Math 114 Summer 2017 **Second Exam** July 27, 2017

Your Name / Ad - Soyad ________ (**75 min.**) Signature / İmza ________

Student ID # / Öğrenci No ☐☐☐☐☐☐☐☐☐ (**mavi tükenmez!**)

Problem	1	2	3	4	Total
Points:	25	23	25	27	100
Score:					

Time limit is **75 minutes**. If you need more room on your exam paper, you may use the empty spaces on the paper. You have to show your. Answers without reasonable-even if your results are true- work will either get zero or very little credit.

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1. (a) (10 Points) Does the series $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n\sqrt{n}}$ converge absolutely, conditionally, or diverge? Justify your answer.

◯ Converges absolutely. ◯ Converges conditionally. ◯ Diverges. Test Used: ________

Solution: Since

$$\sum_{n=1}^{\infty} \left|\frac{\cos(n\pi)}{n\sqrt{n}}\right| = \sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n\sqrt{n}}\right| = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

is a convergent p-series ($p = 3/2 > 1$), the given *series converges absolutely*.

p.573, pr.36

(b) (15 Points) Find the radius and interval of convergence of $\sum_{n=0}^{\infty} \frac{n(x+3)^n}{5^n}$.

Radius of Convergence: ________. Interval of Convergence: ________

Solution: Let $u_n = \frac{n(x+3)^n}{5^n}$. Then

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| < 1 \Rightarrow \lim_{n\to\infty}\left|\frac{\frac{(n+1)(x+3)^{n+1}}{5^{n+1}}}{\frac{n(x+3)^n}{5^n}}\right| < 1 \Rightarrow \lim_{n\to\infty}\left|\frac{(x+3)}{5}\frac{n+1}{n}\right| < 1 \Rightarrow \frac{|x+3|}{5}\underbrace{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)}_{=1} < 1$$

$$\Rightarrow |x+3| < 5$$
$$\Rightarrow -5 < x+3 < 5$$
$$\Rightarrow -8 < x < 2$$

When $x = -8$, we have $\sum_{n=1}^{\infty} \frac{n(-5)^n}{5^n} = \sum_{n=1}^{\infty} (-1)^n n$, a divergent series.

When $x = 2$, we have $\sum_{n=1}^{\infty} \frac{n5^n}{5^n} = \sum_{n=1}^{\infty} n$, a divergent series. So the radius of convergence is $R = 5$; the interval of convergence is $-8 < x < 2$.

p.583, pr.17

2. (a) (10 Points) Find the Maclaurin series for $f(x) = x\mathrm{e}^x$.

Solution: We know that

$$\mathrm{e}^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots \qquad (-\infty < x < +\infty)$$

is the Maclaurin series for e^x with radius of convergence $R = \infty$. Now multiplying by x gives

$$x\mathrm{e}^x = x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^5}{4!} + \cdots + \frac{x^{n+1}}{n!} + \cdots \qquad (-\infty < x < +\infty)$$

is the Maclaurin series for $x\mathrm{e}^x$ with radius of convergence $R = \infty$.

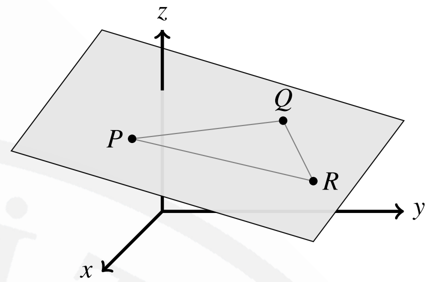
p.588, pr.12

(b) (13 Points) Find the area of the triangle $\triangle(PQR)$ determined by the points $P(2,-2,1)$, $Q(3,-1,2)$, $R(3,-1,1)$.

Solution: First $\vec{PQ} = (3-2)\mathbf{i} + (-1+2)\mathbf{j} + (2-1)\mathbf{k} = \mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\vec{PQ} = (3-2)\mathbf{i} + (-1+2)\mathbf{j} + (1-1)\mathbf{k} = \mathbf{i}+\mathbf{j}$. Then

$$\begin{aligned}\vec{PQ}\times\vec{PR} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} \\ &= -\mathbf{i}+\mathbf{j} \\ \Rightarrow \text{Area} &= \frac{1}{2}|\vec{PQ}\times\vec{PR}| \\ &= \frac{1}{2}|-\mathbf{i}+\mathbf{j}| \\ &= \frac{1}{2}\sqrt{(-1)^2+(1)^2} = \frac{\sqrt{2}}{2}\end{aligned}$$

p.687, pr.17(a)



3. (a) (11 Points) Find the volume of the parallelepiped if four of its vertices are $A(0,0,0)$, $B(0,1,2)$, $C(0,-3,2)$, and $D(3,-4,5)$.

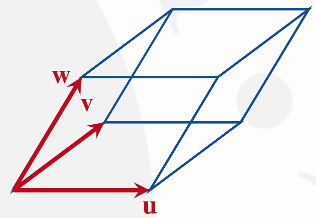
Solution: We have

$$\vec{AB} = \mathbf{j}+2\mathbf{k}, \qquad \vec{AC} = -3\mathbf{j}+2\mathbf{k}, \qquad \vec{AD} = 3\mathbf{i}-4\mathbf{j}+5\mathbf{k},$$

$$\begin{aligned}(\vec{AB}\times\vec{AC})\cdot\vec{AD} &= \begin{vmatrix} 0 & 1 & 2 \\ 0 & -3 & 2 \\ 3 & -4 & 5 \end{vmatrix} \\ &= 0\begin{vmatrix} -3 & 2 \\ -4 & 5 \end{vmatrix} + 0\begin{vmatrix} 1 & 2 \\ -4 & 5 \end{vmatrix} + (3)\begin{vmatrix} 1 & 2 \\ -3 & 2 \end{vmatrix} \\ &= 0+0+3((1)(2)-(-3)(2)) = \boxed{24}\end{aligned}$$

Hence Volume $= |(\vec{AB}\times\vec{AC})\cdot\vec{AD}| = 24$.

p.688, pr.48



(b) (14 Points) Find parametric equations for the line $\mathscr{L}$ through $P_0(2,3,0)$ perpendicular to the vectors $\mathbf{u} = \mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $\mathbf{v} = 3\mathbf{i}+4\mathbf{j}+5\mathbf{k}$.

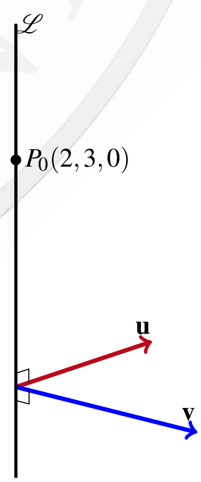
Solution: The direction is

$$\begin{aligned}\mathbf{u}\times\mathbf{v} == &\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix} \\ = &-2\mathbf{i}+4\mathbf{j}-2\mathbf{k}\end{aligned}$$

Hence the parametric equations for the line $\mathscr{L}$ are

$$\boxed{x = 2-2t, \qquad y = 3+4t, \qquad z = -2t}$$

p.694, pr.10



4. (a) (7 Points) Find the value(s) of c if the function

$$f(x,y)=\begin{cases}\dfrac{3x^2y}{x^2+y^2} & (x,y)\neq(0,0)\\ c, & (x,y)=(0,0)\end{cases}$$

is continuous at $(0,0)$.

Solution: We employ the polar coordinates: $x=r\cos\theta$ and $y=r\sin\theta$. Then $x^2+y^2=r^2$ and $r\to 0$ as $(x,y)\to(0,0)$. Hence we have

$$\begin{aligned}\lim_{(x,y)\to(0,0)} f(x,y) &= \lim_{r\to 0}\frac{3r^3\cos^2\theta\sin\theta}{r^2}\\ &= \lim_{r\to 0}(3r\cos^2\theta\sin\theta)\\ &= 0\end{aligned}$$

So $f(x,y)$ is continuous at $(0,0)$ iff $\boxed{\text{c=0}}$.

p.764, pr.68

(b) (10 Points) Find f_x, f_y, and f_z if $f(x,y,z)=yz\ln(xy)$

Solution: We have

$$\begin{aligned}f_x &= yz\cdot\frac{1}{xy}\cdot\frac{\partial}{\partial x}(xy)=\cdot\frac{(yz)y}{xy}=\boxed{\frac{yz}{x}}\\ f_y &= z\ln(xy)+yz\cdot\frac{\partial}{\partial y}(\ln(xy))=z\ln(xy)+\frac{yz}{xy}\cdot\frac{\partial}{\partial y}(xy)=\boxed{z\ln(xy)+z},\\ f_z &= y\ln(xy)+yz\cdot\frac{\partial}{\partial z}(xy)=\boxed{y\ln(xy)}\end{aligned}$$

p.773, pr.30

(c) (10 Points) Suppose $w=2ye^x-\ln z$, $x=\ln(t^2+1)$, $y=\tan^{-1}t$, and $z=e^t$. Using the *Chain Rule* formula, find $\left[\dfrac{dw}{dt}\right]_{t=1}$

Solution:

$$\frac{\partial w}{\partial x}=2ye^x,\ \frac{\partial w}{\partial y}=2e^x,\ \frac{\partial w}{\partial z}=-\frac{1}{z},\ \frac{dx}{dt}=\frac{2t}{t^2+1},\ \frac{dy}{dt}=\frac{1}{t^2+1},\ \frac{dz}{dt}=e^t,$$

$$\Rightarrow \frac{dw}{dt}=\frac{\partial w}{\partial x}\frac{dx}{dt}+\frac{\partial w}{\partial y}\frac{dy}{dt}+\frac{\partial w}{\partial z}\frac{dz}{dt}$$

$$\frac{dw}{dt}=\frac{4yte^x}{t^2+1}+\frac{2e^x}{t^2+1}-\frac{e^t}{z}=\frac{4t(\tan^{-1})(t^2+1)}{t^2+1}+\frac{2(t^2+1)}{t^2+1}-\frac{e^t}{e^t}=4t\tan^{-1}t+1;$$

$$\left[\frac{dw}{dt}\right]_{t=1}=(4)(1)(\frac{\pi}{4})+1=\boxed{\pi+1}$$

p.782, pr.5