Math 114 Summer 2016 **First Exam** July 13, 2016

Your Name / Ad - Soyad (**75 min.**) Signature / İmza

Student ID # / Öğrenci No (**mavi tükenmez!**)

Problem	1	2	3	4	Total
Points:	30	20	25	25	100
Score:					

Time limit is **75 dakika**. Cevap için ayrılan yer yetmiyorsa, sınav kağıdınızdaki boş yerleri kullanabilirsiniz. Cevaplarınızı **AÇIKLAMALISINIZ**. Yeterince açıklanmamış cevaplar –sonuç doğru olsa bile– ya hiç puan alamayacak ya da çok az puan alacak.

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1. Find the following limits.

(a) (10 Points) Let $f(x) = x^3 - 3x^2 - 1$, $x \geq 2$. Find the value of df^{-1}/dx at the point $x = -1 = f(3)$.

Solution:

$$\frac{df}{dx} = 3x^2 - 6x \Rightarrow \left[\frac{df}{dx}\right]_{x=3} = 3(3)^2 - 6(3) = 9 \Rightarrow \left[\frac{df^{-1}}{dx}\right]_{x=-1} = \left[\frac{1}{\frac{df}{dx}}\right]_{x=3} = \boxed{\frac{1}{9}}$$

p.72, pr.15

(b) (10 Points) Evaluate the integral $\displaystyle\int \frac{3\sec^2 t}{6+3\tan t}\,dt$.

Solution: Let $u = 6+3\tan t$ and so $\mathrm{d}u = 3\sec^2 t\,\mathrm{d}t$. Then

$$\int \frac{3\sec^2 t}{6+3\tan t}\,dt = \int \frac{\mathrm{d}u}{u} = \ln|u| + C = \boxed{\ln|6+3\tan t| + C}$$

p.94, pr.10

(c) (10 Points) $\displaystyle\frac{d}{dx}\left(x^3\log_{10} x\right) =$

Solution: By the product rule of derivatives, we have

$$\begin{aligned}\frac{d}{dx}\left(x^3\log_{10} x\right) &= x^3\frac{d}{dx}(\log_{10} x) + (\log_{10} x)\frac{d}{dx}(x^3)\\ &= x^3\frac{1}{x\ln 10} + (\log_{10} x)(3x^2)\\ &= \frac{x^2}{\ln 10} + 3x^2\log_{10} x\end{aligned}$$

p.94, pr.34

2. (a) (12 Points) Find the limit $\lim_{x\to 0^+}\left(\frac{3x+1}{x}-\frac{1}{\sin x}\right)$.

Solution:

$$\begin{aligned}\lim_{x\to 0^+}\left(\frac{3x+1}{x}-\frac{1}{\sin x}\right) &= \lim_{x\to 0^+}\left(\frac{(3x+1)\sin x - x}{x\sin x}\right)\\ &= \lim_{x\to 0^+}\left(\frac{3\sin x+(3x+1)\cos x-1}{\sin x + x\cos x}\right)\\ &= \lim_{x\to 0^+}\left(\frac{3\cos x+3\cos x-(3x+1)\sin x-0}{\cos x+\cos x - x\sin x}\right)\\ &= \frac{3\cos(0)+3\cos(0)-(3(0)+1)\sin(0)-0}{\cos(0)+\cos(0)-(0)\sin(0)}\\ &= \frac{3+3-(1)0-0}{1+1-0}=\frac{6}{2}=\boxed{3}\end{aligned}$$

p.72, pr.8

You may use L'Hôpital's rule wherever applicable.

(b) (8 Points) Find the derivative $\frac{dy}{dv}$ of $y=\ln(\sinh v)-\frac{1}{2}\coth^2 v$ with respect to v.

Solution: We have

$$\begin{aligned}\frac{dy}{dv} &= \frac{d}{dv}\left(\ln(\sinh v)-\frac{1}{2}\coth^2 v\right)\\ &= \frac{d}{dv}\ln(\sinh v)-\frac{1}{2}\frac{d}{dv}\left(\coth^2 v\right)\\ &= \frac{1}{\sinh v}\frac{d}{dv}(\sinh v)-\frac{1}{\not{2}}(\not{2}\coth v)\frac{d}{dv}(\coth v)\\ &= \frac{\cosh v}{\sinh v}-\coth v(-\operatorname{csch}^2 v)=\coth v+\coth v\operatorname{csch}^2 v\\ &= \boxed{\coth v(1+\operatorname{csch}^2 v)}\end{aligned}$$

i

p.82, pr.35

3. (a) (12 Points) Calculate $\sin^{-1}\left(\sin\frac{3\pi}{2}\right)+\sec^{-1}(2)$.

Solution: Note that it would be wrong to give $3\pi/2$ as the answer, since $\sin^{-1}$ is always in the interval $[-\pi/2,\pi/2]$. Work the problem in steps as follows.

$$\sin^{-1}\left(\sin\frac{3\pi}{2}\right)=\sin^{-1}(-1)=\boxed{-\pi/2}.$$

Moreover

$$\sec^{-1}(2)=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}.$$

and so

$$\sin^{-1}\left(\sin\frac{3\pi}{2}\right)+\sec^{-1}(2)=-\frac{\pi}{2}+\frac{\pi}{3}=\boxed{-\frac{\pi}{6}}$$

p.72, pr.8

First find them separately and then add them up.

(b) (13 Points) Evaluate the integral $\int \sec^4(4\theta)\ d\theta$.

Solution: First notice that

$$\begin{aligned}\int \sec^4(4\theta)\ d\theta &= \int \sec^2(4\theta)\sec^2(4\theta)\ d\theta\\ &= \int (1+\tan^2(4\theta))\sec^2(4\theta)\ d\theta\end{aligned}$$

Let $u=\tan(4\theta)$ and so $du=4\sec^2(4\theta)\,d\theta$. Hence

$$\begin{aligned}\int \sec^4(4\theta)\ d\theta &= \int \sec^2(4\theta)\sec^2(4\theta)\ d\theta\\ &= \frac{1}{4}\int \underbrace{(1+\tan^2(4\theta))}_{1+u^2}\underbrace{4\sec^2(4\theta)\ d\theta}_{du}\\ &= \frac{1}{4}\int (1+u^2)\ du\\ &= \frac{1}{4}\left[u+\frac{1}{3}u^3\right]+C\\ &= \boxed{\frac{1}{4}\tan(4\theta)+\frac{1}{12}\tan^3(4\theta)+C}\end{aligned}$$

p.83, pr.52

4. (a) (13 Points) Evaluate the integral $\int x(\ln x)^2\, \mathrm{d}x$.

Solution: We shall integrate by parts. Let $u = (\ln x)^2$ and so $dv = dx$. Then $du = 2(\ln x)\frac{1}{x}\ \mathrm{d}x$ and choose $v = x$. Therefore

$$\begin{aligned} \int x(\ln x)^2\, \mathrm{d}x &= \int u\, \mathrm{d}v = uv - \int v\, \mathrm{d}u \\ &= (\ln x)^2(x) - \int \cancel{(x)} 2(\ln x)\frac{1}{\cancel{x}}\ \mathrm{d}x \\ &= x(\ln x)^2 - 2\int \ln x\ \mathrm{d}x \\ &= x(\ln x)^2 - 2\,(x\ln x - x) + C \\ &= \boxed{x(\ln x)^2 - 2x\ln x + 2x + C} \end{aligned}$$

p.95, pr.68

(b) (12 Points) Evaluate the integral $\int \frac{2x^3 - 2x^2 + 1}{x^2 - x}\, \mathrm{d}x$.

Solution: By long division of polynomials, we have

$$\begin{array}{r|l} & 2x + 0 \\ \hline x^2 - x + 0 & 2x^3 - 2x^2 + 0x + 1 \\ & -2x^3 + 2x^2 + 0x \\ \hline & 0x^2 + 0x + 1 \\ & 0x^2 + 0x + 0 \\ \hline & 0x + 1 \end{array}$$

Therefore,

$$\frac{2x^3 - 2x^2 + 0x + 1}{x^2 - x} = 2x + \frac{1}{x^2 - x}.$$

We decompose the integrand in the following way:

$$\frac{1}{x^2 - x} = \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

Clearing the fractions changes to

$$1 = A(x-1) + Bx \Rightarrow 1 = x(A+B) - A \Rightarrow -A = 1, A + B = 0 \Rightarrow A = -1, B = 1.$$

Thus,

$$\begin{aligned} \int \frac{2x^3 - 2x^2 + 1}{x^2 - x}\, \mathrm{d}x &= \int \left(2x - \frac{1}{x} + \frac{1}{x-1}\right) \mathrm{d}x = \int 2x\ \mathrm{d}x - \int \frac{1}{x}\ \mathrm{d}x + \int \frac{1}{x-1}\ \mathrm{d}x \\ &= \boxed{x^2 - \ln|x| + \ln|x-1| + C} \end{aligned}$$

p.112, pr.26