Math 113 Summer 2018 **Final Exam** August 9, 2018

Your Name / Ad - Soyad (**90 min.**) Signature / İmza

Student ID # / Öğrenci No (**use a blue pen!**)

Problem	1	2	3	4	Total
Points:	32	25	23	30	110
Score:					

Time limit is **90 minutes**. If you need more room on your exam paper, you may use the empty spaces on the paper. You have to show your. Answers without reasonable-even if your results are true- work will either get zero or very little credit.

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1. (a) (12 Points) Find the integral $\int \sqrt{x}\,\sin^2(x^{3/2}-1)\,dx$.

Solution: Let $y = x^{3/2}-1$. Then $dy = \frac{3}{2}x^{1/2}\,dx = \frac{3}{2}\sqrt{x}\,dx$. Therefore

$$\int \sqrt{x}\,\sin^2(x^{3/2}-1)\,dx = \frac{2}{3}\int \underbrace{\sin^2(x^{3/2}-1)}_{\sin^2 y}\underbrace{\frac{3}{2}\sqrt{x}\,dx}_{dy} = \frac{2}{3}\int \sin^2 y\,dy = \frac{2}{3}\int \frac{1}{2}(1-\cos(2y))\,dy$$

$$= \frac{1}{3}\left(y - \frac{1}{2}\sin(2y)\right) + C$$

$$= \frac{1}{3}y - \frac{1}{6}\cos(2y) + C = \boxed{\frac{1}{3}(x^{3/2}-1) - \frac{1}{6}\cos(2(x^{3/2}-1)) + C}$$

p.72, pr.15

(b) (10 Points) Use Logarithmic Differentiation to find the derivative $\frac{dy}{dx}$ if $y = (x+1)^x$.

Solution:

$$y = (x+1)^x \Rightarrow \ln y = \ln(x+1)^x = x\ln(x+1)$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x\ln(x+1))$$

$$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{x+1} + \ln(x+1) \Rightarrow \frac{dy}{dx} = y\left[\frac{x}{x+1} + \ln(x+1)\right] \Rightarrow \boxed{\frac{dy}{dx} = (x+1)^x\left[\frac{x}{x+1} + \ln(x+1)\right]}$$

p.94, pr.10

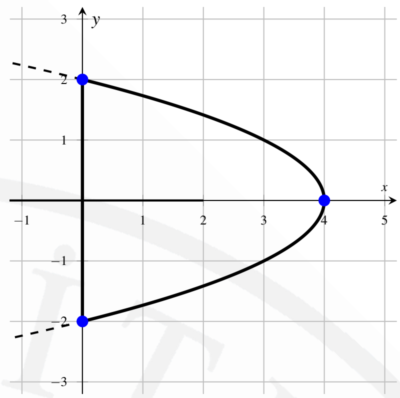
(c) (10 Points) Use L'Hôpital's Rule to find the limit $\lim_{h\to 0}\frac{e^h-(1+h)}{h^2}$.

Solution: Using L'Hôpital's Rule twice gives

$$\lim_{h\to 0}\frac{e^h-(1+h)}{h^2} = \lim_{h\to 0}\frac{e^h-1}{2h} = \lim_{h\to 0}\frac{e^h}{2} = \frac{e^0}{2} = \boxed{\frac{1}{2}}$$

p.94, pr.34

2. (a) (12 Points) Find the area enclosed by $x = 4 - y^2$ and $x = 0$.

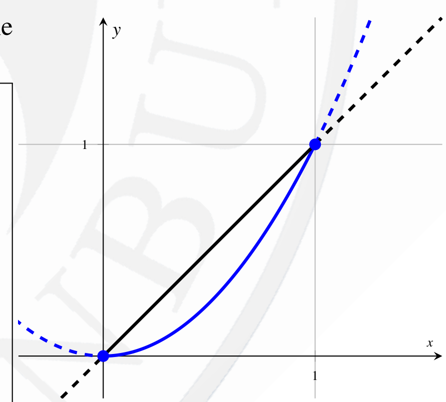


Solution:

$$\begin{aligned}\text{AREA } &= \int_{-2}^{2} \left[4 - y^2\right] \, dy = \left[4y - \frac{1}{3}y^3\right]_{-2}^{2} \\ &= \left[4(2) - \frac{1}{3}(2)^3\right] - \left[4(-2) - \frac{1}{3}(-2)^3\right] \\ &= 8 - \frac{8}{3} + 8 - \frac{8}{3} = 16 - \frac{16}{3} = \boxed{\frac{32}{3}}\end{aligned}$$

(Page 253, problem 3d)

(b) (13 Points) <u>Use the shell method</u> to find the volume of the solid generated by revolving the region bounded by $y = x$ and $y = x^2$ about x-axis.



Solution:

$$\begin{aligned}\text{V } &= 2\pi \int_{0}^{1} y\left(\sqrt{y} - y\right) \, dy = 2\pi \int_{0}^{1} y\left(y^{1/2} - y\right) \, dy \\ &= 2\pi \int_{0}^{1} \left(yy^{1/2} - yy\right) \, dy = 2\pi \int_{0}^{1} \left(y^{3/2} - y^2\right) \, dy \\ &= 2\pi \left[\frac{2}{5}y^{5/2} - \frac{1}{3}y^3\right]_{0}^{1} = 2\pi \left[\frac{2}{5} - \frac{1}{3}\right] = \boxed{\frac{2\pi}{15}}\end{aligned}$$

p.72, pr.55

3. (a) (13 Points) Find the <u>area of the surface</u> generated by revolving the curve $y = \sqrt{x}$, $\frac{3}{4} \le x \le \frac{15}{4}$ about the x-axis.

Solution: The surface area formula we shall use is $S = \int_a^b 2\pi y \sqrt{1 + (dy/dx)^2}\, dy$

$$y = \sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{4x} \Rightarrow S = \int_{3/4}^{15/4} 2\pi(\sqrt{x})\sqrt{1 + \frac{1}{4x}}\, dx$$

$$S = \cancel{2}\pi \int_{3/4}^{15/4} \cancel{\sqrt{x}} \quad \frac{\sqrt{4x+1}}{\cancel{2}\cancel{\sqrt{x}}}\, dx = \pi \int_{3/4}^{15/4} \sqrt{4x+1}\, dy \qquad \boxed{u = 4x+1, \qquad du = 4\, dx \Rightarrow}$$

$$= \frac{\pi}{4} \int_4^{16} \sqrt{u}\, du = \frac{\pi}{4}\left[\frac{u^{3/2}}{3/2}\right]_4^{16} = \frac{\pi}{6}\left[(16)^{3/2} - (4)^{3/2}\right] = \frac{\pi}{6}[64 - 8]$$

$$\longrightarrow S = \boxed{\frac{28\pi}{3}}$$

p.83, pr.52

(b) (10 Points) For $x \ge 2$, let $f(x) = x^3 - 3x^2 - 1$. Find the value of $\frac{df^{-1}}{dx}$ at the point $x = -1 = f(3)$.

Solution:

$$\frac{df}{dx} = 6x^2 + 3 \Rightarrow \left[\frac{df^{-1}}{dx}\right]_{x=6} = \left[\frac{1}{\frac{df}{dx}}\right]_{x=1} = \left[\frac{1}{6x^2+3}\right]_{x=1} = \boxed{\frac{1}{9}}$$

p.83, pr.52

4. (a) (5 Points) $\sec(\cos^{-1}(\frac{1}{2})) =$

Solution: $\sec(\cos^{-1}(\frac{1}{2})) = \sec(\frac{\pi}{3}) = \boxed{2}$

p.212, pr.85

(b) (13 Points) Over the interval $\left[\frac{1}{2e}, \frac{e}{2}\right]$, find the absolute maximum and absolute minimum values of $g(x) = x\ln(2x) - x$.

Solution: $g'(x) = x\left(\frac{2}{2x}\right) + \ln(2x) - 1 = 1 + \ln(2x) - 1 = \ln(2x)$; Solving $g'(x) = 0$, we get $\ln(2x) = 0 \Rightarrow 2x = 1$ and so $x = \frac{1}{2} \in \left[\frac{1}{2e}, \frac{e}{2}\right]$ is the only critical point. For $x > \frac{1}{2}$, we have $y' > 0$ and for $x < \frac{1}{2}$, we have $y' < 0$. Therefore, at $x = \frac{1}{2}$ the graph has a local minimum value $g(\frac{1}{2}) = \frac{1}{2}\underbrace{\ln((2)(\frac{1}{2}))}_{=\ln(1)=0} - \frac{1}{2}. = -\frac{1}{2}$. Moreover, $g(\frac{1}{2e}) = \frac{1}{2e}\ln\frac{2}{2e} - \frac{1}{2e} = -\frac{2}{2e} = -\frac{1}{e}$ and $g(\frac{e}{2}) = \frac{e}{2}\ln\frac{2e}{2} - \frac{e}{2} = 0$. Hence the graph has an absolute maximum value $-\frac{1}{e}$ at $x = \frac{1}{2e}$ and an absolute maximum value at $x = \frac{e}{2}$.

p.212, pr.85

(c) (12 Points) Find the value of the integral $\int_{-1}^{0} \frac{6\,dt}{\sqrt{3-2t-t^2}}$.

Solution:

$$\begin{aligned}\int_{-1}^{0} \frac{6\,dt}{\sqrt{3-2t-t^2}} &= 6\int_{-1}^{0} \frac{dt}{\sqrt{4-(t^2+2t+1)}} = 6\int_{-1}^{0} \frac{dt}{\sqrt{2^2-(t+1)^2}} = 6\left[\sin^{-1}\left(\frac{t+1}{2}\right)\right]_{-1}^{0} \\ &= 6\left[\sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)\right] \\ &= 6\left(\frac{\pi}{6} - 0\right) = \boxed{\pi}\end{aligned}$$

p.212, pr.85