Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

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Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- This exam is closed book.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- Calculators, cell phones are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Do not ask the invigilator anything.
- Use a **ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 80 min.

Do not write in the table to the right.

| Problem | Points | Score |
|---|---|---|
| 1 | 25 | |
| 2 | 25 | |
| 3 | 35 | |
| 4 | 25 | |
| Total: | 110 | |

1. (a) $\boxed{\text{15 Points}}$ $\int t^2 e^{4t}\, dt = ?$

**Solution:** Integrate by parts twice. let $u = t^2$ and $dv = e^{4t}\, dt$. Then $du = 2t\, dt$ and $v = \frac{1}{4}e^{4t}$. Hence

$$\int t^2 e^{4t}\, dt = \int u\, dv = uv - \int v\, du = \frac{t^2}{4}e^{4t} - \frac{1}{2}\int te^{4t}\, dt$$

For the last integral, we let $u = t$ and so $dv = e^{4t}\, dt$. Then $du = dt$ and $v = \frac{1}{4}e^{4t}$.
Therefore, we have

$$\begin{aligned}\int t^2 e^{4t}\, dt &= \frac{t^2}{4}e^{4t} - \frac{1}{2}\left(\frac{t}{4}e^{4t} - \int \frac{1}{4}e^{4t}\, dt\right)\\ &= \frac{t^2}{4}e^{4t} - \frac{2t}{16}e^{4t} + \frac{2}{64}e^{4t} + C = \frac{t^2}{4}e^{4t} - \frac{t}{8}e^{4t} + \frac{1}{32}e^{4t} + C\\ &= \left(\frac{t^2}{4} - \frac{t}{8} + \frac{1}{32}\right)e^{4t} + C\end{aligned}$$

p.441, pr.20

(b) $\boxed{\text{10 Points}}$ $\int \frac{2x+1}{x^2-7x+12}\, dx = ?$

**Solution:**

$$\begin{aligned}\frac{2x+1}{x^2-7x+12} &= \frac{2x+1}{(x-4)(x-3)} = \frac{A}{x-4} + \frac{B}{x-3} \Rightarrow 2x+1 = A(x-3) + B(x-4);\\ x &= 3 \Rightarrow B = \frac{7}{-1} = -7; x = 4 \Rightarrow A = \frac{9}{1} = 9;\\ \int \frac{2x+1}{x^2-7x+12}\, dx &= 9\int \frac{dx}{x-4} - 7\int \frac{dx}{x-3} = 9\ln|x-4| - 7\ln|x-3| + C = \ln\left|\frac{(x-4)^9}{(x-3)^7}\right| + C\end{aligned}$$

p.461, pr.12

2. (a) $\boxed{\text{7 Points}}$ Does the series $\sum_{n=2}^{\infty} \frac{e^n}{e^n+n}$ converge? Give reasons.

**Solution:** Use the $n$th Term Test.

$$\lim_{n\to\infty} \frac{e^n}{e^n+n} = \lim_{n\to\infty} \frac{e^n}{e^n+1} = \lim_{n\to\infty} \frac{e^n}{e^n} = \lim_{n\to\infty} \frac{1}{1} = 1 \neq 0. \Rightarrow \textbf{diverges}.$$

p.551, pr.32

(b) $\boxed{\text{8 Points}}$ Does the series $\sum_{n=1}^{\infty} \frac{1}{3^{n-1}+1}$ converge? Give reasons.

**Solution:** coverges by the Direct Comparison Test: $0 < \frac{1}{3^{n-1}+1} < \frac{1}{3^{n-1}}$, which is the $n$th term of a convergent geometric series.

p.562, pr.37

(c) $\boxed{\text{10 Points}}$ Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{(x+1)^{2n}}{9^n}$ and, within this interval, write the sum of the series as a function of $x$.

**Solution:**

$$\begin{aligned}
\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| &< 1 \Rightarrow \lim_{n\to\infty}\left|\frac{(x+1)^{2n+2}}{9^{n+1}}\frac{9^n}{(x+1)^{2n}}\right| < 1 \Rightarrow \frac{(x+1)^2}{9}\lim_{n\to\infty}(1) < 1 \Rightarrow (x+1)^2 < 9 \Rightarrow |x+1| < 3 \\
&\Rightarrow -3 < x+1 < 3 \Rightarrow -4 < x < 2; \text{ when } x=-4 \text{ we have } \sum_{n=0}^{\infty}\frac{(-3)^{2n}}{9^n} = \sum_{n=0}^{\infty} 1 \text{ which diverges;} \\
\text{at } x &= 2 \text{ we have } \sum_{n=0}^{\infty}\frac{(3)^{2n}}{9^n} = \sum_{n=0}^{\infty} 1 \text{ which also diverges; the interval of convergence is } -4 < x < 2; \text{ the series} \\
\sum_{n=0}^{\infty}\frac{(x+1)^{2n}}{9^n} &= \sum_{n=0}^{\infty}\left(\left(\frac{x+1}{3}\right)^2\right)^n \text{ is a convergent geometric series when } -4 < x < 2 \text{ and the sum is} \\
\frac{1}{1-\left(\frac{x+1}{3}\right)^2} &= \frac{1}{\left[\frac{9-(x+1)^2}{9}\right]} = \frac{9}{9-x^2-2x-1} = \frac{9}{8-2x-x^2}
\end{aligned}$$

p.583, pr.44

3. (a) $\boxed{\text{10 Points}}$ Find the point in which the line $x=-1+3t, \quad y=-2, \quad z=5t$ meets the plane $2x-3z=7$.

**Solution:**

$2x-3z=7 \Rightarrow 2(-1+3t)-3(5t)=7 \Rightarrow -9t-2=7 \Rightarrow t=-1 \Rightarrow x=-1-3, y=-2, z=-5 \Rightarrow (-4,-2,-5)$ is the point we wished to find.

p.695, pr.56

(b) $\boxed{\text{10 Points}}$ $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^2-y}{x+y} = ?$

**Solution:**

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along } y=kx\\ k\neq -1}}\frac{x^2-y}{x+y} = \lim_{x\to 0}\frac{x^2-kx}{x+kx} = \lim_{x\to 0}\frac{x-k}{1+k} = \frac{-k}{1+k} \Rightarrow \text{ different limits for different values of } k, k\neq 1.$$

Therefore the original limit does not exist.

p.762, pr.46

(c) $\boxed{\text{15 Points}}$ Find the value of $\dfrac{\partial z}{\partial x}$ at the point $(1,1,1)$ if the equation $xy+z^3x-2yz=0$ defines $z$ as a function of the two independent variables $x$ and $y$ and the partial derivative exists.

**Solution:**

$$\begin{aligned}
xy+z^3x-2yz &= 0 \Rightarrow \frac{\partial}{\partial x}(xy+z^3x-2yz) = \frac{\partial}{\partial x}(0) \\
&\Rightarrow y+\left(3z^2\frac{\partial z}{\partial x}\right)x+z^3-2y\frac{\partial z}{\partial x}=0 \\
&\Rightarrow (3xz^2-2y)\frac{\partial z}{\partial x} = -y-z^3 \\
&\Rightarrow \text{ at } (1,1,1) \text{ we have } (3-2)\frac{\partial z}{\partial x} = -1-1 \text{ or } \frac{\partial z}{\partial x} = -2.
\end{aligned}$$

p.773, pr.65

4. (a) $\boxed{\text{15 Points}}$ Use *only* the method of **Lagrange multipliers** to find the maximum aod minimum values of $f(x,y,z) = x-2y+5z$ on the sphere $x^2+y^2+z^2=30$.

**Solution:** $\nabla f = \mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$ and $\nabla g = 2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$ so that $\nabla f = \lambda \nabla g \Rightarrow \mathbf{i} - 2\mathbf{j} + 5\mathbf{k} = \lambda(2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}) \Rightarrow 1 = 2x\lambda$,

$-2 = 2y\lambda$ and $5 = 2z\lambda \Rightarrow x = \dfrac{1}{2\lambda}$, $y = -\dfrac{1}{\lambda} = -2x$, and $z = \dfrac{5}{2\lambda} = 5x$

$\Rightarrow x^2 + (-2x)^2 + (5x)^2 = 30 \Rightarrow x = \pm 1$.

Thus, $x = 1$, $y = -2$, $z = 5$ or

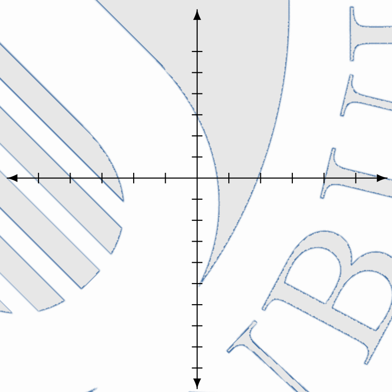
$x = -1$ $y = 2$, $z = -5$. Therefore $f(1,-2,5) = 30$ is the maximum value and $f(-1,2,-5) = -30$ is the minimum value.

p.818, pr.23

(b) $\boxed{\text{10 Points}}$ Evaluate the **double integral** of $f(x,y) = x^2+y^2$ over the triangular region with vertices $(0,0)$, $(1,0)$, and $(0,1)$.

**Solution:**

$$\begin{aligned}
\int_0^1 \int_0^{1-x} (x^2+y^2)\, dy\, dx &= \int_0^1 \left[x^2 y + \frac{y^3}{3}\right]_0^{1-x} dx \\
&= \int_0^1 \left[x^2(1-x) - \frac{(1-x)^3}{3}\right] dx \\
&= \int_0^1 \left[x^2 - x^3 + \frac{(1-x)^3}{3}\right] dx \\
&= \left[\frac{x^3}{3} - \frac{x^4}{4} - \frac{(1-x)^4}{12}\right]_0^1 \\
&= \left(\frac{1}{3} - \frac{1}{4} - 0\right) - \left(0 - 0 - \frac{1}{12}\right) \\
&= \frac{1}{6}
\end{aligned}$$



p.848, pr.26