

## MATH 216

### Homework 1

**A.** Write the type of the following equations:

Example:  $y'' + 2x^3y^2 = 0$  is a second order nonlinear homogeneous ordinary differential equation.

1.  $y''' + y(y')^3 = 2x$ .
2.  $y'' + 2e^{3x}y' + 2y = (x^2 + 5)^3$ .
3.  $\frac{dy}{dx} + 3x^2y = 0$ .
4.  $x''(t) - x^2t^2 = 0$ .

**B.** Show that the following functions are the solutions of the following differential equations:

1.  $x'' - 2x' + x = 0$ ;  $x(t) = te^t$ .
2.  $y'' + 4y = 0$ ;  $y(x) = \cos 2x$

**C.** Find the equilibrium points  $\left(\frac{dy}{dx} = 0\right)$  of the following differential equations and determine if they are attractive or repulsive.

1.  $\frac{dy}{dx} = -2 - y$ .
2.  $\frac{dy}{dx} = 2 - y$ .
3.  $\frac{dy}{dx} = (y - 2)y$ .
4.  $\frac{dy}{dx} = (4 - y)y$ .
5.  $\frac{dy}{dx} = (y^2 - 2y - 8)y$ .

**D.** Find the general solutions of the following differential equations:

1.  $9yy' + 4x = 0$ .
2.  $y' + (x + 1)y^3 = 0$ .
3.  $\frac{dx}{dt} = 3t(x + 1)$ .
4.  $y' + \csc y = 0$ .
5.  $x' \sin 2t = x \cos 2t$ .
6.  $y' + y = 5$ .
7.  $y' = (y - 1) \cot x$ .

8.  $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}.$
9.  $(3x^2 + y^2)dx - 2xydy = 0.$
10.  $y' = \frac{y}{x} + \tan\left(\frac{y}{x}\right).$
11.  $e^{\frac{x}{y}}(y - x)\frac{dy}{dx} + y(1 + e^{\frac{x}{y}}) = 0.$
12.  $y' = \sqrt{(x + y + 1)}.$
13.  $y' = (x + y + 1)^2 - (x + y).$
14.  $(2x + 3y)dx + (3x + 2y)dy = 0.$
15.  $(x^3 + \frac{y}{x})dx + (y^2 + \ln x)dy = 0.$
16.  $(e^x \sin y + \tan y)dx + (e^x \cos y + x \sec^2 y)dy = 0.$
17.  $(3x^2y + 2xy + y^3)dx + (x^2 + y^2)dy = 0.$
18.  $ydx + (2x - ye^y)dy = 0.$
19.  $xy' + y = y^{-2}.$
20.  $y' = y(xy^3 - 1).$
21.  $(1 + x^2)y' = 2xy(y^3 - 1).$
22.  $xy'' = y'.$
23.  $x^2y'' + 3xy' = 2.$
24.  $y'' = 2y(y')^3.$
25.  $yy'' + (y')^2 = yy'.$

**E.** Solve the following initial value problems:

1.  $y' = x^3e^{-y}; y(2) = 0.$
2.  $y\frac{dy}{dx} = 4x(y^2 + 1)^{\frac{1}{2}}; y(0) = 1.$
3.  $y' + 3y = 12; y(0) = 6.$
4.  $y' = y \cot x; y(\frac{\pi}{2}) = 2.$
5.  $y' + 3(y - 1) = 2x; y(0) = 1.$
6.  $\frac{dy}{dx} = \frac{10}{(x+y)e^{x+y}} - 1; y(0) = 0.$
7.  $(4x^2 - 2y^2)y' = 2xy; y(3) = -5.$
8.  $(x - y)dx + (3x + y)dy = 0; y(3) = -2.$

9.  $\frac{dy}{dx} = \frac{x^3 - xy^2}{x^2y}$ ;  $y(1) = 1$ .
10.  $(xy + 1)ydx + (2y - x)dy = 0$ ;  $y(0) = 3$ .
11.  $y' - \frac{1}{x}y = y^2$ ;  $y(1) = 2$ .
12.  $y'' + 4y = 0$ ;  $y(0) = 2$ ,  $y'(0) = -2$ .
13.  $xy'' + y' = 4x$ ;  $y(1) = -1$ ,  $y'(1) = 3$ .