Your Name / Adınız - Soyadınız | Signature / İmza

(**süre:60 dak**)

Student ID # / Öğrenci No

(**mavi tükenmez!**)

Soru	1	2	3	4	Toplam
Puan	30	22	22	26	100
Puanınız					

1. (a) (15 Puan) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+\sqrt{n+1}}$ serisi mutlak yakınsak mıdır? Koşullu yakınsak mıdır? Iraksak mıdır? Açıklayınız.

Solution: This series *converges conditionally* since $\left\{\frac{1}{\sqrt{n}+\sqrt{n+1}}\right\}_{n=1}^{\infty}$ is a decreasing sequence of positive terms converging to $0 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+\sqrt{n+1}}$ converges by the Alternating Series Test; but

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}+\sqrt{n+1}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n}+\sqrt{n+1}} = \lim_{n\to\infty} \frac{1}{1+\sqrt{1+\frac{1}{n}}} = \frac{1}{2}$$

so that $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$ diverges by the Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ which is a divergent p-series.

p.573, pr.44

(b) (15 Puan) $\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{4^n}$ serisinin yakınsaklık aralığını bulunuz. Bu aralıkta seriyi x'in bir fonksiyonu olarak yazınız.

Solution:
$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| < 1 \Rightarrow \lim_{n\to\infty} \left|\frac{(x-1)^{2n+2}}{4^{n+1}} \frac{4^n}{(x-1)^{2n}}\right| < 1 \Rightarrow \frac{(x-1)^2}{4} \lim_{n\to\infty} |1| < 1 \Rightarrow (x-1)^2 < 4 \Rightarrow |x-1| < 2 \Rightarrow -2 < x-1 < 2 \Rightarrow -1 < x < 3$; at $x=-1$ we have $\sum_{n=0}^{\infty} \frac{(-2)^{2n}}{4^n} = \sum_{n=0}^{\infty} \frac{4^n}{4^n} = \sum_{n=0}^{\infty} (1)$, which diverges; at $x=3$ we have $\sum_{n=0}^{\infty} \frac{2^{2n}}{4^n} = \sum_{n=0}^{\infty} \frac{4^n}{4^n} = \sum_{n=0}^{\infty} (1)$, a divergent series; the interval of convergence is $- < x < 3$; the series $\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{4^n} = \sum_{n=0}^{\infty} \left(\left(\frac{x-1}{2}\right)^2\right)^n$ is a convergent geometric series when $-1 < x < 3$ and the sum is

$$\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{4^n} = \sum_{n=0}^{\infty} \left(\left(\frac{x-1}{2}\right)^2\right)^n = \frac{1}{1-\left(\frac{x-1}{2}\right)^2} = \frac{1}{\left[\frac{4-(x-1)^2}{4}\right]} = \frac{4}{4-x^2+2x-1} = \frac{4}{3+2x-x^2}$$

p.583 pr.43

2. (a) (11 Puan) $x^2+y^2+(z+3)^2 = 25$ ve $z=0$ denklemlerini sağlayan noktalar kümesinin geometrik tarifini yapınız.

Solution:
$x^2+y^2+(z+3)^2 = 25$ and $z=0$ imply that $x^2+y^2+(0+3)^2 = 25 \Rightarrow x^2+y^2 = 25-9 = 16$ so we have $x^2+y^2=16$ is the circle in the xy-plane.

p.663, pr.11

(b) (11 Puan) Orijin merkezli 1 yarıçaplı üst yarı-küreyi belirleyen eşitliği yazınız.

Solution: The required equality is $z = \sqrt{1-x^2-y^2}$.

p.664, pr.38

3. (a) (11 Puan) $P_0(3,-2,1)$'den geçen $x=1+2t, \quad y=2-t, \quad z=3t$ doğrusuna paralel olan doğrunun parametrik denklemlerini yazınız.

Solution: The direction is $\mathbf{v}=2\mathbf{i}-\mathbf{j}+3\mathbf{k}$. The line will have the parametric equations as $x=3+2t$, $y=-2-t$, $z=1+3t$ where $-\infty<t<+\infty$.

p.694, pr.6

(b) (11 Puan) $A(1,1,-1), \quad B(2,0,2), \quad C(0,-2,1)$ noktalarından geçen düzlem denklemini yazınız.

Solution: First we need vectors. We have $\vec{AB}=(2-1)\mathbf{i}+(0-1)\mathbf{j}+(2-(-1))\mathbf{k}=\mathbf{i}-\mathbf{j}+3\mathbf{k}$ and $\vec{AC}=(0-1)\mathbf{i}+(-2-1)\mathbf{j}+(1-(-1))\mathbf{k}=-\mathbf{i}-3\mathbf{j}+2\mathbf{k}$. Now

$$\Rightarrow \mathbf{n}=\vec{AB}\times\vec{AC}=\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 3 \\ -1 & -3 & 2 \end{vmatrix}=\begin{vmatrix} -1 & 3 \\ -3 & 2 \end{vmatrix}\mathbf{i}-\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix}\mathbf{j}+\begin{vmatrix} 1 & -1 \\ -1 & -3 \end{vmatrix}\mathbf{k}=7\mathbf{i}-5\mathbf{j}-4\mathbf{k}$$

is normal to the plane. Hence the plane has the equation

$$7(x-2)+(-5)(y-0)+(-4)(z-2)=0 \Rightarrow \boxed{7x-5y-4z=6.}$$

p.695, pr.23

4. (a) (12 Puan) $x=2, \quad y=3+2t, \quad z=-2-2t$ doğrusunun $6x+3y-4z=-12$ düzlemini kestiği noktayı bulunuz.

Solution: $6x+3y-4z=-12 \Rightarrow 6(2)+3(3+2t)-4(-2-2t)=-12 \Rightarrow 14t+29=-12 \Rightarrow t=-\dfrac{41}{14} \Rightarrow x=2, y=3-\dfrac{41}{7}$ and $z=-2+\dfrac{41}{7}$ so $(2,-\dfrac{20}{7},\dfrac{27}{7})$ is the point we we are asked.

p.695, pr.54

(b) (14 Puan) Orijinden geçen ve $2x-y-z=4$ düzlemine dik olan doğrunun $3x-5y+2z=6$ düzlemiyle kesim noktasını bulunuz.

Solution: The line containing $(0,0,0)$ normal to the plane is represented by $x=2t$, $y=-t$, and $z=-t$. This line intersects the plane $3x-5y+2z=6$ when $3(2t)-5(-t)+2(-t)=6 \Rightarrow t=\dfrac{2}{3} \Rightarrow$ the point of intersection is $\left(\dfrac{4}{3},-\dfrac{2}{3},-\dfrac{2}{3}\right)$.

p.703, pr.54