2018–19 **MATH216 Mathematics IV – Exercise Sheet 4** N. Course

Exercise 19 (Homogeneous Second Order Linear ODEs with constant coefficients). Find the general solution of the following ODEs:

(a) $y'' - 2y' + 2y = 0$

(b) $y'' + 2y' + 2y = 0$

(c) $y'' + 2y' - 8y = 0$

(d) $y'' - 2y' + 6y = 0$

(e) $y'' + 6y' + 13y = 0$

(f) $9y'' + 16y = 0$

(g) $y'' - 2y' + y = 0$

(h) $9y'' + 6y' + y = 0$

(i) $4y'' + 12y' + 9y = 0$

(j) $4y'' - 4y' - 3y = 0$

(k) $y'' - 2y' + 10y = 0$

(l) $y'' - 6y' + 9y = 0$

(m) $4y'' + 17y' + 4y = 0$

(n) $4y'' + 20y' + 25y = 0$

(o) $25y'' - 20y' + 4y = 0$

(p) $2y'' + 2y' + y = 0$

Solve the following IVPs:

(q) $\begin{cases} 9y'' + 6y' + 82y = 0 \\ y(0) = -1 \\ y'(0) = 2 \end{cases}$

(r) $\begin{cases} y'' - 6y' + 9y = 0 \\ y(0) = 0 \\ y'(0) = 2 \end{cases}$

Exercise 20 (Reduction of Order). In each of the following problems:

(i) Check that y_1 solves the ODE;

(ii) Use the method of reduction of order to find a second, linearly independent solution, y_2 [HINT: Start with $y_2(t) = v(t)y_1(t)$.];

(iii) Check that your y_2 solves the ODE; and

(iv) Calculate the Wronskian of y_1 and y_2.

(a) $t^2y'' + 2ty' - 2y = 0, \quad t > 0; \quad y_1(t) = t$

(b) $t^2y'' - 4ty' + 6y = 0, \quad t > 0; \quad y_1(t) = t^2$

(c) $t^2y'' + 3ty' + y = 0, \quad t > 0; \quad y_1(t) = t^{-1}$

(d) $t^2y'' - t(t+2)y' + (t+2)y = 0, \quad t > 0; \quad y_1(t) = t$

(e) $xy'' - y' + 4x^3y = 0, \quad x > 0; \quad y_1(x) = \sin x^2$

(f) $(x-1)y'' - xy' + y = 0, \quad x > 1; \quad y_1(x) = e^x$