FORENAME:

SURNAME:

STUDENT NO:

DEPARTMENT:

TEACHER: ☐ Neil Course ☐ Vasfı Eldem ☐ Asuman Özer ☐ Sezgin Sezer

SIGNATURE:

Question	Points	Score
1	65	
Total:	65	

- The time limit is 90 minutes.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- All communication between students, either verbally or non-verbally, is strictly forbidden.
- Calculators, mobile phones, smart watches, and any digital means of communication are forbidden. The sharing of pens, erasers or any other item between students is forbidden.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- Please do not write in the table above.

1. (a) [10 points] For what value of a is $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$ continious at every x?

Solution: The functions $x^2 - 1$ and $2ax$ are continious on which they are defined. In order to the function f be continious, left and right hand limit at $x = 3$ should be equal:

$$\lim_{x\to 3^-} (x^2 - 1) = 8$$

$$\lim_{x\to 3^-} (2ax) = 6a$$

$$6a = 8 \Rightarrow a = \frac{4}{3}$$

(b) [10 points] Show that the equation $x^4 + 2x^2 - 2 = 0$ has exactly one solution on $[0, 1]$.

Solution:

$$f(x) = x^4 + 2x^2 - 2$$

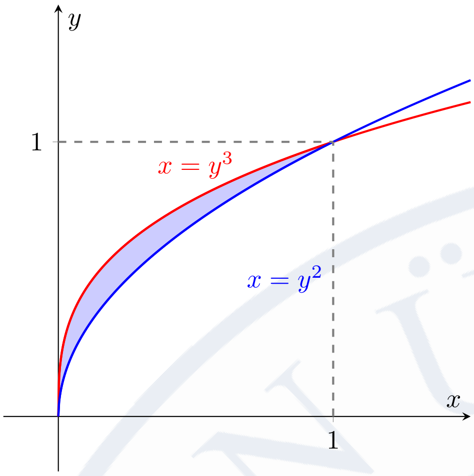
$$f(0) = -2 < 0, \quad f(1) = 1 > 0.$$

So, the equation has at least one solution at $[0, 1]$. Moreover, the equation

$$f'(x) = 4x^3 + 4x$$

is increasing on $[0, 1]$ because $f'(x) > 0$ between $x = 0$ and $x = 1$. So, equalion has only one solution at on $[0, 1]$.

2. 15 points Find the area of the region enclosed by the curves $x = y^3$ and $x = y^2$.



Solution:

$$A = \int_0^1 (y^2 - y^3)dy$$

$$= \frac{y^3}{3} - \frac{y^4}{4}\Big|_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

3. (a) 15 points Find the length of the curve $y = \dfrac{x^5}{5} + \dfrac{1}{12x^3}$, $\frac{1}{2} \leq x \leq 1$.

Solution:

$$\frac{dy}{dx} = x^4 - \frac{1}{4x^4} \Rightarrow (\frac{dy}{dx})^2 = x^8 - \frac{1}{2} + \frac{1}{16x^8}$$

$$\Rightarrow \sqrt{1 + (\frac{dy}{dx})^2} = \sqrt{x^8 + \frac{1}{2} + \frac{1}{16x^8}} = \sqrt{(x^4 + \frac{1}{4x^4})^2} = x^4 + \frac{1}{4x^4}$$

$$\int_{-\frac{1}{2}}^{1} (x^4 + \frac{1}{4x^4})dx = \frac{x^5}{5} + \frac{1}{12x^3}\Big|_{-\frac{1}{2}}^{1} = \frac{153}{160}$$

(b) 30 points Find the volumes of the solids generated by revolving the region in the first quadrant bounded by the curve $x = y - y^3$ and the y axis about

(i). the x-axis

(ii). the line $y = 1$.

Solution:

(i). **the shell method** Shell radius: $r = y$
Shell high: $x = y - y^3$

$$V = \int_0^1 2\pi y(y - y^3)dy = 2\pi \int_0^1 (y^2 - y^4)dy$$

$$= 2\pi \left(\frac{y^3}{3} - \frac{y^5}{5}\right)\Big|_0^1 = 2\pi \left(\frac{1}{3} - \frac{1}{5}\right) = \frac{4\pi}{15}$$

(ii). **the shell method** Shell radius: $r = (1 - y)$
Shell high: x=y-y^3

$$V = \int_0^1 2\pi(1 - y)(y - y^3)dy = 2\pi \int_0^1 (y^4 - y^3 - y^2 + y)dy$$

$$= 2\pi \left(\frac{y^5}{5} - \frac{y^4}{4} - \frac{y^3}{3} + \frac{y^2}{2}\right)\Big|_0^1 = 2\pi \left(\frac{1}{5} - \frac{1}{4} - \frac{1}{3} + \frac{1}{2}\right) = \frac{7\pi}{30}$$

4. [20 points] Let $y = x^5 - 5x^4 = x^4(x-5)$ be given.

   1. Identify the set on which the function defined.
   2. Find local and absolute maximum, local minimum and saddle points.
   3. Identify the interval on which the function is increasing, decreasing, concave up and concave down.
   4. Sketch the graph.

Solution:

1. Critical Points: $x = 0$ and $x = 4$.
2. Local max: $f(0) = 0$, Local min:$f(4) = -256$, inf. point: $f(3) = -162$
3. There is no asymptotes!

x	$-\infty$		0		3		4		∞
$f'(x)$		$+$		-		-		$+$	
$f''(x)$		-		-		$+$		$+$	
$f(x)$