Your Name / Adınız - Soyadınız

Signature / İmza

Student ID # / Öğrenci No

(**mavi tükenmez!**)

Problem	1	2	3	4	Total
Points:	22	22	29	27	100
Score:					

1. (a) (11 Points) $\int_1^2 x\ln x\ dx = ?$

Solution: Let $u = \ln x$, $dv = x\,dx$. Then $du = \frac{1}{x}$ and $v = \frac{1}{2}x^2$. Integrating *by parts* gives

$$\int_1^2 x\ln x\ dx = \left[\frac{x^2}{2}\ln x\right]_1^2 - \int_1^2 \frac{x^2}{2}\frac{1}{x}\ dx = 2\ln 2 - \frac{3}{4} = \boxed{\ln 4 - \frac{3}{4}}$$

p.441, pr.5

(b) (11 Points) $\int x^2 \sin x^3\ dx = ?$

Solution: Let $u = x^3$ and so $du = 3x^2\ dx$.

$$\int x^2 \sin x^3\ dx = \frac{1}{3}\int \sin x^3\ (3x^2)\ dx = \frac{1}{3}\int \sin u\ du = -\frac{1}{3}\cos u + C = \boxed{-\frac{1}{3}\cos x^3 + C}$$

p.442, pr.40

2. (a) (11 Points) $\int \frac{3\ dx}{\sqrt{1+9x^2}} = ?$

Solution: Let $u = 3x$ and so $du = 3\ dx$. Then we have $\int \frac{3\ dx}{\sqrt{1+9x^2}} = \int \frac{du}{\sqrt{1+u^2}}$. Now we use the *trigonometric substitution* $u = \tan\theta$ with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Therefore $du = \sec^2\theta\ d\theta$. Now we have

$$\int \frac{du}{\sqrt{1+u^2}} = \int \frac{d\theta}{\sqrt{1+\tan^2\theta}} = \int \frac{d\theta}{\sqrt{\sec^2\theta}} = \int \frac{d\theta}{\sec\theta} = \int \cos\theta\ d\theta = \sin\theta + C = \frac{u}{\sqrt{u^2+1}} + C = \boxed{\frac{3x}{\sqrt{9x^2+1}} + C}$$

p.452, pr.2

(b) (11 Points) $\int \frac{x+4}{x^2+5x-6}\ dx = ?$

Solution: We employ the *partial fraction decomposition* and write the integrand as

$$\frac{x+4}{x^2+5x-6} = \frac{x+4}{(x-1)(x+6)} = \frac{A}{x-1} + \frac{B}{x+6} \Rightarrow x+4 = A(x+6) + B(x-1)$$

Now we determine A and B. Put $x = -6$. Then we get $-6+4 = B(-6-1) \Rightarrow B = \frac{2}{7}$. And similarly, for $x = 1$, we have $1+4 = A(1+6) + B(1-1) \Rightarrow A = \frac{5}{7}$. Hence

$$\int \frac{x+4}{x^2+5x-6}\ dx = \frac{2}{7}\int \frac{dx}{x+6} + \frac{5}{7}\int \frac{dx}{x-1} = \frac{2}{7}\ln|x+6| + \frac{5}{7}\ln|x-1| + C = \boxed{\frac{1}{7}\ln|(x+6)^2(x-1)^5| + C}.$$

p.461, pr.11

3. (a) (18 Points) Evaluate the improper integral $\int_{-\infty}^{+\infty} 2xe^{-x^2}\ dx$.

Solution:

$$\begin{aligned}\int_{-\infty}^{+\infty} 2xe^{-x^2}\,dx &= \int_{-\infty}^{0} 2xe^{-x^2}\,dx + \int_{0}^{+\infty} 2xe^{-x^2}\,dx = \lim_{b\to-\infty}\int_{b}^{0} 2xe^{-x^2}\,dx + \lim_{c\to\infty}\int_{0}^{c} 2xe^{-x^2}\,dx \\ &= \lim_{b\to-\infty}\left[-e^{-x^2}\right]_b^0 + \lim_{c\to\infty}\left[-e^{-x^2}\right]_0^c = \lim_{b\to-\infty}\left[-1-(-e^{-b^2})\right] + \lim_{c\to\infty}\left[-(-e^{-c^2})-(-1)\right] \\ &= (-1-0)+(0+1)=0.\end{aligned}$$

Then we get

p.487, pr.24

(b) (11 Points) Determine if the improper integral $\int_2^\infty \frac{x\,dx}{\sqrt{x^4-1}}$ converges or diverges.

Solution:

$$\lim_{x\to\infty}\frac{\dfrac{x}{\sqrt{x^4-1}}}{\dfrac{x}{\sqrt{x^4}}} = \lim_{x\to\infty}\frac{\sqrt{x^4}}{\sqrt{x^4-1}} = \lim_{x\to\infty}\frac{1}{\sqrt{1-\dfrac{1}{x^4}}} = 1.$$

Now

$$\int_2^\infty \frac{x\,dx}{\sqrt{x^4}} = \int_2^\infty \frac{1}{x}\,dx = \lim_{b\to\infty}[\ln x]_2^b = \infty,$$

which diverges $\Rightarrow \int_2^\infty \frac{x\,dx}{\sqrt{x^4-1}}$ *diverges by the Limit Comparison Test.*

p.487, pr.54

4. (a) (17 Points) Find a formula for the nth partial sum of $\sum_{n=1}^{\infty}\left(\ln\sqrt{n+1}-\ln\sqrt{n}\right)$ and use it to determine if the series converges or diverges. If it converges, find its sum.

Solution:

$$\begin{aligned} s_k &= \left(\ln\sqrt{2}-\ln\sqrt{1}\right)+\left(\ln\sqrt{3}-\ln\sqrt{2}\right)+\left(\ln\sqrt{4}-\ln\sqrt{3}\right)+\cdots+\left(\ln\sqrt{k}-\ln\sqrt{k-1}\right)+\left(\ln\sqrt{k+1}-\ln\sqrt{k}\right) \\ &= \ln\sqrt{k+1}-\ln\sqrt{1}=\ln\sqrt{k+1}\end{aligned}$$

Hence the formula is $\boxed{s_n=\ln\sqrt{n+1}}$. Therefore $\lim_{n\to\infty}s_n=\lim_{n\to\infty}\ln\sqrt{n+1}=\infty$. Hence the series *diverges*.

p.551, pr.37

(b) (10 Points) Does the series $\sum_{n=1}^{\infty}\frac{2^n-1}{3^n}$ converge or diverge? If it converges, find its sum.

Solution: This is a difference of two geometric series

$$\sum_{n=1}^{\infty}\frac{2^n-1}{3^n}=\sum_{n=1}^{\infty}\frac{2^n}{3^n}-\sum_{n=1}^{\infty}\frac{1}{3^n}=\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n-\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n$$

The sum is

$$s=\frac{\frac{2}{3}}{1-\frac{2}{3}}-\frac{\frac{1}{3}}{1-\frac{1}{3}}=3-\frac{1}{2}=\frac{5}{2}$$

p.552, pr.59