Your Name / Ad - Soyad

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Student ID # / Öğrenci No

(mavi tükenmez!)

Problem	1	2	3	4	Total
Points:	25	25	25	25	100
Score:					

Time Limit: 60 min.

1. (a) (14 Points) $\int_1^2 x \ln x \, dx = ?$

Solution: Let $u = \ln x$, $dv = x\,dx$. Then $du = \frac{1}{x}$ and $v = \frac{1}{2}x^2$. Integrating *by parts* gives

$$\int_1^2 x \ln x \, dx = \left[\frac{x^2}{2}\ln x\right]_1^2 - \int_1^2 \frac{x^2}{2}\frac{1}{x}\,dx = 2\ln 2 - \frac{3}{4} = \boxed{\ln 4 - \frac{3}{4}}$$

p.441, pr.5

(b) (11 Points) Let $f(x) = x^2 - 4x - 5$, $x > 2$. Find the value of df^{-1}/dx at the point $x = 0 = f(5)$.

Solution: Method I:Even though we can find a formula for f^{-1} in this case, we note that $y = 0$ corresponds $x = 5$, and since, $f'(x) = 2x - 4$To find the value of the derivative of f^{-1} at $x = 0$, we use the inverse function theorem (namely the Theorem 7.1).

$\left(f^{-1}\right)'(0) = \frac{1}{f'(5)} == \frac{1}{2(5)-4} = \boxed{\frac{1}{6}}$

The figure on the right is the graph of f^{-1} with the tangent at the point $x = 0$.

Method II: We shall find the formula for f^{-1}. Let $y = x^2 - 4x - 5$. Then $y = (x^2 - 4x + 4) - 4 - 5 \Rightarrow y + 9 = (x-2)^2 \Rightarrow x - 2 = \pm\sqrt{y+9} \Rightarrow x = 2 \pm \sqrt{y+9}$. Therefore, since $x > 2$ is the given restriction on the domain for f^{-1}, the formula for f^{-1} is $f^{-1}(x) = 2 + \sqrt{x+9}$. Hence the derivative we want to find is $\left(f^{-1}\right)'(x) = \frac{1}{2\sqrt{x+9}}$ and its value is

$$\left(f^{-1}\right)'(0) = \frac{1}{2\sqrt{0+9}} = \frac{1}{2\sqrt{9}} = \frac{1}{6}.$$

p.442, pr.40

2. (a) (14 Points) $\displaystyle\int \frac{3\sec^2 t}{6+3\tan t}\,dt = ?$

Solution: Let $u = 6+3\tan t$ and so $du = 3\sec^2 t\,dt$. Now

$$\int \frac{3\sec^2 t}{6+3\tan t}\,dt = \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|6+3\tan t| + C}$$

p.452, pr.2

(b) (11 Points) $\displaystyle\frac{d}{dx}\left(x^3\log_{10} x\right) = ?$

Solution: We have

$$\begin{aligned}\frac{d}{dx}\left(x^3\log_{10} x\right) &= x^3\frac{d}{dx}(\log_{10} x) + (\log_{10} x)\frac{d}{dx}(x^3) \\ &= \boxed{x^3\frac{1}{x\ln 10} + 3x^2(\log_{10} x)}\end{aligned}$$

p.461, pr.11

3. (a) (13 Points) $\lim_{x\to 0^+}\left(\frac{3x+1}{x}-\frac{1}{\sin x}\right)=?$

Solution: This limit has the indeterminate $\infty-\infty$.

$$\begin{aligned}\lim_{x\to 0^+}\left(\frac{3x+1}{x}-\frac{1}{\sin x}\right) &= \lim_{x\to 0^+}\frac{(3x+1)\sin x - x}{x\sin x}\\ &= \lim_{x\to 0^+}\frac{3\sin x+(3x+1)\cos x-1}{\sin x + x\cos x}\\ &= \lim_{x\to 0^+}\frac{3\cos x+(3)\cos x-(3x+1)\sin x}{\cos x+\cos x - x\sin x}\\ &= \frac{3\cos 0+(3)\cos 0-(3(0)+1)\sin 0}{\cos 0+\cos 0-0\sin 0}\\ &= \frac{3+3-0}{1+1-0} = \boxed{\frac{6}{2}=3}\end{aligned}$$

p.402, pr.40

(b) (12 Points) $\int \frac{dx}{x\sqrt{25x^2-2}}=?$

Solution: Let $u=5x$ and so $du=5dx$.

$$\int\frac{dx}{x\sqrt{25x^2-2}}=\int\frac{du}{u\sqrt{u^2-2}}=\frac{1}{\sqrt{2}}\sec^{-1}\left|\frac{u}{\sqrt{2}}\right|+C=\boxed{\frac{1}{\sqrt{2}}\sec^{-1}\left|\frac{5x}{2}\right|+C}$$

p.413, pr.47

4. (a) (10 Points) $y = (x^2+1)\cosh(\ln x) \quad \Rightarrow \quad dy/dx = ?$

Solution:

$$y = (x^2+1)\cosh(\ln x) = y = (x^2+1)\frac{e^{\ln x} + e^{-\ln x}}{2} = y = (x^2+1)\frac{x+1/x}{2} = (x^2+1)\frac{x^2+1}{2x} = \frac{(x^2+1)^2}{2x}$$

So we can now differentiate by means of quotient rule.

$$\frac{dy}{dx} = \frac{(2x)(2)(x^2+1)(2x) - 2(x^2+1)^2}{(2x)^2} = \boxed{\frac{2(x^2+1)(4x-(x^2+1))}{4x^2}}$$

p.442, pr.13

(b) (15 Points) $\int 8\cos^3(2\theta)\sin(2\theta)\,d\theta = ?$

Solution: Let $u = \cos(2\theta)$ and so $du = -2\sin 2\theta d\theta$. Then, we have

$$\int 8\cos^3(2\theta)\sin(2\theta)\,d\theta = -\int 8u^3\,du = -\left[8\frac{u^4}{4}\right] + C = -2u^4 + C = \boxed{-2\cos^4(2\theta) + C}$$

p.448, pr.21