Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- Calculators, cell phones off and away!.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place a box around your answer to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 90 min.

Do not write in the table to the right.

Problem	Points	Score
1	30	
2	12	
3	33	
4	25	
Total:	100	

1. (a) [8 Points] Find the value of $\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$.

   (b) [12 Points] Find the integral $\displaystyle\int_1^2 \frac{8\,\mathrm{d}x}{x^2-2x+2}$.

Solution:

$$\int_1^2 \frac{8\,\mathrm{d}x}{x^2-2x+2} = 8\int_1^2 \frac{\mathrm{d}x}{1+(x^2-2x+1)} = 8\int_1^2 \frac{\mathrm{d}x}{1+(x-1)^2}$$
$$= 8\left[\tan^{-1}(x-1)\right]_1^2$$
$$= 8\left[\tan^{-1}(2-1)-\tan^{-1}(1-1)\right]_1^2 = 8\left(\frac{\pi}{4}-0\right) = \boxed{2\pi}$$

p.115, pr.11

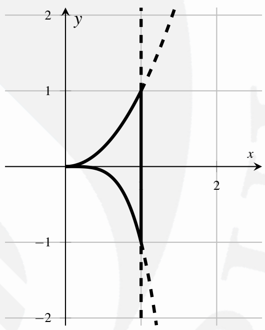
   (c) [10 Points] Find the derivative $\frac{dy}{dx}$ if $y=\tan^{-1}\sqrt{x^2-1}+\csc^{-1}x$ where $x>1$.

Solution:

$$y=\tan^{-1}\sqrt{x^2-1}+\csc^{-1}x=\tan^{-1}(x^2-1)^{1/2}+\csc^{-1}x$$
$$\frac{dy}{dx}=\frac{\frac{1}{2}(x^2-1)^{-1/2}2x}{1+((x^2-1)^{1/2})^2}-\frac{1}{|x|\sqrt{x^2-1}}=\frac{\frac{x}{\sqrt{x^2-1}}}{x^2}-\frac{1}{|x|\sqrt{x^2-1}}=\frac{1}{x\sqrt{x^2-1}}-\frac{1}{|x|\sqrt{x^2-1}}=\boxed{0}\text{ as } x>1$$

p.192, pr.87

2. [12 Points] Find the volume of the solid generated by revolving the region bounded by $y=x^2$, $y=-x^4$ and $x=1$ about y-axis.



Solution: If we use the shells, we slice vertically and so we have

$$V=\int_a^b 2\pi\begin{pmatrix}\text{shell}\\\text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\\text{height}\end{pmatrix}\mathrm{d}x$$
$$=\int_0^1 2\pi x(x^2-(-x^4))\ \mathrm{d}x$$
$$=2\pi\int_0^1 (x^3+x^5)\ \mathrm{d}x = 2\pi\left[\frac{x^4}{4}+\frac{x^6}{6}\right]_0^1$$
$$=2\pi(\frac{1}{4}+\frac{1}{6})=\boxed{\frac{5\pi}{6}}.$$

p.191, pr.22

3. (a) 11 Points Find $\frac{dy}{dx}$ if $y = \int_{\sec x}^{2} \frac{1}{t^2+1}\,\mathrm{d}t$.

Solution: First let $u = \sec x$ and so $du = \sec x \tan x\, dx$. Now we have

$$\frac{dy}{dx} = \frac{d}{dx}\left(\int_{\sec x}^{2} \frac{dt}{1+t^2}\right) = \frac{d}{dx}\left(-\int_{2}^{\sec x} \frac{dt}{1+t^2}\right) = -\frac{d}{dx}\left(\int_{2}^{u} \frac{dt}{1+t^2}\right) = -\frac{d}{du}\left(\int_{2}^{u} \frac{dt}{1+t^2}\right)\frac{du}{dx}$$

$$= -\frac{1}{1+u^2}\sec x \tan x$$

$$= -\frac{1}{1+\sec^2 x}\sec x \tan x = \boxed{-\frac{\sec x \tan x}{1+\sec^2 x}}$$

p.191, pr.22

(b) 10 Points Evaluate the integral $\int_{0}^{\pi/2} \frac{5\sin x \cos x}{\sqrt{4+5\cos^2 x}}\,\mathrm{d}x$.

Solution: Let $u = 4+5\cos^2 x$ and so $\mathrm{d}u = -10\cos x \sin x\,\mathrm{d}x$. When $x = 0$, we have $u = 9$ and when $x = \pi/2$, we have $u = 4$. Therefore

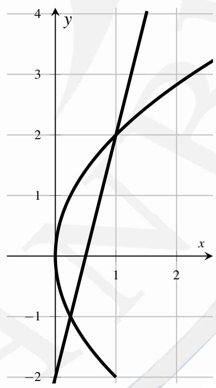
$$\int_{0}^{\pi/2} \frac{5\sin x \cos x}{\sqrt{4+5\cos^2 x}}\,\mathrm{d}x = -\frac{5}{10}\int_{0}^{\pi/2} \frac{1}{\sqrt{4+5\cos^2 x}}(-10)\sin x \cos x\,\mathrm{d}x$$

$$= -\frac{1}{2}\int_{9}^{4} \frac{1}{\sqrt{u}}\,\mathrm{d}u$$

$$= \int_{4}^{9} \frac{1}{2\sqrt{u}}\,\mathrm{d}u = \left[\sqrt{u}\right]_4^9 = \sqrt{9}-\sqrt{4} = 3-2 = \boxed{1}$$

p.94, pr.10

(c) 12 Points Find the area of the region enclosed by $y^2 = 4x$ and $y = 4x-2$.



Solution: Let us find the intersection points:

$\frac{y^2}{4} = \frac{y+2}{4} \Rightarrow y^2 - y - 2 = 0 \Rightarrow (y-2)(y+1) = 0 \Rightarrow y = -1$ or $y = 2 \Rightarrow\ c = -1\ , d = 2$;

$f(y) = \frac{y+2}{4}$ and $g(y) = \frac{y^2}{4}$ and so

$$\text{AREA } = \int_{c}^{d}[f(y)-g(y)]\,\mathrm{d}y = \int_{-1}^{2}\left(\frac{y+2}{4} - \frac{y^2}{4}\right)\mathrm{d}y = \frac{1}{4}\int_{-1}^{2}(y+2-y^2)\,\mathrm{d}y = \frac{1}{4}\left[\frac{y^2}{2}+2y-\frac{y^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{4}\left[\left(\frac{4}{2}+4-\frac{8}{3}\right)-\left(\frac{1}{2}-2+\frac{1}{3}\right)\right] = \boxed{\frac{9}{8}}$$

p.321, pr.21

4. (a) 11 Points For what value(s) of k will the curve $y = x^3 + kx^2 + 3x - 4$ have *exactly one* horizontal tangent?

Solution: $f'(x) = 3x^2 + 2kx + 3 = 0$ (slope must be zero, for horizontal tangent)

To determine the types and number of roots of a quadratic equation $ax^2 + bx + c = 0$, we must examine the discriminant $\Delta = b^2 - 4ac$:

- If $b^2 - 4ac > 0$ then there are two real (unique) roots
- If $b^2 - 4ac = 0$ then there is one (double) real root
- If $b^2 - 4ac < 0$ then there are two imaginary roots

For single tangent, $b^2 - 4ac = 0$,
So, $4k^2 - 36 = 0$
$k^2 - 9 = 0$
$\Rightarrow k = \pm 3$.

p.257, pr.4

(b) 14 Points Sketch the graph of $y = \dfrac{2x}{x+5} = 2 - \dfrac{10}{x+5}$. Find the asymptotes. Find all local maximum/minimum points and inflection points, if any. You may assume $y' = \dfrac{10}{(x+5)^2}$ and $y'' = -\dfrac{20}{(x+5)^3}$.

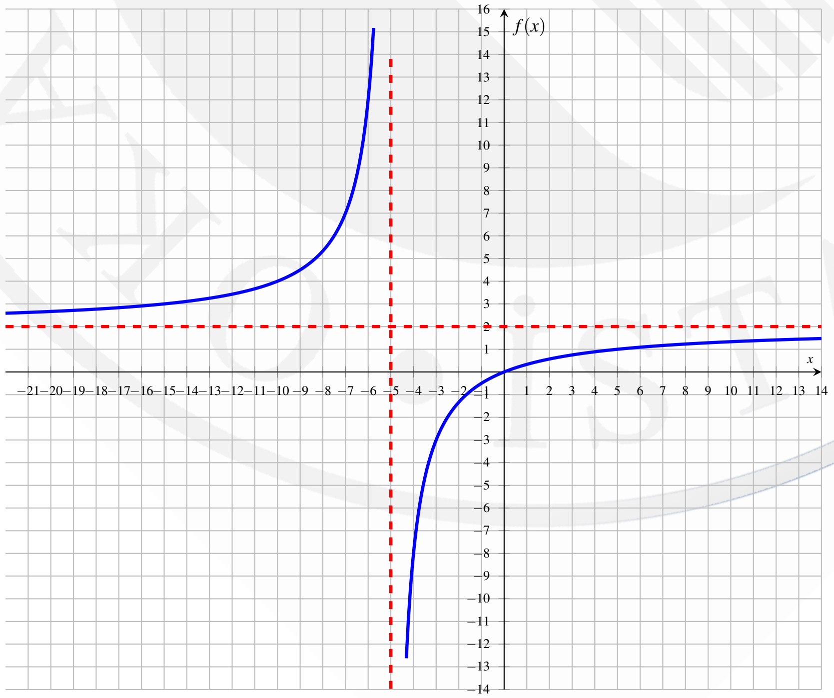
Solution: First $\lim_{x\to -5^+} \dfrac{2x}{x+5} = -\infty$, $\lim_{x\to -5^-} \dfrac{2x}{x+5} = +\infty$. So the line $x = -5$ *is a vertical asymptote*. Since $\lim_{x\to\pm\infty} \dfrac{2x}{x+5} = 2$, the line $y = 2$ is a horizontal asymptote.

Since $y' = \dfrac{10}{(x_5)^2} > 0$ for all $x \neq -5$, the graph is increasing on $(-\infty,-5)\cup(-5,\infty)$. Therefore no local maxima or minima occurs.

Moreover, we have $y'' = \dfrac{20}{(x+5)^3}$ and so

$$y'' \begin{cases} > 0, & \text{on } (-\infty,-5) \quad \text{y is concave up} \\ < 0, & \text{on } (-5,\infty) \quad \text{y is concave down} \end{cases}$$

Hence f is concave up on $(-\infty,-5)$ and concave down on $(-5,\infty)$. Also graph has *no point of inflection* as there is no tangent line at $x = 3$.



p.212, pr.85