

Exercise 21 (The Method of Undetermined Coefficients). Find the general solutions of the following ODEs:

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|-------------------------------------|---------------------------------|---------------------------------------|
| (a) $y'' - 2y' - 3y = 3e^{2t}$ | (d) $y'' + 2y' = 3 + 4 \sin 2t$ | (g) $2y'' + 3y' + y = t^3 + 3 \sin t$ |
| (b) $y'' + 2y' + 5y = 3 \cos 2t$ | (e) $y'' + 9y = t^2 e^{3t} + 6$ | (h) $y'' + y = 3 \sin 2t + t \cos 2t$ |
| (c) $y'' - 2y' - 3y = 2 - 3te^{-t}$ | (f) $y'' + 2y' + y = 2e^{-t}$ | (i) $y'' + y' + 4y = 2 \sinh t$ |

Exercise 22 (The Method of Undetermined Coefficients). Solve the following IVPs:

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|---|---|
| (a) $\begin{cases} y'' + y' - 2y = 2t \\ y(0) = 0 \\ y'(0) = 1 \end{cases}$ | (c) $\begin{cases} y'' + 4y = t^2 + 3e^t \\ y(0) = 0 \\ y'(0) = 2 \end{cases}$ |
| (b) $\begin{cases} y'' - 2y' + y = te^t + 4 \\ y(0) = 1 \\ y'(0) = 1 \end{cases}$ | (d) $\begin{cases} -y'' + 6y' - 16y = 1 + 6e^{3t} \sin(2t) \\ y(0) = \frac{15}{16} \\ y'(0) = -1 \end{cases}$ |

Exercise 23 (The Method of Variation of Parameters). Find the general solutions of the following ODEs:

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|---|---|
| (a) $y'' + y = \tan t, \quad 0 < t < \frac{\pi}{2}$ | (c) $y'' + 4y' + 4y = t^{-2}e^{-2t}, \quad t > 0$ |
| (b) $y'' + 4y = 3 \operatorname{cosec} 2t, \quad 0 < t < \frac{\pi}{2}$ | (d) $y'' - 2y' + y = \frac{e^t}{1 + t^2}$ |

Exercise 24 (Going Backwards). Find linear, homogeneous ODEs with constant coefficients, which have general solutions equal to the functions given below. The first one is done for you.

(ω) $y(t) = c_1 e^t + c_2 e^{2t} + c_3 e^{3t}.$

Clearly $r_1 = 1$, $r_2 = 2$ and $r_3 = 3$. We need to give an ODE which has characteristic equation $0 = (r - r_1)(r - r_2)(r - r_3) = (r - 1)(r - 2)(r - 3) = r^3 - 6r^2 + 11r - 6$. One possible answer is $y''' - 6y'' + 11y' - 6y = 0$.

- (a) $y(t) = c_1 + c_2 t + c_3 e^{3t} \sin t + c_4 e^{3t} \cos t + c_5 e^{3t} \sin 2t + c_6 e^{3t} \cos 2t$
- (b) $y(t) = c_1 e^t + c_2 t e^t + c_3 e^{2t} \sin t + c_4 e^{2t} \cos t + c_5 e^{2t} t \sin t + c_6 e^{2t} t \cos t$
- (c) $y(t) = c_1 e^{2t} + c_2 t e^{2t} + c_3 t^2 e^{2t} + c_4 e^{-t} \sin 3t + c_5 e^{-t} \cos 3t$

Exercise 25 (Higher Order Linear ODEs).

- (a) Given that $\sin t$ is a solution of $y^{(4)} + 2y''' + 6y'' + 2y' + 5y = 0$, find the general solution of this ODE.
- (b) Find the general solution of $y^{(4)} + y'' = 3x^2 + 4 \sin x - 2 \cos x$.

(c) Solve $\begin{cases} \frac{d^3 y}{dx^3} - 2 \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} - 8y = 0 \\ y(0) = 2 \\ y'(0) = 0 \\ y''(0) = 0. \end{cases}$