Your Name / Adınız - Soyadınız (**time:60**) Signature / İmza

Student ID # / Öğrenci No **(mavi tükenmez!)**

Problem	1	2	3	4	Total
Points:	26	24	26	24	100
Score:					

1. (a) (13 Points) Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converge absolutely, conditionally, or diverge? Justify your answer.

Solution: This series *converges conditionally* by the Alternating Series Test since

$$\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}} > 0 \quad \text{and} \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0.$$

But the convergence is not absolute since

$$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$$

is a divergent p-series (as $p = \frac{1}{2} \leq 1$).

p.606, pr.27

(b) (13 Points) $\int \sin^3 x \cos^4 x \, dx = ?$

Solution:

$$\int \sin^3 x \cos^4 x \, dx = \int \cos^4 x (1 - \cos^2 x) \sin x \, dx = \int \cos^4 x \sin x \, dx - \int \cos^6 x \sin x \, dx = \boxed{-\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C}$$

p.490 pr.37

2. (a) (11 Points) Find an equation for the plane that passes through the point $(3, -2, 1)$ normal to the vector $\mathbf{n} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$.

Solution: $P_0(3, -2, 1)$ and $\mathbf{n} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$ imply that $2(x-3) + (1)(y - (-2)) + (z-1) = 0 \Rightarrow \boxed{2x + y + z = 5}$

p.702, pr.35

(b) (13 Points) Find parametric equations for the line in which the planes $x + 2y + z = 1$ and $x - y + 2z = -8$ intersect.

Solution: The direction of the line is

$$\mathbf{n_1} \times \mathbf{n_2} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} \mathbf{k} = 5\mathbf{i} - \mathbf{j} - 3\mathbf{k}.$$

Since the point $(-5, 3, 0)$ is on both planes, the desired line is $x = -5 + 5t, y = 3 - t, z = -3t$.

p.702, pr.43

3. (a) (14 Points) Find equations for the *tangent plane* and *normal line* at $P_0(1, -1, 3)$ on the surface $x^2 + 2xy - y^2 + z^2 = 7$.

Solution: (a) $\nabla f = (2x + 2y)\mathbf{i} + (2x - 2y)\mathbf{j} + 2z\mathbf{k} \Rightarrow \nabla f(1, -1, 3) = 4\mathbf{j} + 6\mathbf{k} \Rightarrow$ Tangent plane: $4(y+1) + 6(z-3)0 \Rightarrow 2y + 3z = 7$;
(b) Normal line: $x = 1 y + -1 + 4t, z = 3 + 6t$.

p.799, pr.4

(b) (12 Points) Find all the local maxima, local minima, and saddle points of $f(x, y) = e^{2x} \cos y$.

Solution: $f_x(x,y) = 2e^{2x}\cos y = 0$ and $f_y(x,y) = -e^{2x}\sin y = 0 \Rightarrow$ no solution since $e^{2x} \neq 0$ and the functions $\cos y$ and $\sin y$ cannot equal 0 for the same $y \Rightarrow$ no critical points $\Rightarrow$ no extrema and no saddle points.

p.809, pr.24

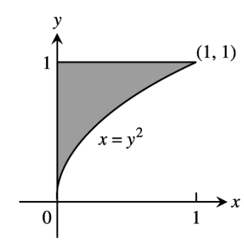
4. (a) (12 Points) Find the point on the plane $x+2y+3z = 13$ closest to the point $(1,1,1)$.

Solution: Let $f(x,y,z) = (x-1)^2+(y-1)^2+(z-1)^2$ be the square of the distance from $(1,1,1)$. Then $\nabla f = 2(x-1)\mathbf{i}+2(y-1)\mathbf{j}+2(z-1)\mathbf{k}$ and $\nabla g = \mathbf{i}+2\mathbf{j}+3\mathbf{k}$ so that $\nabla f = \lambda \nabla g \Rightarrow 2(x-1)\mathbf{i}+2(y-1)\mathbf{j}+2(z-1)\mathbf{k} = \lambda\mathbf{i}+2\mathbf{j}+3\mathbf{k} \Rightarrow 2(x-1) = \lambda$, $2(y-1) = 2\lambda$, $2(z-1) = 3\lambda \Rightarrow 2(y-1) = 2[2(x-1)]$ and $2(z-1) = 3[2(x-1)] \Rightarrow x = \frac{y+1}{2} \Rightarrow z+2 = 3\left(\frac{y+1}{2}\right)$ or $z = \frac{3y-1}{2}$; thus $\frac{y+1}{2}+2y+3\left(\frac{3y-1}{2}\right)-13 = 0 \Rightarrow y = 2 \Rightarrow x = \frac{3}{2}$ and $z = \frac{5}{2}$. Therefore the point $\left(\frac{3}{2},2,\frac{5}{2}\right)$ is closest (since no point on the plane is farthest from the point $(1,1,1)$).

p.818, pr.17

(b) (12 Points) Sketch the region of integration and evaluate $\int_0^1\int_0^{y^2} 3y^3 e^{xy}\,dx\,dy$.

Solution:



$$\int_0^1\int_0^{y^2} 3y^2 e^{xy}\,dx\,dy = \int_0^1 \left[3y^2e^{xy}\right]_0^{y^2}\,dy = \int_0^1 \left(3y^2e^{y^3}-3y^2\right)\,dy = \left[e^{y^3}-y^3\right]_0^1 = e-2$$

p.848, pr.23