regardless the values of a and b. For continuity at $x = -1$, we require that the one-sided limits of $f(x)$ at $x = -1$ be equal. But $\lim_{x \to -1^-} f(x) = -2$ and $\lim_{x \to -1^+} f(x) = a(-1) + b = -a + b$.
Similarly, for continuity at $x = 1$, we require that the one-sided limits of $f(x)$ at $x = 1$ be equal. But $\lim_{x \to 1^-} f(x) = a(1) + b = a + b$ and $\lim_{x \to 1^+} f(x) = 3$.
Equality of one-sided limits is equivalent to

$$\begin{aligned} -2 &= -a + b \text{ and } a + b = 3 \\ \Longrightarrow & \quad a = \frac{5}{2} \text{ and } b = \frac{1}{2}. \end{aligned}$$

6. 15 Points Assume that $f(x)$ and $g(x)$ are differentiable functions satisfying

$$\begin{array}{llll} g(0) = 1 & f(0) = 1 & f(1) = 3 & g(1) = 5 \\ g'(0) = \frac{1}{2} & f'(0) = -3 & f'(1) = \frac{1}{2} & g'(1) = -4 \end{array}$$

Let $h(x) = f(x + g(x))$. Evaluate $h'(0)$. p.148, pr.74

First, by the Chain Rule, we have $h'(x) = f'(x + g(x))(1 + g'(x))$. Then $h'(0) = f'(0 + g(0))(1 + g'(0)) = f'(g(0))(1 + g'(0))$.
Hence $h'(0) = f'(1)(1 + \frac{1}{2}) = (\frac{1}{2})(\frac{3}{2}) = \frac{3}{4}$.

7. 10 Points Find the volume of the solid generated by revolving the region bounded by

$$y = x - x^2 \text{ and } y = 0$$

about the x-axis. p.317, pr.22

When $f(x) = x + \frac{1}{x}$ for $\frac{1}{2} \leq x \leq 2$, then

$$\frac{f(2) - f(1/2)}{2 - 1/2} = f'(c) \Rightarrow 0 = 1 - \frac{1}{c^2} \Rightarrow c = \pm 1$$

But $-1 \notin [\frac{1}{2}, 2]$, so $c = 1$.

Hasan Özekes:
Page 212: #83

$$y = x - x^2, \; y = 0$$

about the x-axis.
Page 219: #4
Page 282: #42
Page 307: #29
Page 317: #22

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Page 325: #24(c)
Page 298: #39
Page 307: #29(a)
Page 291: #43
Page 241: #51(a)